Search Opportunities of Swarming Particles Methods in Irregular Multi-Extreme Enviroments

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***Abstract*— The paper studies the actual task of developing and setting up the algorithms for multi-extreme objects search optimization. To solve such problems, the heuristic methods are effectively used, in particular applying the swarming particles method. The mathematical base for the modified swarming particles method, which is oriented to solve the multi-extreme search problems, is developed and described in detail. The modified algorithm is applied to the irregular multi-extreme Lambda test functions, considered to be a very difficult test case. The developed "Lambda Function" software is created to control the generation, editing and investigation of multidimensional multi-extreme test functions. The applied Lambda function is a multiplicative function developed by R. Neydorf, with fundamental extremes, multidimensionality and isolation in the factor space, which completely exclude the influence of the computed results. The application of this fundamentally new test function shows that such modified method of swarming particles is suitable for solving rather complex multi-extreme search problems. In addition, the developed "Lambda Function" software shows wide range of application possibilities when developing and researching the test functions for other related applications.**

Keywords-search optimization; multi-extreme; method of swarming particles; test functions; irregularity; software.

I. INTRODUCTION

Many modern technical and scientific problems are complex, as they need to solve optimization problems [1]. Today, most of the known search engine optimization methods are designed and used to find one optimum, which is often the global one. However, the goal is not always to find only the global optimal solution. In many cases, there are many suboptimal and close to the global optimal extreme solutions, which are quite acceptable. To study such problems and find solutions applying the Multi-Extreme (ME) optimization, subject-oriented methods, as well as tools for testing and evaluation, are required.

When making decisions regarding ME, it is necessary to take into account that the deterministic search methods are usually very sensitive to their essential nonlinear continuum dependencies (in particular to discontinuity of their derivatives and variables). When searching the discrete quotient spaces, ME problems are often NP-complete [1]. In this regard, to solve complex (multidimensional and ME) optimization problems, more and more often various effective heuristic methods are applied.

The most important advantages of the heuristic algorithms, over other types of optimization algorithms, are in analogies that generated them. They consider the adaptation processes found in living and inanimate nature. Methodologically, they are based on processes found in the knowledge areas as decision-making theory, fuzzy logic, neural networks, evolutionary-genetic mechanisms, fleece behavior, etc. These processes partially repeat and in many ways supplement each other [2][3]. The disadvantages of these methods are that they are not adaptable to support analytical research and evaluation.

Today, the heuristic methods are used to solve problems of high computational complexity. One of the most promising representatives of such methods is the Method of Swarming Particles (MSP) [2]. However, the peculiarity of research and practical development of ME optimization algorithms are coming with their own complexity, cumbersomeness and significant development times, when a large number of extremes in the factor space of the related problem has to be solved.

The impossibility in performing theoretical studies of qualitative properties and numerical settings of heuristic algorithms implies that their performance and efficiency are most often checked with so-called Test functions (TFs) [4]-[6]. When algorithms for investigating ME objects are in development, the selection of effective testing tools is the problem. It is well known that TFs have either one global extreme, or they have a regular character with respect to the extremes location, and the magnitude of their respective amplitudes [7]-[10]. Thus, to have a more effective testing, the irregular multidimensional ME functions need to be created.

The most famous and widely used ME optimization TFs are: Rosenbrock [7], Rastrigin [8], Himmelblau [9], De Jong [7], Griewank [7], Schwefel [7], etc. In addition, many papers describe other variants of TFs that generates ME functions [10]. They ensure a good verification of the ME optimization algorithms, the quality of the structural and parametric setup has to be controlled to study the factor space. In this context, a structural evaluation means the determination of the number of extremes and their spatial arrangement (coordinates). The parametric estimation means the determination of the extremes magnitudes (taking into account their signs).

The disadvantage of most TFs is their regular and analytical character. The absence of no differentiable or poorly differentiable areas greatly facilitates the work on the algorithm, by evaluating the surface shape under investigation. The real search is made difficult due to the fact that their coordinates are usually close to each other. The presence of noticeable surface curvature at the respective extreme distance facilitates its search. Therefore, the TF extreme should be as close as possible to the impulse form, as in such case its neighborhood is minimally curved. A sufficiently developed adaptive algorithm can easily identify the period of the extremes alternation.

