# Correlated M/G/1 Queue modelling of Jitter Buffer in TDMoIP

Usha Rani Seshasayee and Manivasakan Rathinam
Department of Electrical Engineering
Indian Institute of Technology Madras
Chennai-600 036, India.
e-mail: ee08d001, rmani@ee.iitm.ac.in

***Abstract*— Time Division Multiplexing over Internet Protocol (TDMoIP) is a pseudowire technology for emulating TDM circuits over Internet Protocol (IP) networks. In such networks, timing and synchronization plays a key role in achieving the required jitter in terms of variance of interdeparture interval. A jitter buffer is used at the receiver, to circumvent the impairment of the packet networks: delay, jitter and loss. But, out of these, delay and loss can't be compensated for, while QoS in IP networks is used to minimize them. The jitter (or variance of packet delay) can be reduced to a tolerable level at the receiving Inter Working Function. A tradeoff between delay and jitter is required to achieve the desired jitter. This paper presents the condition under which the jitter buffer at the receiver is to be operated for minimum output variance in a TDMoIP framework, to achieve minimum slip rate and thus better voice quality. The receiver jitter buffer is modeled as a correlated M/G/1 queueing system with EARMA correlations between the interarrival and the service times. The motivation for the above correlation structure is that, given the correlations within the service intervals, the EARMA correlation results in reduction of variance in the interdeparture interval. This is a step towards achieving CBR upstream. The key advantage of using EARMA correlation is that the analysis of such a correlated queue is analytically tractable. The variance of the interdeparture times of the above queue is presented. The analysis of the departure process, the waiting times of incoming packets of this correlated queue and the relevant simulations show that if the variance of the interdeparture time process constituting output TDM stream is to be less than that of the interarrival time process of the jitter buffer, which is modeled as M/G/1 queue, then the mean waiting time of the packets in the jitter buffer would be greater than that of independent (M/M/1) case. The values of the parameters of the M/G/1 queue which minimizes the variance of interdeparture interval are identified. Our study also included a G/G/1 queue in which interarrivals are also correlated. Extensive simulations demonstrate our analytical results.**

***Keywords-TDMoIP; Jitter; Correlated queue.***

## I. INTRODUCTION

Time Division Multiplexing (TDM) circuits have been the backbone of voice communications over the past several decades. TDM is a reliable, hard partitioned circuit switched technology and offers low delay services for real time interactive digital telephony. But, the bandwidth is used inefficiently in TDM. For efficient bandwidth utilization, there has evolved a 'converged' network catering to all services that is packet based. In such a network, digitized voice is carried over packet switched infrastructure. The transition from circuit switched to packet switched would take place in a phased manner, as legacy networks could not be replaced overnight. Industries and academia are trying to provide solution to this as TDM over Packet Switched Network infrastructure. In this work, we address synchronization in Time Division Multiplexing over Internet Protocol (TDMoIP).

Time Division Multiplexing over Internet Protocol [1, 2, 3] is a technology wherein a logical circuit is realized in an IP network, which links two TDM islands. The TDM traffic at the transmitter is packetized into constant bit sized frames and transmitted across an IP network. When the packet carrying the TDM payload traverses the IP network, it experiences random delay due to the queueing at the intermediate routers. It is because of this, that the packets at the receiver arrive randomly, an effect called as packet delay variation/jitter. To compensate for this, jitter buffer of large enough size is used. A mismatch between the read and the write clocks at the input and the output of this buffer, due to large delay variation will cause overflow or underflow of the jitter buffer. Such clock mismatch can lead to observable defects on the end service, e.g. frame slips when interworking with existing PSTN and narrowband ISDN (N-ISDN) networks [2]. Synchronization in the data link layer of the ISO stack is therefore an important issue in such networks.

