Enhanced Stability of Three-Users Multirate Distributed Haptic Cooperation via Coordination to Average Peer Position

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***Abstract*—Distributed networked haptic cooperation may become unstable when the number of interacting users increases because the effective coordination gain for the shared virtual object increases. The average position coordination strategy maintains the coordination gain of the shared virtual object constant regardless of the number of cooperating participants. Therefore, the average position strategy is expected to increase the stability region of networked haptic cooperation among multiple users. This paper confirms through analysis and experiments that AP coordination maintains the three-users haptic cooperation stable for larger coordination gains than traditional virtual coupling coordination. The stability analysis is performed in a multirate control framework. Multirate control is deployed to support high sampling rate of the peer force feedback loops in the presence of a low network update rate. The experiments report a one degree of freedom manipulation of a virtual cube by three cooperating users.**

***Keywords*-Networked haptic cooperation; distributed control; multirate control; coordination to averaged position.**

I. INTRODUCTION

The rapid growth of the computer networking infrastructure has inspired haptics researchers to develop methods to add the sense of touch to applications like defence [6], cooperative industrial design [10], surgical teletraining [18], telerehabilitation [22], emotion recognition [3] and multi-user on-line computer games [12]. Such applications demand that multiple networked users be enabled to cooperate in a shared virtual environment, i.e., be enabled to simultaneously interact with the virtual environment and with each other. Simultaneous haptic interaction in a shared virtual environment can be supported through client-server communications and centralized control of the interaction, or through peer-to-peer communications and distributed haptic control. The client-server connectivity maintains better consistency among the interacting users and is typically deployed to support cooperation among more than two users [4], [5], [16]. Nonetheless, it incurs double communication delay and can render only much lower contact stiffness even for small network delay [9]. Peer-to-peer communications and distributed control of the interaction are preferable for realistic force feedback in rigid shared virtual environments [9].

Existing architectures for distributed control of networked haptic cooperation [9], [19], [20] typically coordinate each peer's local copy of the shared virtual object (SVO) to all other SVO copies. The coordination of each SVO copy to the other SVO copies may lead to instability because the impedances of the many coordinating controllers compound and may exceed the Z-width [8] of the users' haptic interfaces, i.e., the maximum impedance that the interfaces can stably display to their respective users. However, little research addresses this difficulty. Recent work in [14], [15] introduces a passivity-based framework for optimizing the distributed connectivity and for computing virtual coupling [1], [7] parameters that guarantee stable cooperation among a fixed number of operators. The work in [14], [15] hinges on a passive mechanical integrator [13] whose extension to rigid body interaction is unclear.

In prior work [17], we have introduced a distributed haptic control architecture whose SVO coordination gain at all peer sites is independent of the number of participants involved in the cooperation. In that architecture, the local SVO copy at each user has been coordinated to a SVO representative whose motion has been computed by averaging the motion of all other SVO copies. In the preliminary study [17], the average position (AP) coordination strategy has been contrasted through simulations to virtual coupling coordination. In this paper, we present analytical and experimental support that AP coordination maintains three-users networked haptic cooperation stable for larger coordination gains than traditional virtual coupling coordination. The paper considers operators connected across a network with limited bandwidth and constant and relatively small communication delay [9]. It deploys multirate control in order to support a high sampling rate of the peer force feedback loops in the presence of a low network update rate. The paper uses lifting [2], [9] to derive the state transition matrix of the multirate three-users haptic cooperation system with AP coordination and to carry out an eigenvalue-based analysis of its stability. Experimental one degree of freedom (DOF) cooperative manipulations of a virtual cube by three cooperating users are presented to validate the analytical results. Although the analysis and experiments presented in the paper involve one DOF networked haptic cooperation, the AP coordination strategy is readily applicable to cooperative rigid body manipulations. This is because the AP coordination is independent of the SVO simulation algorithms and because virtual coupling control is suitable for rigid body coordination [8].

Section II overviews the traditional virtual coupling and the AP coordination strategies for three-users networked haptic cooperation. Section III presents the steps involved in developing the multirate state space model of three-users haptic cooperation across a network with low update rate and small and constant network delay, and uses the multirate state transition matrix to compare the stability regions of the two coordination techniques. Section IV validates the analytical results through experiments in which three networked users manipulate a shared virtual cube together. Section V presents the conclusions drawn from this work and the directions for future work.