In Section 2, the problem described in this paper is formulated. Section 3 contains a description of the Multiplicatively Allocating Function (MAF) and its characteristics. Section 4 illustrates the features of the developed special software (SW) for MAF building. Section 5 describes the mathematical model (MM) of modified MSP for ME search. Section 6 shows the result of experiments on the generated Lambda TFs. Section 7 contains the conclusion of the conducted research and future work.

II. PROBLEM FORMULATION

Following the above described issues, the goal of this paper is to develop and study the MSP modification, aiming to solve different ME search problems. For testing and setting a highly efficient solution to treat these problems, it is necessary to test the MSP on TFs, which are coming with disadvantages, as already described in the introduction. Thus, it is necessary to implement algorithmically and programmatically the TF generator, which is theoretically presented in [4], and to conduct and process statistically the representative experiments, when setting up the modified MSP.

III. SCALABLE MAF FOR EXTREME FORMING

R. Neydorf et al. developed the general principles for constructing the universal irregular ME TFs, based on the application of MAF suitable constructed to approximate problems [4]-[6].

MM of such MAF for N-dimensional ME TF, with a number of K extremes, has the form:

$$\lambda(\vec{x}) = \sum_{k=1}^{K} [a_k \prod_{i=1}^{N} \lambda_{x_i k}(x_i, x_{ik}, \Delta x_{ik}, e_{ik})] \qquad (1)$$

where: *x vector* – is an N-dimensional values vector, *x vector* ϵ $\{x_1, x_2, x_3, \ldots, x_n\}$; α_k – is a coefficient specifying the extreme value.

Figure 1 demonstrates the modeling of 3 λ-functions maxima in 2-dimensional space having different pulse fronts edge steepness (2). Variant A is impulse extreme (e_{ik}=0.1), B is intermediate variant (e_{ik}=0.5), C is shelving extreme (e_{ik}=1).

$$\begin{aligned} &\lambda_{x_i k}(x_i, x_{ik}, \Delta x_{ik}, e_{ik}) = \\ &= [x_i - x_{idk} + \sqrt{(x_i - x_{idk})^2 + e_{ik}^2}\,]* \\ &*[x_{iuk} - x_i + \sqrt{(x_{iuk} - x_i)^2 + e_{ik}^2}\,]/ \\ &/(4\sqrt{[(x_i - x_{idk})^2 + e_{ik}^2]\cdot[(x_{iuk} - x_i)^2 + e_{ik}^2]}) \end{aligned} \qquad (2)$$

where: $\{x_{ik}, \Delta x_{ik}, e_{ik}\}$ – is the set of TF parameters; $x_{idk} = x_{ik} - \Delta x_{ik}$, $x_{iuk} = x_{ik} + \Delta x_{ik}$ – are the initial and final coordinates of extreme pulse for *x vector*; e_{ik} – is the edge steepness parameter.

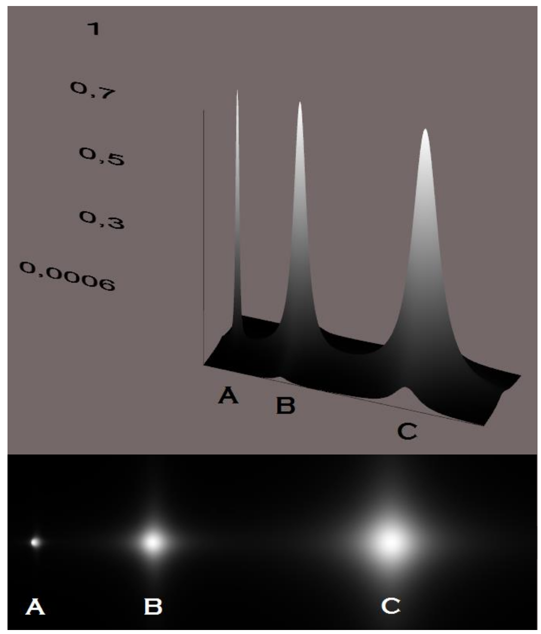

Figure 1. Different pulse fronts steepness of λ-function extrema

The graphs are constructed from (1) and (2).