The area of timing and synchronization is well researched. Clock recovery schemes can be categorized into synchronous and asynchronous. In case of synchronous schemes [4], the sender delivers via the outgoing stream, the information on the frequency difference between a source clock and reference clock, which is generated out of a common clock available to both sender and the receiver. The original source clock is recovered using this frequency difference. Asynchronous schemes recover the source clock from one of the following: buffer level [5], time difference of arrival of packets [2]. Hybrid techniques which combine any

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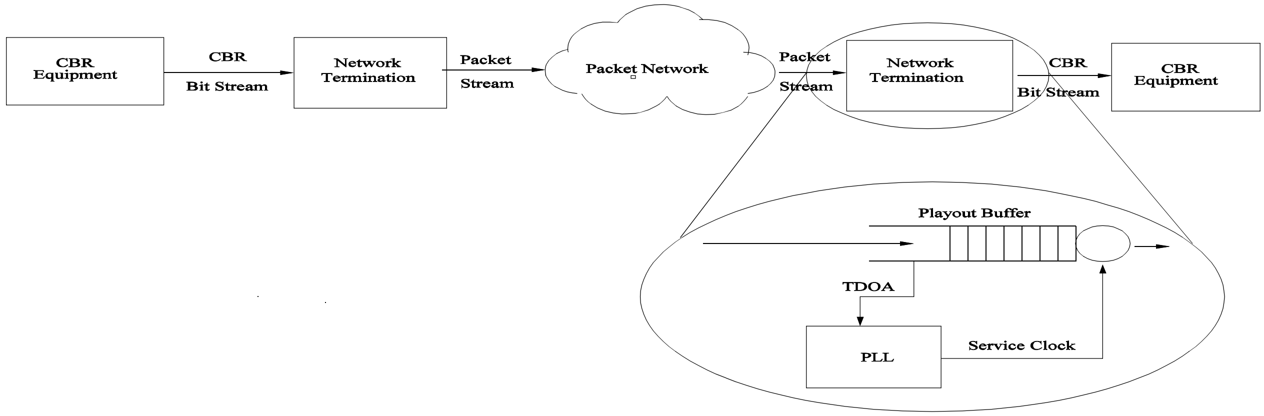


Figure 1. A conventional model for clock recovery for TDMoIP

of two methods have also been studied [6]. All of the above schemes attempt to recover the source clock in the physical layer. For achieving synchronization at the data link layer of the ISO stack, in this paper, we use [7] a queueing model, where frames arriving from the transmitter are queued in the jitter buffer and served such that the variance of the interdeparture process of the outgoing frame stream is minimum.

The rest of the paper is organized as follows. Section 2 briefly describes conventional and proposed model for TDMoIP. Section 3 discusses the proposed model in detail. An analytical expression for variance of the departure process at the receiver is also presented. We are the first one to look at the departure process of a correlated queue, to characterize jitter in packet networks. Section 4 gives the simulation results for interarrival times being independent as well as, correlated. In Section 5, conclusions are drawn and future work is outlined.

## II. CONVENTIONAL MODEL FOR TDMOIP

TDMoIP is a technology wherein TDM frames such as E1/T1 are packetized and sent across a layer 2/3 virtual circuit, in which the QoS is provisioned such that these TDM encapsulated packets are given the highest priority. These packets upon traversing the packet network experiences delay variability, which is caused by the queueing delays at the intermediate routers. These packets having the TDM payload, on arriving at the receiver are stored in a jitter buffer, which is meant to mitigate the effect of the packet network.

### A. Schematic for TDMoIP

The scheme for clock recovery at the physical layer [2] is shown in Fig. 1. For TDMoIP, there is no common clock present at the transmitter and receiver. The receiver has to estimate the transmitter clock from the received data stream. This is accomplished using a phase-locked loop (PLL). The PLL locks the phase of the receiver clock to that of the transmitter or would manipulate the arrival pattern to get accurate receiver clock. Thus the function of the PLL is to compensate for any frequency deviation. The main problem encountered in packet networks is the queueing delay (which is variable for each packet), introduced by packet routers and switches. It is because of this varying delay that the constant bit rate TDM stream does not appear periodic. A frequency estimate of the transmitter clock is done based on the measurements of the packet arrivals to mitigate the effects of packet delay variations using various techniques. This frequency estimate is used as reference for the PLL. The PLL locks the receiver clock's phase to that of the transmitter. The clock from the PLL is used to derive the Constant Bit Rate (CBR) bit stream. Therefore, the better the estimate of frequency, the more periodic will the received stream be. The variance of the frequency estimate should be low for proper synchronization.

