Analysing the effect of Renewable Energy Sources on Electricity Prices in Spain. A Maximum Entropy Econometric Approach.

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Abstract— **There is a controversial debate about the effects of the promotion of renewable energy and electric power sector reforms on electricity prices. This paper explores the impact of renewable energies and other environmental and economic variables on electricity prices in Spain. However, information available regarding renewable energies is scarce so when trying to estimate the electricity price model through regression procedures a dimensionality problem arises. Therefore we use a Maximum Entropy Econometric approach which allows estimating models when information is limited.**

Keywords-electricity price, renewable energy, Entropy measures

I. Introduction

The promotion of renewable energies is a key concept in European Union by environmental and economic reasons. This type of energy contributes to obtain the objectives established by Kyoto Protocol. Besides, it allows the obtaining of various social-economic advantages, such as the diversification of energy offer, the improvement of opportunities in regional and local development, and the creation of a domestic industry and employment[1].

In contrast, the renewable energies also have some costs related to the adjustments in production, prices and transportation systems.

Regarding the price effects, since the majority of renewable energy technologies are not profitable at current energy prices, they are expected to increase energy costs. In order to make them profitable, there are several public supports. The support scheme used in Spain is the feed-in tariffs. This system requires that distributors acquire, in first place, the energy produced by renewable energy sources at a determined price established by regulator during a specific time period (generally, around fifteen years). From generator view point, it operates as a subsidy.

In addition, environmental costs related to CO_2 emissions in electricity generation in general have a significant negative effect on energy costs as a CO_2 emission trading scheme exists [4].The substitution of conventional electricity generation by renewable energies could reduce the cost derived from environmental emissions and the electricity price. Besides, it is necessary to consider that a higher use of renewable energies could reduce even the final electricity prices because its promotion stimulates the generation of renewable energy which is characterized by variable costs lower than fossil conventional technologies [5].

Therefore the cost disadvantage of renewable compared to conventional energies is crucially dependent on future prices of energies used in power plants as well as on the amount of CO_2 emission permits. The expansion of renewable energy could affect electricity price.

Moreover, the regulatory reforms in the Spanish Electricity Market also could have its effect on electricity prices. In fact, the introduction of liberalization in the retailing activity could reduce electricity price as a consequence of the competition system.

Since in Spain there is a part of the electricity price paid by the government, it could be interesting to study the effect of the general economic activity on electricity price.

Therefore, there are several variables affecting electricity prices.

In this paper, we explore the effect of electricity generated by renewable energy and other factors (matrix X) on household electricity prices in Spain (y). Our data sets are provided by Eurostat during the period 2002-2007.

During the last years, several studies have been developed using cross-sectional or temporal data to explain the effect of several variables on electricity price. Some of them include explanatory variables related to energy use or technology [6][7] or market liberalization [8][9]. Our empirical study takes into consideration all effects together.

Traditional parametric methods require an elevated sample size for the efficient estimation of the coefficients in the models. Thus, $y = X\beta + u$ its estimation by regression

[1] Several studies have reviewed the effects of the introduction of renewable energies at EU country levels. This is the case of [1] for United Kingdom or [2] for Germany. Other studies as [3] have analysed the macroeconomic impact of renewable energy at local level.

 ISBN: 978-1-61208-138-0

techniques requires that the number of observations was superior to the number of independent variables.

However, information available regarding renewable energy and electricity market liberalization is scarce so it limits the sample date. Therefore, when trying to estimate the electricity price model through regression procedures a dimensionality problem arises.

As an alternative to estimate the model, when a dimensionality problem arises, we propose a Maximum Entropy Econometric approach, which has been defined by [10] as "a sub-discipline of processing information from limited and noisy data with minimal a priori information on the data-generating process". This approach has its roots in Information Theory and builds on the entropy-information measure [11], the classical maximum entropy principle [12],[13], which was developed to recover information from underdetermined models, and the Generalized Maximum Entropy Theory [14].

The maximum entropy Econometric approach was developed to estimate models using limited or incomplete data. Therefore, we investigate its possibilities in the estimation of household electricity price.

This paper is divided into two more sections. The first of them (Section II) makes a revision of the functioning of Spanish electricity market and analyzes the important role of the renewable energies in such electricity system.

The next section (Section III) assesses the problem of electricity price model estimation through regression-based procedures when a dimensionality problem arises. It also contains a brief description of some of the uncertainty measures provided by the Information Theory and the requirements to optimize their values. Furthermore, we describe the Maximum Entropy Econometric procedure in order to estimate the model. Moreover, we present an empirical application of the proposed method to Spanish household electricity prices over the period 2002-2007. Some concluding remarks complete the paper.

II. ELECTRICY MARKET IN SPAIN. AN OVERVIEW

In this section, we analyze the characteristics of the Spanish electricity market after the liberalization process and the role of renewable energies in such market.

Liberalization of the Spanish electricity system is begun with Law 54/1997 [15] where the key element is the creation of the wholesale electricity market (pool[2]).