IV. SOFTWARE IMPLEMENTATION OF Λ-GENERATOR

The SW for TFs creation is developed with C# programming language. It is a MAF research tool. The SW is a desktop application with third-party library for visualization, which is a part of the executable file to simplify its execution.

The "Lambda Function" features are:

- Russian and English interface languages;
- Create / load / save / delete the test. The test is saved in the XML format. This feature allows user to use the resulting TF to effectively check the optimization algorithm within the third-party program without the use of additional technologies;
- Multidimensionality;
- Adding (editing) extremes in 2 modes: 1st - manual input and 2nd - pseudo-random generation of parameter values in the specified ranges;
- Display and save the resulting TF equation in analytical form;

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- Validation of all input data;
- Visualization of the TF graph with the cut-off points to display multidimensional TF and 2-display modes for 2D and 3D graphs.

Figures 2 and 3 illustrate the SW capabilities (2D and 3D models). Figure 2 shows a user function with 50 maxima (equal in magnitude of amplitudes, increments and steepness of pulse fronts). Figure 3 demonstrates a generated function with 31 maxima (different amplitudes, increments and steepness of the pulse fronts).

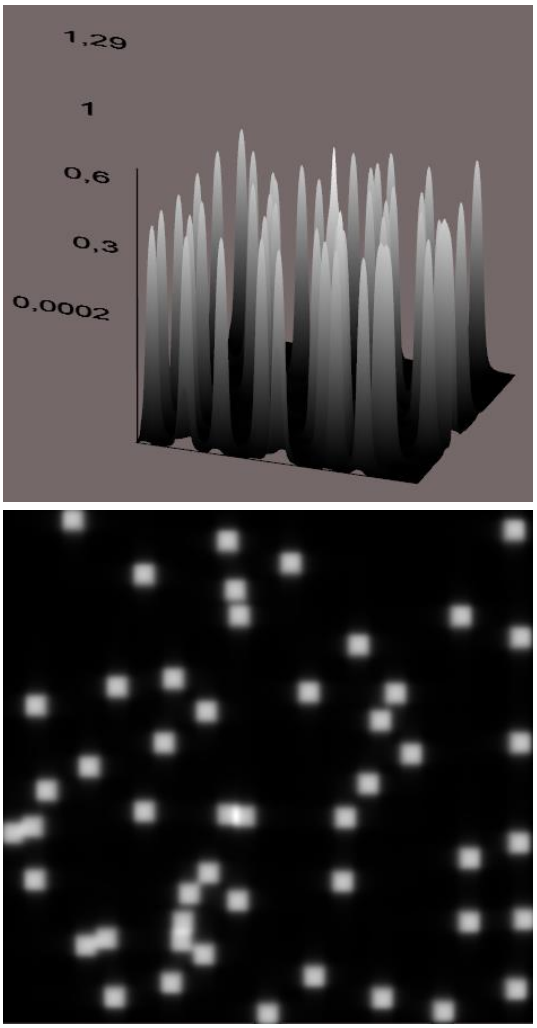

Figure 2. Demonstration of different steepness of pulse fronts of λ-function extrema

V. MSP MM, MODIFIED FOR ME SEARCH

The essence for using MSP in search optimization problems is well known [1][2]. The classical MSP algorithm imitates the real group behavioral insects, birds, fish, many protozoa, etc. However, the ME optimization requires some specific algorithm properties. Therefore, the canonical MSP version has been significantly revised and modified by the authors [2][3]. The hybrid algorithm includes basic algorithm laws of mechanics, dynamics, gravitation and stochastic "blurring" of the method parameters, which are used in the swarm prototype. In particular, its modification has been developed for solving ME problems in multidimensional spaces.