### B. Literature survey

When there is no common network clock, the asynchronous clock recovery schemes are applicable. They are also called as adaptive clock recovery schemes. An adaptive clock recovery method based on jitter buffer level was studied in [5]. In this jitter buffer level based method, the clock is recovered by calculating the receiving rate of the IP packets. In another adaptive scheme, the time difference of arrivals of the packets is filtered and source clock is recovered with minimum variance as in [2]. There are hybrid techniques, which combine two of the above methods to minimize the error in clock recovery and also to increase its convergence rate [6]. The schemes mentioned here recover clock at the physical layer. Minimizing variance of the inter-packet time at either of the two layers, physical or data link of the ISO stack will reduce jitter in the outgoing stream.

For achieving synchronization at the data link layer of the ISO stack, in this paper we use [7] a queueing model, where payloads are extracted from the TDM encapsulated IP packets arriving from the transmitter, are queued in the jitter

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buffer. They are served such that the variance of the inter-departure process of the outgoing packet stream is minimum.

### C. *The Jitter buffer model*

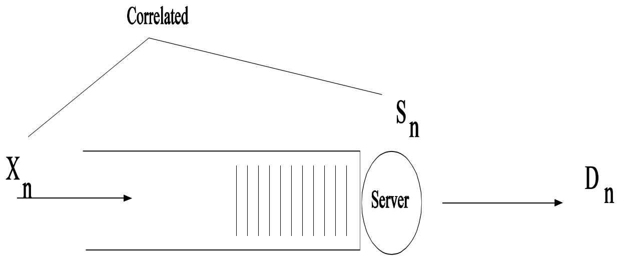


Figure 2. FIFO, Single server queue

The receiver jitter buffer is modeled as a single server, first in first out (FIFO) queue where $\{X_n\}$ is the interarrival time process, $\{S_n\}$ is the service time process, with correlation between the two processes, as shown in Fig. 2. The interdeparture time process of this queue should have minimum variance so that there is proper synchronization at the receiver.

Jacobs [7] gives the waiting time distribution for a correlated queue having correlations between interarrival and interdeparture times under heavy traffic conditions. We use the above model for modelling the jitter buffer and study the interdeparture process in such a queue. We are the first one to look at departure process, its statistics and to use the same in a jitter buffer model. For the analysis of the simulation data of the queue [8] is used. We include here in our studies a queue with non-Poissonian arrival process, to be more realistic.

## III. OUR QUEUE MODEL

### A. *Research solution: The EARMA queue model*

A M/G/1 queue is a queueing system with a single server, arrival process being Poisson, service times following a general distribution and having an infinite buffer space. The jitter buffer is modeled as a M/G/1 queue with exponential autoregressive and moving average (EARMA) [7] correlations between the interarrival times and the service times. The following table gives the notations used for the correlated queue.

TABLE I. THE EARMA QUEUE

| Symbols | Description |
|---|---|
| $\{X_n\}$ | Sequence of independent exponentially distributed random variables with positive finite mean $\lambda^{-1}$, where $X_n$ is the interarrival time between the n th and the n-1 th arrival of the packet at the receiver jitter buffer |
| $\{E_n\}$ | Sequence of independent exponential random variables with positive finite mean $\mu^{-1}$ |
| $\{J_n\}$ | Sequence of Bernoulli random variables with $P(J_n=1)=1-\beta$ ; $0\leq\beta\leq1$ |
| $\{K_n\}$ | Sequence of Bernoulli random variables with $P(K_n=1)=1-\rho_{ar}$ ; $0\leq\rho_{ar}<1$ |
| $\{S_n\}$ | The service time process, where $S_n$ represents the service time of the n th packet, following a general distribution and having a positive finite mean $\mu^{-1}$ |
| $\{B_n\}$ | An auto regressive process, where $B_0$ has an exponential distribution with mean $\lambda^{-1}$ |

We consider a single server queue with FIFO discipline, as the packets that are sent from a single source arrive in the order transmitted inspite of the random delay. The $0^{th}$ packet arrives at t=0 and finds the server free. Packets are of constant size, an E1 frame per packet. Henceforth, the word packet goes synonymous with the word frame and treated as an entity arriving at the queue.