In daily market, electricity companies determine, for every generation unit, the offered amount and price. In parallel, electricity consumers establish the demanded amount and the maximum prices that they are ready to pay. In this context, the use of an algorithm based on auctions of first price establishes the wholesale electricity price (see Fig.1).

On the other hand, liberalization of Spanish electricity system entails too the creation of a retail electricity market. In this market, consumer can choose distributor freely and negotiate with them the price and the conditions of the electricity supply. Therefore, consumers have two options of supply: regulated (by means of the payment of a "integral" tariff where all supply costs are included) and competitive or of market (by means of the payment of an access tariff to the networks plus the energy contracting costs and other services).

Regarding the role of renewable energies in the electricity generation the European Community Directives 2001/77/EC [16] and 2009/28/EC [17] give freedom to each member state for choosing the support mechanism of renewable resources.

In the case of Spain, the current legal framework of renewable energy is the Royal Decree 661/2007 [18] whose aim is to minimize the environmental impact of electricity supply. Like that, the basic instrument introduced is feed-in tariff. This mechanism entails two possibilities in the sale of electricity, generated by renewable energies: a) to sell the electricity to distributor at a regulated tariff or b) to sell the electricity directly in the market where the remuneration is given by the negotiated price in the market plus a feed-in tariff.

This legal framework has allowed that Spain was a pioneering and leader country in the integration of renewable energies as the wind or solar energy in the electricity system (Spain is the second European country in terms of installed capacity and production of these types of energy, only behind Germany).

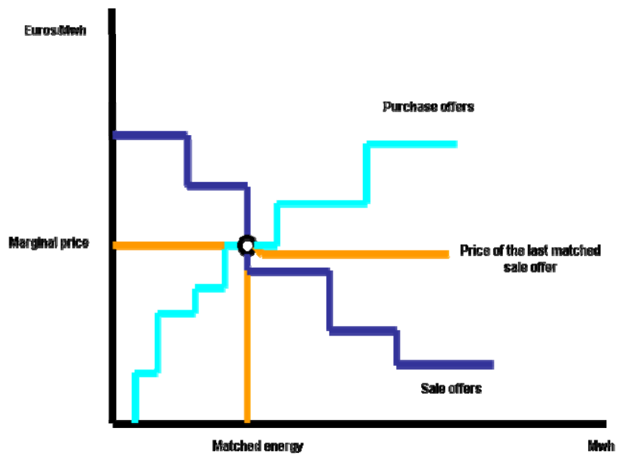

Fig. 1.Establishment of Wholesale electricity prices €/mWh

[2] Spanish electricity market is structured on various markets where electricity demand and offer is adjusted. Daily market is emphasized because the electricity price shaped in it supposes around the 89% of final electricity price. In this market, electricity energy is negotiated for every one of the twenty-four hours of the following day.

III. Modelling the Impact of Renewables on Electricity Prices. A Maximum Entropy Econometric Approach.

Our goal is to estimate the effect of several variables (m) on electricity prices (*y*) by using data of several years (T).

$y = X\beta + u$, being X a matrix $T_X m$, *y* a matrix $T_X 1$, β the vector of coefficients to be estimated (vector $m_X 1$) and vector of disturbances u.

A. *Maximum Entropy Econometric Approach.*

The estimation of $y = X\beta + u$ by regression techniques requires that the number of observations (T) has to be superior to the number of independent variables (m).

Nevertheless, information available regarding renewable energy and electric liberalization market is scarce. Therefore, in a situation of limited sample data the estimation of the model by regression procedures (OLS) is not possible as the problem is undetermined or ill-posed.

However, when these circumstances of small amount of information available make it unfeasible to combine forecasts through OLS procedures the Maximum Entropy Econometric approach allows us to recover the estimates of $\beta_1, \beta_2, ..., \beta_m$ in the corresponding parameterized model without making distributional assumptions. The approach consists of developing a non-linear inversion procedure [14] which requires the application of the tools provided by the Information Theory ([11], [12], [13]).

Consider a regression-based method: $y = X\beta + u$ in a situation of limited sample data where m>T. A probability distribution should be used in order to represent partial and limited information regarding the individual observations so they are consistent with the observed sample data. Therefore, following [14] it is possible to define an inverse general problem for recovering β defined as: $y = X\beta + u = XP + u$, where $P:(p_1,\dots,p_m)'$ is a m-dimensional vector of unknown terms related to the probability distribution. The main objective is to estimate a probability distribution P given the limited information and minimal distributional assumptions and therefore recover β as $\hat{P} = \hat{\beta}$.

However, as the number of observations (T) is smaller than the number of independent variables (m), if we attempt to recover P by using traditional procedures of mathematical inversion, there is more than one vector P making the solution feasible. Therefore the problem is ill-posed and we have no basis for picking a particular solution vector for P from the feasible set. Thus, if we ask for a particular set of probabilities considered as most likely, it seems reasonable to favour the one that could have been generated in the greatest number of ways given the available data.