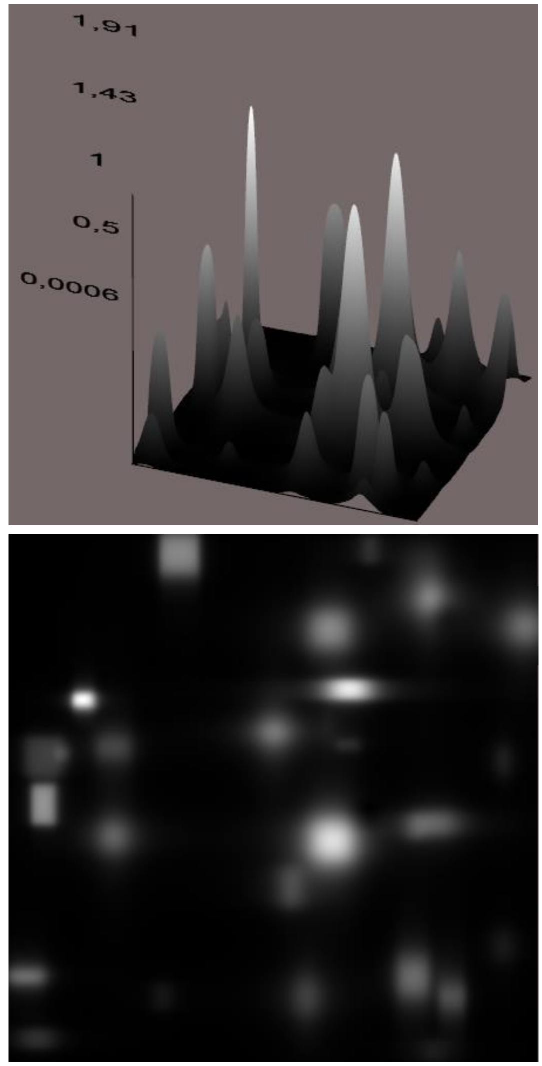

Figure 3. Example of generated ME TF with impulses of different steepness

MSP MM is constructed from the basic kinematic motion equations of the material point for the particle position and velocity:

$$\vec{X}_{ti} = \vec{X}_{(t-\Delta t)i} + \vec{V}_{(t-\Delta t)i} \cdot \Delta t \qquad (3)$$

$$\vec{V}_{ti} = \vec{V}_{(t-\Delta t)i} + \vec{A}_{(t-\Delta t)i} \cdot \Delta t \qquad (4)$$

where: $\vec{X}_{(t-\Delta t)i}$ - is previous particle position; $\vec{V}_{(t-\Delta t)i}$ - is previous particle velocity; Δt - is time interval (iteration); $\vec{A}_{(t-\Delta t)i}$ - is particle acceleration at previous iteration, where:

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$$\vec{A}_i = \sum \frac{\hat{D}_i^Q g^Q m_i^Q}{(r_i^Q)^2 + (\varepsilon^Q)^2} - \\ - \mu_{vis}\vec{V}_{(t-\Delta t)i} - \mu_{tur}\left|\vec{V}_{(t-\Delta t)i}\right|\vec{V}_{(t-\Delta t)i} \quad (5)$$

where: $\sum \frac{\hat{D}_i^Q g^Q m_i^Q}{(r_i^Q)^2 + (\varepsilon^Q)^2}$ - is the acceleration caused by the bio-analog of particles gravitational attraction to the extreme point, $Q \in \{G, L_i\}$, G - is the particle attraction to the global swarm extreme; L_i - the best found position by particle for all time; $\hat{D}_i^Q$ - is the unit director vector towards the point of attraction; g^Q - is the gravitational constant prototype; m_i^Q - is the gravity center mass; r_i^Q - is the distance between particle position and diffuse position of the attraction target point; ε^Q - is a natural acceleration limiter that excludes the passage of any material point at $\Delta X<\varepsilon$ distance; $-\mu_{vis}\vec{V}_{(t-\Delta t)i}$ - is the viscosity friction; $-\mu_{tur}\left|\vec{V}_{(t-\Delta t)i}\right|\vec{V}_{(t-\Delta t)i}$ - is the turbulent friction; μ_{vis}, μ_{tur}- are the coefficients of viscosity and turbulent friction, respectively.