In order to reduce the variance of the departure process, we introduce a positive correlation between service intervals and interarrival times (X and Y are said to be positively correlated when X increases Y also increases). This would mean that a queue with large buffer occupancy would be served faster and vice versa. All these would reduce variance of interdeparture times. A correlation structure which satisfies our requirement is the EARMA correlation. Hence, the service time of the n th packet, $S_n$ is taken to be:

$$S_n = \beta E_n + J_n(\lambda\mu^{-1}B_n) \tag{1}$$

where, $B_n = \rho_{ar} B_{n-1} + K_n X_n$ (2)

Also, note that statistical correlation is also imposed within the service times themselves. These models have the advantage that the marginal distributions and correlation structure of the sequences are specified separately, so that we can compare them with an independent (M/M/1) case with the same distribution. The covariance within service times is obtained from (1) and (2) as

$$COV(S_n, S_{n+k}) = \mu^{-2}\rho_{ar}^k(1-\beta)^2; k = 1,2,3.......... \tag{3}$$

The covariance between the interarrival and service times is obtained from (1) and (2) as

$$COV(S_n, X_{n-k}) = (\lambda\mu)^{-1}\rho_{ar}^k(1-\rho_{ar})(1-\beta); k = 0,1,2,3.........., n-1 \tag{4}$$

The variance of the interdeparture time process should be as less as possible so that the output data stream containing the TDM payload would be closer to CBR as was sent by the transmitter. Larger variance in the inter-departure times would lead to underrun or overrun of the jitter buffer in the upstream node causing clock slips. In effect, all these would result in low quality of voice signal carried by the TDM pipe. By using more sophisticated clock recovery algorithms, recovered TDM clocks can be made to comply with ITU-T G.823 and G.824 specifications for T1/E1 jitter and wander control while simultaneously delivering optimal latency.

### B. Departure process

Departure process is important in the analysis of TDMoIP synchronization algorithm. The variance of the interdeparture time should be minimum as possible so that the output stream would appear periodic with minimum packet delay variation. Table II gives the variables related to the departure process.

TABLE II. DEPARTURE PROCESS OF THE QUEUE

| Symbols | Description |
|---|---|
| $\{D_n\}$ | Sequence of random variables with positive finite mean $\lambda^{-1}$, where $D_n$ is the interdeparture time between the n th and the n-1 th packet at the output of the receiver jitter buffer |
| $\{W_n\}$ | The waiting time process, where $W_n$ is the waiting time of the nth packet in the Queue |

From the queueing theory results [9, 10, 11], the inter-departure time process can be written in terms of the interarrival, waiting and service times. Let $\{D_n\}$ be the inter departure time process, where $D_n$ is the interdeparture between the n th and the n-1 th departure of packets at the output of the receiver jitter buffer, then

$$D_n = X_n + W_n - W_{n-1} + S_n - S_{n-1} \tag{5}$$

From the above equation, the variance of the interdeparture time process is written in terms of the variance and covariance terms as given below. Other covariance terms which are not present in this equation tend to zero because the two variables in them are independent. So we have,

$$\begin{aligned}Var(D_n) &= Var(X_n) + 2.Var(S_n) + 2.COV(S_n, X_n) - 2.COV(S_n, S_{n-1}) \\ &\quad - 2.\mathrm{E}[X_n - S_n].\mathrm{E}[W_n] \\ &= \lambda^{-2} + 2\mu^{-2} + 2(1-\rho_{ar})(1-\beta)(\lambda\mu)^{-1} - 2\mu^{-2}\rho_{ar}(1-\beta)^2 \\ &\quad - 2[\lambda^{-1} - \mu^{-1}].\mathrm{E}[W_n]\end{aligned} \tag{6}$$