II. COORDINATION OF THREE-USERS DISTRIBUTED NETWORKED HAPTIC COOPERATION

In distributed networked haptic cooperation, the users interact through manipulating a SVO together. The control of such cooperation is achieved through coordinating all peers' local copies of the SVO. Virtual coupling control [1], [7] has typically been selected to coordinate the SVO copies [9], [19], [20]. The extension of virtual coupling coordination to three-users networked haptic cooperation is schematically depicted in Fig. 1. This figure illustrates that the virtual environment at each peer includes a copy of the SVO and an avatar of the peer's haptic interface. The SVO damping is assigned to each local copy, and the SVO mass is equally divided among all copies. The avatar inherits the dynamics of the haptic interface. The dynamics of two-users distributed networked haptic cooperation with virtual coupling coordination are presented in [21], and their extension to three-users interaction inserts a virtual coupler between each pair of peers, as shown in Fig. 1. One problem with this control architecture is that the effective coordination gain for each SVO copy becomes larger as the number of users increases and may exceed for Z-width of the haptic interface and thus, destabilize the cooperation. To address this shortcoming, we have proposed the AP coordination strategy [17].

In the AP scheme, the SVO copy of each peer is coordinated to the average position of all other SVO copies. The AP coordination strategy is illustrated schematically in Fig. 2 for the local SVO copy of Peer 1 and for three-users networked haptic cooperation. In this figure, notation is used as follows: $m_{\text{HD}1}$ and $b_{\text{HD}1}$ are the mass and damping of the haptic device of Peer 1; $m_{\text{O}1}$ and $b_{\text{O}1}$ are the mass and damping of the SVO copy of Peer 1's; $K_{\text{C}1}$ and $B_{\text{C}1}$ are the stiffness and damping of the contact between Peer 1 and its local SVO copy, respectively; K_{T} and B_{T} are the stiffness and damping gains of the virtual coupler which coordinates the distributed SVO copies. As in traditional virtual coupling coordination, the SVO mass m_{O} is equally distributed among the SVO copies and the SVO damping b_{O} is assigned to each SVO copy, i.e., $m_{\text{O}1} = \frac{m_{\text{O}1}}{3}$ and $b_{\text{O}1} = b_{\text{O}}$.

According to Fig. 2, the dynamics of three-users distributed networked haptic cooperation with AP coordination are:

- for the haptic device of Peer i:

$$m_{\text{HD}i}\ddot{x}_{\text{HD}i} + b_{\text{HD}i}\dot{x}_{\text{HD}i} = F_{\text{h}i} - F_{\text{C}i}, \tag{1}$$

  where $F_{\text{h}i}$ is the force applied by Peer i to its haptic interface and $F_{\text{C}i}$ is the contact force between Peer i and its SVO copy;
- for Peer i's copy of the SVO:

$$m_{\text{O}i}\ddot{x}_{\text{O}i} + b_{\text{O}i}\dot{x}_{\text{O}i} = F_{\text{C}i} - F_{\text{T}i}, \tag{2}$$

  where $F_{\text{T}i}$ is the coordination force applied by the AP coordination controller to Peer i's copy of the SVO.

In Equations (1) and (2), the contact and coordination forces are computed using:

$$F_{\text{C}i} = K_{\text{C}i}(x_{\text{HD}i} - x_{\text{O}i}) + B_{\text{C}i}(\dot{x}_{\text{HD}i} - \dot{x}_{\text{O}i}) \tag{3}$$

and:

$$F_{\text{T}i} = K_{\text{T}}(x_{\text{O}i} - x_{\text{O}i_d}) + B_{\text{T}}(\dot{x}_{\text{O}i} - \dot{x}_{\text{O}i_d}). \tag{4}$$

In Equation (4), $x_{\text{O}i_d}$ and $\dot{x}_{\text{O}i_d}$ are the desired position and velocity of the SVO copy of Peer i and, according to the AP strategy, are computed through averaging the motion commands coming from the other two peers:

$$x_{\text{O}i_d} = \frac{\sum_{j=1,j\neq i}^{3} x_{\text{O}j_n}}{2} \quad \text{and} \quad \dot{x}_{\text{O}i_d} = \frac{\sum_{j=1,j\neq i}^{3} \dot{x}_{\text{O}j_n}}{2} \tag{5}$$

Lastly, the motion commands are the delayed position and velocity of the sending peer:

$$x_{\text{O}i_n}(t) = x_{\text{O}i}(t - T_d) \quad \text{and} \quad \dot{x}_{\text{O}i_n}(t) = \dot{x}_{\text{O}i}(t - T_d), \tag{6}$$

and T_d is the network delay. In this paper, the network delay is assumed constant and equal in all communication channels, and an integer multiple of the network packet update interval T_n.