To take into account the MM stochastic behavioral components, the equation of parameters random fluctuation (distortion) is included:

$$\lambda^{\xi}(\varphi) = \lambda \cdot (1 + 2\varphi \cdot (rnd(1) - 0.5)) \quad (6)$$

where: λ - is the nominal value of fluctuating parameter; φ - is the coefficient of parameter distortion, relative to the nominal value; $rnd(1)$ - is the random float number in [0;1] range. This law applies to the following collective parameters of a swarm and particles:

- Prototypes of gravitational constants - g^Q;
- Coefficients of viscosity and turbulent friction - μ_{vis} and μ_{tur};
- Dissipation coefficient - μ_{dis}.

VI. MSP MODIFICATION FOR Λ TFS APPLICATION

To study and adjust the ME modification of MSP, 3 demonstration Lambda TFs are generated using the "Lambda Function" SW; see Figures 4(a), 4(b) and 4(c). In addition, to test the MSP modification, an appropriate "MMSP" (Modified MSP) SW was developed. For its development, the C # programming language was used.

For all experiments, the same particle number (P) and iteration (I) settings were used, to obtain a more general picture of MSP operation on various generated functions. At the same time, the dynamics parameters were settings dynamically, with respect to the region under consideration.

Figures 4(d), 4(e) and 4(f) show Lambda functions and localized MSP regions (red squares) and extremes (blue dots), which are found and evaluated. Each function has a specific feature that allows you to identify the positive aspects and disadvantages of the optimization algorithm being developed.

Figures 4(a) and 4(d) show a TF with 5 minima and 3 maxima. The functions are steep near extremes and moderately canopies at the bases, and being located at a considerable distance from each other. However, the amplitude of the extremes is not high (-1), and it is not easy to identify the whole set of extremes from the first pass.

Figures 4(b) and 4(e) illustrate the generated function with 20 maxima. This function is complicated by the fact that the extremes have steep pulses. Outside the extremes region, agents have virtually no information about remote impulses. Only the mechanism of interaction between particles in the swarm, as part of the MSP MM modification, has enabled to investigate this type of function.

Figures 4(c) and 4(f) show a Lambda function, which has 8 minima and 8 maxima to identify and estimate the minima. The shape of this function is similar to the bends of "peaks" and "gorges," which can be smooth, but may have sharp cliffs. By localizing one of the extremes, the multi-agent system is not exploring the rest of the search space. However, this does not happen in the modified MSP.

Tables I-III show the experiments results of a successful search for the modified-MSP. These results were obtained from the basic MM motion of the swarm (preceding the MM clustering mechanism [11], which divided the search space into subspaces and found in each an extreme, and which was replaced by the dynamic clustering caused by the behavioral model of the swarm itself). The agents localize extreme areas, under the influence of attraction forces (not only global, but also local). The increase in the influence of local attraction is caused, in particular, by the introduction of a turbulent deceleration in the MM. The removal of the non-dynamic clustering mechanism from MM also enabled to exclude the "cluster" attraction of the swarm particles to the closest previously created clusters, which allowed the particles to behave in a more similar way to the real prototype.

As a result, the minimum error of the obtained approximation in experiment 1 (see Table I), relative to the standard, was ~0.01%, average ~0.03%. MSP successfully isolated the extreme regions and obtained the described results due to the smooth motion of the particles to the extreme values found at the moment (based on (3)-(5)). This allowed the particles not to jump through the extremes.

The minimum error of the obtained approximation in experiment 2 (see Table II), relative to the standard, turned out to be ~0.001%, mean ~0.01%. The parameter of the slope of the pulse fronts of a given Lambda function for all extremes is 0.01, which implies the complexity in finding them. However, since the number of extremes is 20, the particles interact with each other and receive an additional opportunity to study the neighboring extremes. This effect is due to the fact that, when the particle is found close to a extreme, then, in the next step, it will get a large acceleration (see (5)), which will allow the particle to escape from this extreme attraction zone and visit the extreme region of the neighboring one.