The variance of Poisson arrival process is $\lambda^{-2}$.Therefore, the condition for the variance of inter-departure time process, which is stationary, to be less than the variance of the interarrival time process from (6) is obtained as:

$$\mathrm{E}[W_n] > \frac{\lambda}{\mu(\mu-\lambda)}\left[1 + (1-\rho_{ar})(1-\beta)\mu\lambda^{-1} - \rho_{ar}(1-\beta)^2\right] \tag{7}$$

Equation (7) is satisfied only when the parameters are related as: $\rho_{ar} > (2-\beta)^{-1}$, especially under heavy traffic limit conditions. But under such a condition, the mean waiting time would be definitely be greater than the independent case. Thus, there is a tradeoff between the mean waiting time and the variance of the interdeparture time process. The study of the variance of the departure process is done more precisely here, than in [12].

### C. Correlated interarrivals

The queue was studied with interarrival times being correlated as in [13], as it was found in [14], that the interarrivals are correlated in packet networks. The jitter buffer is modeled as a G/G/1 queue, with correlation within the interarrrival times. Table III gives the variables related to the correlated arrival process.

TABLE III. CORRELATED INTERARRIVAL TIMES

| Symbols | Description |
|---|---|
| $\{G_n\}$ | Sequence of independent exponentially distributed random variables with positive finite mean $\lambda^{-1}$ |
| $\{H_n\}$ | Sequence of Bernoulli random variables with $P(H_n=1)=1-\alpha$; $0\le\alpha<1$ |

The interarrival time between the n-1 th and the n th arrival, $X_n$ is given by:

$$X_n = \alpha\, X_{n-1} + G_n H_n \tag{8}$$

## IV. SIMULATION RESULTS

The correlated M/G/1 queue with EARMA correlations was simulated for different traffic intensity and correlation parameter values in MATLAB. The graph for the ratio of mean waiting time of this correlated queue to the mean waiting time of the independence (M/M/1) case was plotted. Also, the ratio of the variance of the departure process to the arrival process was plotted, in the same graph. The simulation results are interpreted clearly than in [12].

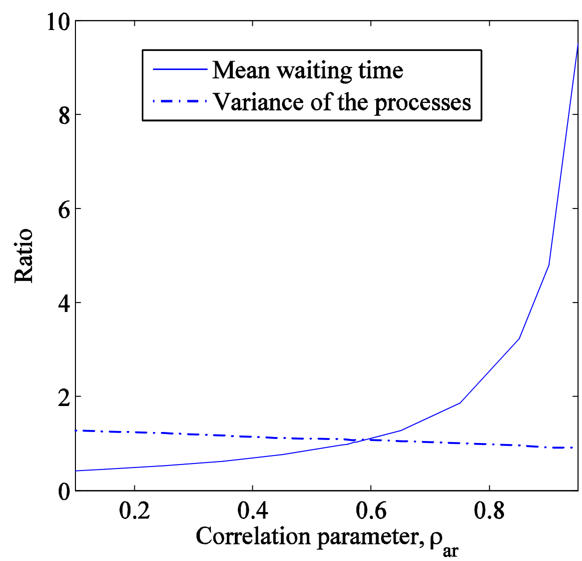


Figure 3. Ratio of the mean waiting time of the correlated M/G/1 to the M/M/1 queue and ratio of the variance of the departure process to the arrival process, for various correlation parameter values, $\rho_{ar}$; $\beta$=0.25; traffic intensity, $\rho$=0.9 Erlangs.

As seen above, under heavy traffic condition, the ratio of the variance of departure process to the arrival process of this queue decreases when strong correlation is imposed between the interarrival and service times and goes well below a factor of 1. But the ratio of the waiting time of this queue to the independent case increases when there is strong correlation between the interarrival and service times. The correlation parameter, $\rho_{ar}= (2-\beta)^{-1}=0.57$, when $\beta=0.25$ is the point from where the variance of the departure is less than that of the arrival process. Therefore, as seen from Fig. 1, high correlation parameter values yields lesser variance, but the waiting time is more.