III. STABILITY ANALYSIS

Because of network bandwidth limitations, the packet update rate is generally lower than the typical 1 KHz haptic rate required for realistic force feedback, especially in rigid virtual environments. Therefore, three-users networked haptic cooperation is a system with two sampling intervals: the (fast) sampling interval of the local force feedback loops at the peer users, T_c, typically equal to 0.001 s; and the (slow) sampling interval of the network updates, T_n, typically equal to 0.008 s [9]. Its stability can be investigated using eigenvalue analysis of its multirate state transition matrix.

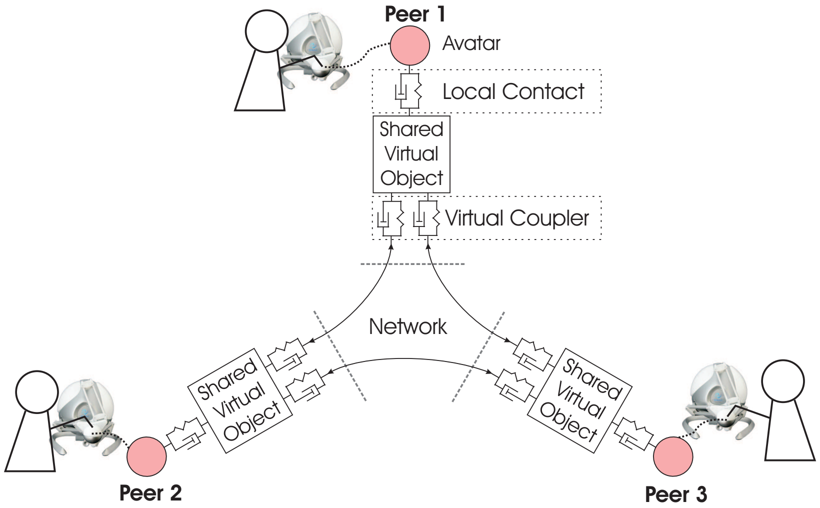

Figure 1. Traditional virtual coupling coordination of distributed haptic cooperation among three users.

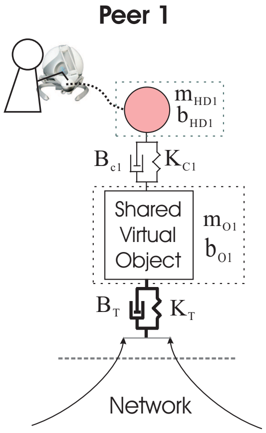

Figure 2. AP coordination of haptic cooperation among three users as applied at Peer 1.

In turn, this matrix can be computed through deriving the state space model of the multirate haptic cooperation system using the lifting approach in [2] and [9]. The derivations are briefly presented in the following sections. They assume a communication delay in each direction equal to one network update interval, i.e., $T_d = T_n = 0.008$ s.

A. Open-loop Continuous-time State-space Representation

The continuous-time state-space representation of the open-loop three-users networked haptic cooperation system is obtained from the dynamics of the users, of the haptic interfaces and of the SVO copies by grouping the system inputs and outputs into fast and slow sub-vectors, hereafter denoted with c and n indices respectively. Specifically, the system inputs comprise the contact forces, updated at the fast haptic rate (Equation (3)), and the SVO coordination forces, including both fast and slow updated components (Equation (4)):

$$\mathbf{u}^T = (\mathbf{u}_c^T \quad \mathbf{u}_n^T)^T \tag{7}$$

where:

$$\mathbf{u}_c^T = (F_{\mathrm{C1}} \quad F_{\mathrm{T1}_c} \quad F_{\mathrm{C2}} \quad F_{\mathrm{T2}_c} \quad F_{\mathrm{C3}} \quad F_{\mathrm{T3}_c})^T \tag{8}$$

$$\mathbf{u}_n^T = (F_{\mathrm{T1}_n} \quad F_{\mathrm{T2}_n} \quad F_{\mathrm{T3}_n})^T \tag{9}$$

$$F_{\mathrm{T}i_c} \quad = \quad K_{\mathrm{T}} x_{\mathrm{O}i} + K_{\mathrm{T}} x_{\mathrm{O}i} \tag{10}$$

and:

$$F_{\mathrm{T}i_n} \quad = \quad -K_{\mathrm{T}} x_{\mathrm{O}i_d} - K_{\mathrm{T}} x_{\mathrm{O}i_d}. \tag{11}$$