The minimum error of the approximation obtained in experiment 3 (see Table III), relative to the standard, was ~0.09%, average ~3%.

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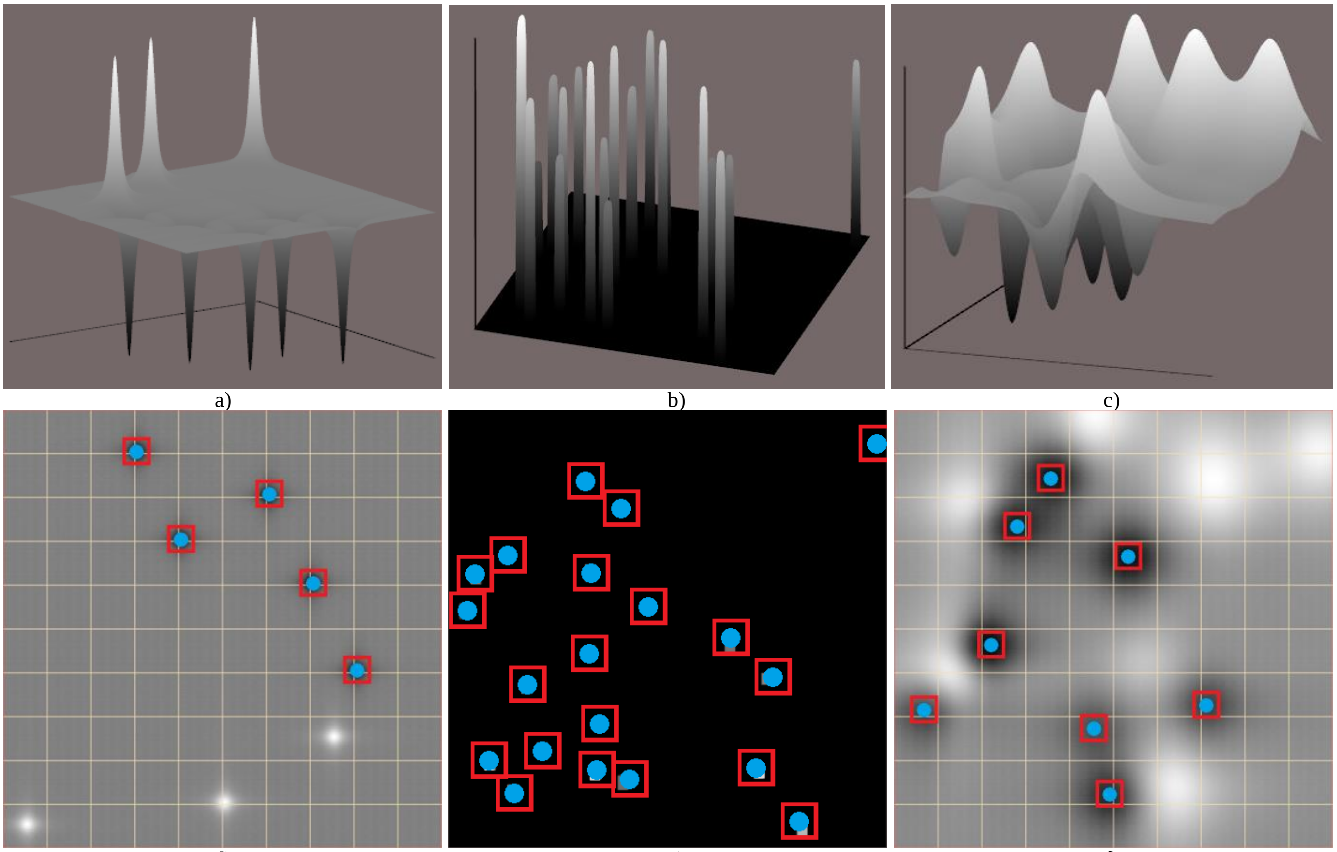