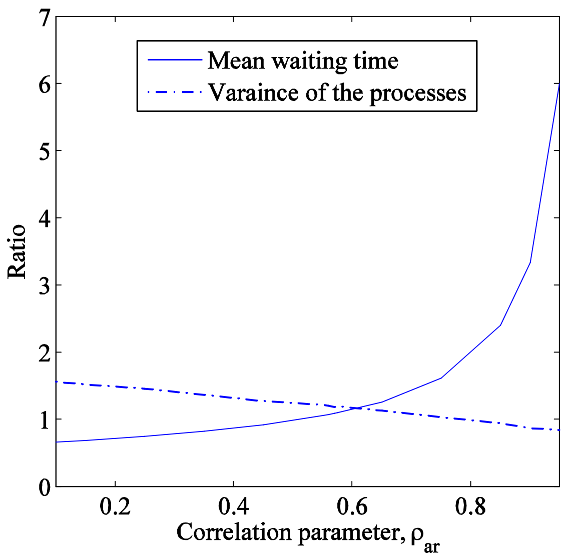


Figure 4. Ratio of the mean waiting time of the correlated M/G/1 to the M/M/1 queue and ratio of the variance of the departure process to the arrival process, for various correlation parameter values, $\rho_{ar}$; $\beta=0.25$; traffic intensity, $\rho=0.6$ Erlangs.

As is seen from Fig. 4, even under light traffic condition, the ratio the variance of the departure process is less than arrival process for strong correlation, that is, higher values of $\rho_{ar.}$ The correlation parameter values at which the mean waiting time ratio and the variance ratio are unity are same, as for the heavy traffic condition as in Fig. 3. That is, the correlation parameter, $\rho_{ar}= (2-\beta)^{-1}=0.57$, when $\beta=0.25$ is the point from where the variance of the departure is less than that of the arrival process.

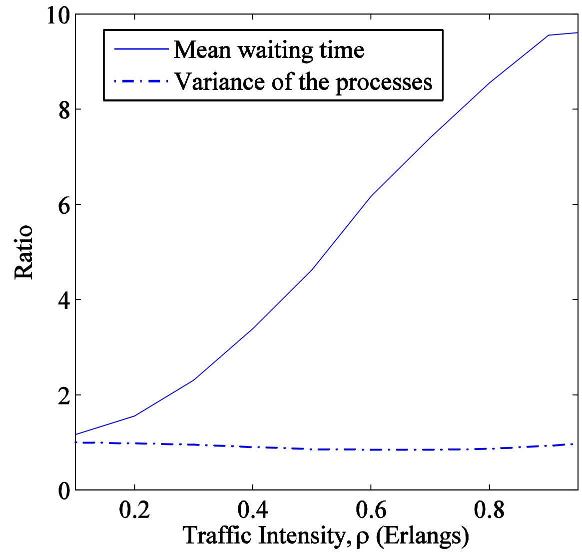


Figure 5. Ratio of the mean waiting time of the correlated M/G/1 to the M/M/1 queue and ratio of the variance of the departure process to the arrival process for various traffic intensity values, $\rho$ (Erlangs); $\beta=0.25$; correlation parameter, $\rho_{ar}=0.95$.

Fig. 5 depicts a condition where the correlation parameter is selected such that the variance of the departure process is less compared to the arrival process but the mean waiting time is more compared to the independent case. That is, our objective of minimum jitter variance is achieved.