The state vector comprises the states of all haptic interfaces and SVO copies:

$$\mathbf{x}^T = (\mathbf{x}_{\text{peer1}} \quad \mathbf{x}_{\text{peer2}} \quad \mathbf{x}_{\text{peer3}})^T \tag{12}$$

where:

$$\mathbf{x}_{\text{peeri}}^T = (x_{\mathrm{HD}i} \quad \dot{x}_{\mathrm{HD}i} \quad x_{\mathrm{O}i} \quad \dot{x}_{\mathrm{O}i})^T; \quad i = 1, 2, 3. \tag{13}$$

The output vector is:

$$\mathbf{y}^T = (\mathbf{y}_{\mathrm{c}}{}^T \quad \mathbf{y}_{\mathrm{n}}{}^T)^T, \tag{14}$$

where:

$$\mathbf{y}_{\mathrm{c}}{}^T = \mathbf{x}^T, \tag{15}$$

$$\mathbf{y}_n^T = (\mathbf{y}_{\text{peer1}_n} \quad \mathbf{y}_{\text{peer2}_n} \quad \mathbf{y}_{\text{peer3}_n})^T \tag{16}$$

and

$$\mathbf{y}_{\text{peeri}_n}^T = (x_{\mathrm{O}i_d} \quad \dot{x}_{\mathrm{O}i_d})^T; \quad i = 1, 2, 3. \tag{17}$$

Hence, the continuous-time state-space model of open-loop three-user networked haptic cooperation with AP coordination is:

$$\begin{aligned} \dot{\mathbf{x}}_{12\times1} &= \mathbf{A}_{12\times12}\mathbf{x}_{12\times1} + \mathbf{B}_{12\times9}\mathbf{u}_{9\times1} \\ \mathbf{y}_{18\times1} &= \mathbf{C}_{18\times12}\mathbf{x}_{12\times1} \end{aligned} \tag{18}$$

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B. Discrete-Time State-Space Representation

Following the approach [2] and assuming that the network sampling interval is an integer multiple of the sampling interval of the peers' force control loops and that the force feedback and network update sampling times are synchronized, the discrete-time state-space representation of the open-loop system can be written in the form:

$$\begin{aligned} \mathbf{x}_{\mathrm{D}_{96\times1}}[k+1] &= \mathbf{A}_{\mathrm{D}_{96\times96}}\ \mathbf{x}_{\mathrm{D}_{96\times1}}[k] + \mathbf{B}_{\mathrm{D}_{96\times51}}\ \mathbf{u}_{\mathrm{D}_{51\times1}}[k] \\ \mathbf{y}_{\mathrm{D}_{102\times1}}[k] &= \hat{\mathbf{C}}_{\mathrm{D}_{102\times96}}\ \mathbf{x}_{\mathrm{D}_{96\times1}}[k] + \hat{\mathbf{D}}_{\mathrm{D}_{102\times51}}\ \mathbf{u}_{\mathrm{D}_{51\times1}}[k] \end{aligned} \tag{19}$$

where k is the k-th network update interval and more details about the derivations of the system matrices A_{D}, B_{D}, $\hat{C}_{\mathrm{D}}$ and $\hat{D}_{\mathrm{D}}$ can be found in [9]. Furthermore, by augmenting the state vector with the delayed inputs [9], computational and communication delays are incorporated into the discrete-time open-loop model in Equation (19).

C. Stability Analysis

For three-users networked haptic cooperation, the feedback matrix F_{D} comprises the contact and SVO coordination forces and is computed using the approach [2]. Thereafter, the stability of the multirate closed-loop system can be derived through eigenvalue analysis of the closed-loop state transition matrix A_{D}^{cl}, calculated via:

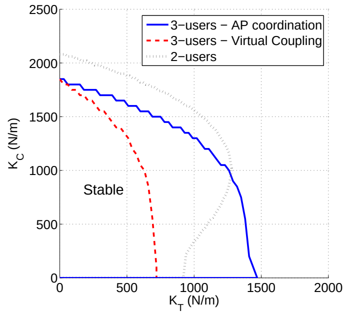
$$A_{\mathrm{D}}^{cl} = A_{\mathrm{D}_{aug}} + B_{\mathrm{D}_{aug}}\ F_{\mathrm{D}}\ (I - D_{\mathrm{D}_{aug}} F_{\mathrm{D}})^{-1} C_{\mathrm{D}_{aug}} \tag{20}$$

where $A_{\mathrm{D}_{aug}}$, $B_{\mathrm{D}_{aug}}$, $C_{\mathrm{D}_{aug}}$ and $D_{\mathrm{D}_{aug}}$ are the state transition matrices obtained after augmentation with computational and communication delays. Specifically, the three-users networked haptic cooperation system is stable if and only if all eigenvalues of A_{D}^{cl} are inside the unit circle:

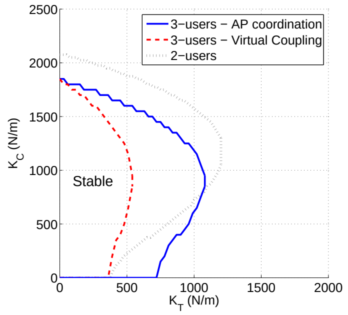
$$\left\| eig(A_{\mathrm{D}}^{cl}) \right\| < 1 \tag{21}$$

The stability regions for cooperation with AP and with virtual coupling coordination are presented: (i) in Fig. 3 for the case of Proportional coordination, when damping is injected as SVO damping ($B_{\mathrm{T}} = B_{\mathrm{C}i} = 0$ Ns/m); and (ii) in Fig. 4 for the case of Proportional-Derivative coordination, when damping is injected as coordination and local contact damping ($B_{\mathrm{T}} = B_{\mathrm{C}i} = 2$ Ns/m). The numerical computations are carried out considering that the haptic devices has mass $m_{\mathrm{HD}_i} = 0.1$ kg and physical damping $b_{\mathrm{HD}_i} = 5.0$ Ns/m and for a SVO with mass $m_{\mathrm{O}} = 0.6$ kg and damping $b_{\mathrm{O}} = 5.0$ Ns/m. The sampling interval of the force feedback loops at all peers is $T_{\mathrm{c}} = 0.001$ s and the network update interval is $T_{\mathrm{n}} = 0.008$ s. The computational delay is equal to one sampling interval of the force feedback loop, i.e., $T_{VE} = T_c = 0.001$ s. The communication delay is equal to one network update interval, i.e., $T_d = T_n = 0.008$ s.

Fig. 3 illustrates that the Proportional AP coordination maintains the cooperation stable for K_T gain twice as large as the K_T gain of Proportional virtual coupling coordination. The increase in the K_T gain afforded by the AP strategy

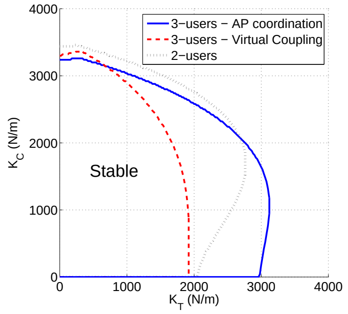


(a) Computational delay.

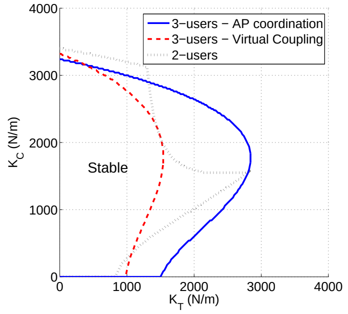


(b) Computational and communication delay.

Figure 3. Stability region for three-users haptic cooperation with Proportional SVO coordination.



(a) Computational delay.



(b) Computational and communication delay.

Figure 4. Stability region for three-users haptic cooperation with Proportional-Derivative SVO coordination.

is smaller compared to the virtual coupling strategy when Proportional-Derivative coordination is deployed. This result confirms the negative impact of delayed damping on stability. Both figures also attest to the negative impact on system stability of the communication (network) delay.

IV. EXPERIMENTS

This section contrasts the AP coordination strategy to traditional virtual coupling coordination through controlled three-users cooperative manipulations. The controlled experimental cooperations are performed on a platform that enables point interaction between the networked users and the shared virtual environment. The platform comprises three FALCON NOVINT haptic devices, each connected to a respective computer and providing 3DOF force feedback to users. All computers run Windows XP on an Intel Core 2 Duo CPU at 2.67 Ghz with 2 GB RAM. The computers are in the same laboratory and communicate over the network via the UDP protocol. Given the proximity of the three computers, the network delay is negligible and a Wide Area Emulator (WANem) running on a separate computer implements the network environment. The shared virtual environment is generated as a C++ console application. It comprises a shared virtual cube in a rigid enclosure that constrains the cube to move horizontally along a single direction. To ensure the "same" users during successive experiments, the user-applied forces are replaced by forces commanded to the actuators through software. Since the haptic devices are impedance-type interfaces, the forces commanded through software eliminate the adaptive damping associated with the user manipulations of the haptic devices and represent a worst-case scenario for stability [11].