Figure 4. Generated TF graphs and MSP result on different scenes: *a* – I on 3D, *b* – II on 3D, *c* – III on 3D, *d* – I on 2D, *e* – II on 2D, *f* – III on 2D

TABLE I. I Λ TF STANDARD AND MSP RESULT

Standard			**MSP**		
x	*y*	*f(x, y)*	*x*	*y*	*f(x, y)*
70.5	60.5	-1	70.4811	60.4904	-0.9998
30.5	90.5	-1	30.5206	90.5171	-0.9998
60.5	80.5	-1	60.4901	80.5104	-0.9999
80.5	40.5	-1	80.5486	40.5173	-0.9993
40.5	70.5	-1	40.5158	70.4641	-0.9996

The complexity of this experiment consists in mixing maxima and minima. This means that the particles will be located more often in positions that may be worse than their previous ones. However, the method is also effective in such a case.

TABLE II. II Λ TF STANDARD AND MSP RESULT

Standard			**MSP**		
x	*y*	*f(x, y)*	*x*	*y*	*f(x, y)*
6.4254	4.6182	0.4543	6.4253	4.6182	0.4543
3.2322	4.4678	0.7859	3.2322	4.4698	0.7859
9.7602	9.2187	0.6206	9.7505	9.2172	0.6206
1.3463	6.6313	0.5903	1.3412	6.6291	0.5903
3.9888	1.4936	0.4183	3.99	1.4981	0.4183
…					
4.5307	5.4641	0.7796	4.5327	5.4604	0.7796
2.1397	2.2475	0.5101	2.1423	2.2536	0.5101
3.3574	1.6594	0.8659	3.3518	1.6565	0.8659
0.9593	1.9634	0.9677	0.9597	1.9561	0.9677
1.4741	1.2706	0.7321	1.473	1.2718	0.7321
7.2637	3.8593	0.5123	7.2631	3.8621	0.5123

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TABLE III. III Λ TF STANDARD AND MSP RESULT

Standard			MSP		
x	*y*	*f(x, y)*	*x*	*y*	*f(x, y)*
3.6721	8.491	-0.5598	3.6181	8.451	-0.5651
5.3615	6.6256	-0.5653	5.343	6.6296	-0.5658
0.7982	3.1601	-0.2936	0.7953	3.1288	-0.2901
4.9938	1.1940	-0.4426	4.8912	1.1833	-0.455
4.5671	2.7411	-0.2833	4.5894	2.6884	-0.2849
2.025	4.5505	-0.5831	2.1631	4.6302	-0.6375
2.6129	7.4111	-0.4821	2.769	7.3498	-0.5187
7.0786	3.2646	-0.3418	7.1416	3.2602	-0.3451

The attraction of particles to the global extreme allows improving the result of the whole swarm, even in a situation where the best position of the particle itself is not a local extreme (which forces the particle to swarm in the pseudo-local area). With additional sub-optimization of the parameters of the swarm and particles, the error can be significantly reduced [2][3].

VII. CONCLUSION AND FUTURE WORK

The developed Lambda Function SW has proven to be an effective tool for the generation of the irregular multi-dimensional ME TFs. The easy-to use and convenient interface to access the multi-functional SW allows the fast generation and qualitative analysis of TFs. The demonstrated SW functions do not have an obvious regular and analytical character, like the set of the today existing ME optimization TFs.

The experiments carried out on TFs showed that the developed MSP modification allows localizing the extreme areas of the nonstandard irregular ME Lambda function, having the approximation error from ~0.001% to ~3%.

The experimentally obtained results allow the validation of the developed MSP modification, and prove that the developed Lambda Function SW is an effective tool in searching the extremes of heterogeneous generated TFs.

The main research outcome is the modified heuristic method of swarming particles applied to the author's Cut-Glue approximation for highly nonlinear dependencies.

The developed generator, in addition to the presented advantages has the possibility of creating irregular multidimensional ME TFs with MAF modification. Thus, it further helps to investigate the properties of TFs, when they are applied to different domains, and allowing more accurate results analysis of the overall "Cut-Glue" approximations approach.

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