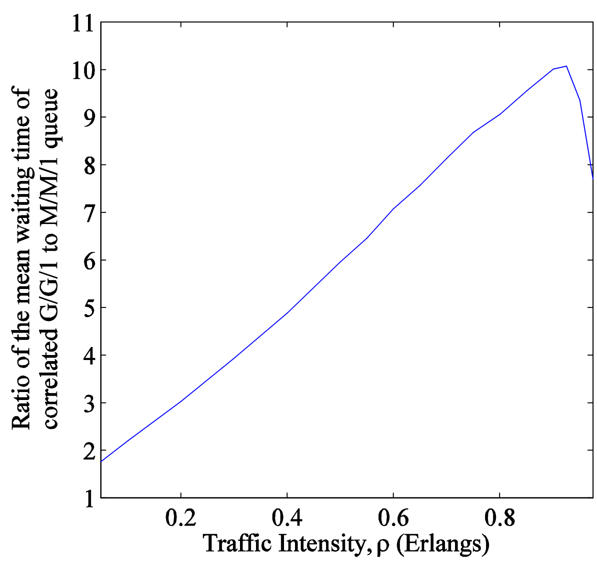


Figure 6. Ratio of the mean waiting time of the correlated G/G/1 to the M/M/1 queue for various traffic intensity values, $\rho$ (Erlangs); $\beta=0.25$; correlation parameter, $\rho_{ar}=0.95$; interarrival time correlation parameter, $\alpha=0.65$.

Fig. 6 depicts a condition where the correlation parameters $\beta$ and $\rho_{ar}$ are selected for a particular interarrival time correlation parameter, $\alpha$ such that the mean waiting time is more compared to the independent case. High mean waiting time renders low variance of the departure process. This aids in achieving lesser jitter. Under heavy traffic conditions, the ratio of the mean waiting time of the correlated G/G/1 to the M/M/1 queue reduces. For this particular value of the interarrival time correlation parameter, it is better not to operate the queue under heavy traffic condition, so as to achieve our objective of lesser variance.

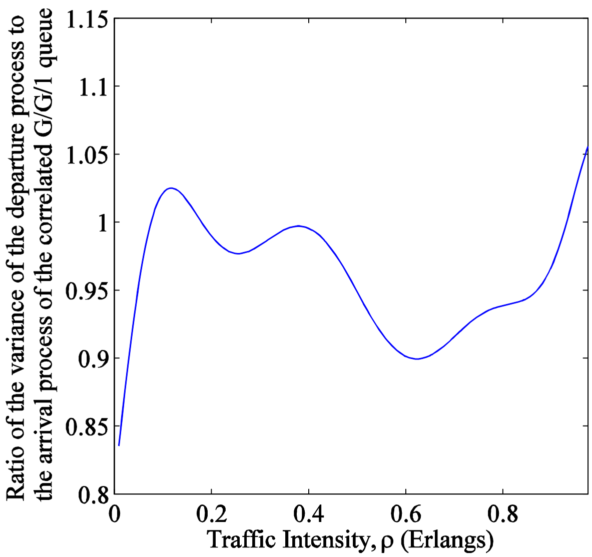


Figure 7. Ratio of the variance of the departure process to the arrival process for various traffic intensity values, $\rho$ (Erlangs); $\beta$=0.25; correlation parameter, $\rho_{ar}$=0.95; interarrival time correlation parameter, $\alpha$=0.65.

Fig. 7 depicts a condition where the correlation parameters are selected such that the variance of the departure process is less than the independent case, for a particular value of the interarrival time correlation parameter.

The queue behaviour, when there is auto-regressive correlation (correlation value, $\alpha$=0.65) between interarrival times shows that the mean waiting time of this correlated queue is greater than the mean waiting time of the M/M/1 queue at a correlation parameter value, between the interarrival and the service time, which is different from the one presented earlier. Whereas, the variance of the departure process of this queue is less than the variance of the arrival process. Only when the correlation between the interarrival times is less, the objective of lesser variance of the departure sequence is achieved.

## V. CONCLUSION AND FUTURE WORK

A correlated M/G/1 is used for modelling the jitter buffer in TDMoIP and a set of parameters of the queue is identified to achieve our objective of minimum variance of the departure process. The correlation parameters are obtained to achieve the desired output variance. This work is extended to arrivals being correlated, as interarrival times are correlated in packet networks and their simulation results are presented. Obtaining the analytical expressions relating the correlation parameter of the interarrival time and the queue statistics can be a future work. A feedback mechanism to control the various correlation parameters to achieve the desired voice quality can also be considered.

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