In the experiments, Peer 1 is located on the right side of the cube and pushes the cube to the left with constant force $F_{\mathrm{h1}} = 4$ N. Peer 2 and Peer 3 are on the left side of the cube and push the cube to the right with constant and equal forces $F_{\mathrm{h2}} = F_{\mathrm{h3}} = 2$ N. The experiment starts with the users not in contact with the virtual cube. Fig. 5 depicts the snapshot of initial experimental conditions displayed at Peer 1. The virtual cube has mass $m_{\mathrm{O}} = 0.3$ kg and hence, the local cube copies at the three peers have mass at each peer $m_{\mathrm{O}i} = 0.1$ kg. The sampling interval of the force control loop is $T_{\mathrm{c}} = 0.001$ s, and the network update interval is $T_{\mathrm{n}} = 0.008$ s. The network delay is considered negligible. The stiffness and damping of the local contacts are $K_{\mathrm{C}} = 3900$ N/m and $B_{\mathrm{C}} = 0.05$ Ns/m, respectively. The SVO coordination gains are $K_{\mathrm{T}} = 2000$ N/m and $B_{\mathrm{T}} = 1.0$ Ns/m.

Experimental three-users networked haptic cooperation results are shown in Fig. 6a for AP coordination of the SVO, and in Fig. 6b for traditional virtual coupling coordination. These results illustrate that the cooperation is stable for AP coordination, and unstable for virtual coupling coordination. Thus, they validate the ability of the AP coordination strategy to maintain the networked cooperative manipulation stable for stiffer coordination gains.

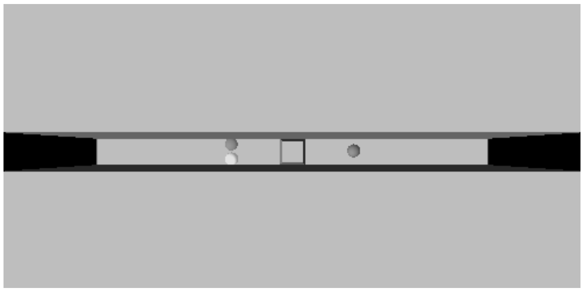
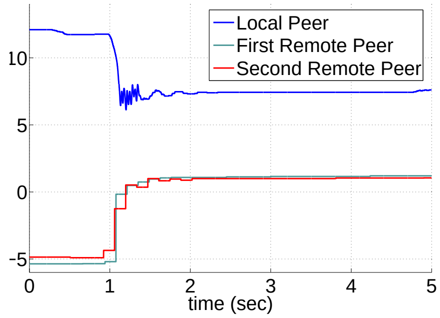
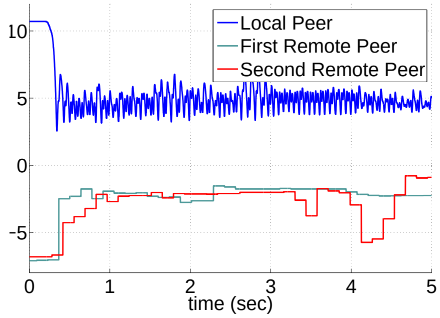


Figure 5. Snapshot of initial experimental conditions displayed to Peer 1.



(a) AP coordination.



(b) Virtual coupling coordination.

Figure 6. Controlled experimental three-users cooperation.

V. CONCLUSION

This paper has investigated the stability of the average position (AP) coordination strategy for three-users networked haptic cooperation. The AP strategy maintains the SVO coordination gain constant regardless of the number of cooperating participants. For three-users networked haptic cooperation, the paper has validated through analysis and experiments that AP coordination maintains the interaction stable for larger coordination gains. The stability analysis has been based on the eigenvalues of the state transition matrix of the three-users multirate haptic feedback system. The experiments have illustrated a one DOF cooperative manipulation of a virtual cube. Regardless of the one DOF analysis and experiments, the AP coordination strategy is readily applicable to rigid body cooperative haptic manipulations.

Upcoming work investigates the transparency of AP coordination and techniques to improve it in the presence of significant network delay, as well as its stability for cooperation across networks with variable delay and packet loss.

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