The definition of the entropy measure H(P) and the formulation of the Entropy Maximization problem can help us to estimate a unique P distribution since the principle of Maximum Entropy provides a basis for transforming the sample information into a probability distribution that reflects our uncertainty about the individual outcomes.

The measures of entropy H(P) quantify the uncertainty associated with a random experiment. In particular, given a random variable X with values xi and probability distribution $P = (p_1, ..., p_n)$ with $p_i \geq 0$ (i=1..., n) and $\sum_{i=1}^{n} p_i = 1$, Shannon's measure of entropy ([11]) is defined as:

$$H_S(P) = H_S(p_1, ..., p_n) = -\sum_{i=1}^{n} p_i \log p_i .$$

The value of the entropy is maximum when all the values xi have the same probability (and then P is a uniform distribution). This situation would be justified by the Laplace Indifference Principle, according to which the uniform distribution is the most suitable representation of our knowledge when the random variable is completely unknown. Nevertheless, sometimes the ignorance of the probability distribution of X is not absolute and we have some partial information on the distribution such as the mean, variance, moments or some characteristics which can be formulated as equality constraints. In such a case, it is possible to estimate the probability distribution through the application of the Maximum Entropy principle ([12], [13]) choosing the distribution for which the available information is just sufficient to obtain the probability assignment.

Thus, if we know certain values a_r (r=1..., s) associated with functions $g_r(X)$ of the values of X but we do not know its distribution, the problem consists of estimating a nonnegative distribution that fulfils the conditions $p_i \geq 0$ for i=1..., n and $\sum_{i=1}^{n} p_i = 1$, maximizing the value of the entropy.

By solving the maximization problem we can obtain the estimated probabilities $\hat{P} = \{\hat{p}_1, \dots, \hat{p}_n\}$. We pointed out that the maximum entropy distribution does not have a closed-form solution and therefore numerical optimization techniques must be used to compute the probabilities.

Working towards a criterion for recovering the parameters of the regression model related to electricity price in the general inverse problem $y = X\beta + u = XP + u$, if there is no evidence that a specific independent variable is more significant than others, the related probability distribution (P) would be the uniform (according to Laplace Indifference Principle). However, the principle of maximum entropy provides a basis for using the sample information in a probability distribution P that reflects our uncertainty about the individual independent variable. Therefore, the problem consists of estimating a nonnegative distribution P by maximizing the value of the entropy *H(P)* subject to the available information. By solving the optimization problem the estimated probability distribution $\hat{P}$ is obtained.

We consider a general inverse problem $y = X\beta + u = XP + u$ where we wish to determine the unknown and unobservable frequencies $P = (p_1, ..., p_m)'$, representing the data generating process. Then, within the possible sets of probabilities fulfilling $\sum_{i=1}^{m} p_i = 1$, $p_i \geq 0$, we must choose to assign a single vector. Through the application of the principle of maximum entropy we maximize $H(P)$ under the restrictions of information consistency $y = X\beta + u$, and the adding up-normalization constraint for P: $P'\ell = 1$.

If the vector of disturbances, u, is assumed to be a random vector with finite location and scale parameters, we can represent our uncertainty about it by treating each u_t (t=1, ..., T) as a finite and discrete random variable with $2 \leq J \leq \infty$ possible outcomes.

Thus, it is assumed that each u_t is limited by an interval (v_{t1}, v_{tJ}), whose probability, $\Pr(v_{t1} < u_t < v_{tJ})$, can become as small as we want. For example, for J=2, the error can be defined as: $u_t = w_t v_{t1} + (1 - w_t) v_{tJ}$ where each $w_t \in [0,1]$ is a vector of error weights. Furthermore, $J \geq 2$ can be used to assume certain characteristics of symmetry and kurtosis about the error distribution.

Because there may be different levels of uncertainty underlying each β_i, for more general inferential purposes, point estimates may be limiting and unrealistic. Consequently, it is possible to generalize the maximum entropy problem to permit a discrete probability distribution to be specified and obtained for each β_i. Rather than search for the point estimates of β, each β_i is viewed as the mean value of some well defined random variable z.

Then, for each β_i, we assume there exists a discrete probability distribution that is defined over a parameter space $\mathbb{R}^K$ by a set of equally distanced discrete points $z_i = [z_1, \ldots, z_K]'$ with corresponding probabilities $p_i = [p_{i1}, \ldots, p_{iK}]'$ and with $K \geq 2$. Therefore: $\beta_i = E_{P_i}[z_i]$ or $\beta = E_P[z]$.

Using the Maximum entropy econometric approach, one investigates how "far" the data pull the estimates away from a state of complete ignorance (uniform distribution). In order to measure the reduction in the initial uncertainty, the information index entropy measure R is defined ([19], [20], [21]) and where $R \in [0,1]$. A high value of R implies the data tell us something about the estimates, or similarly, there is valuable information in the data.

Moreover, we can define measures to evaluate the information in each one of the variables i = 1,2,...,m as the normalized entropy: $S(\hat{p}_i)$.

These variable-specific information measures reflect the relative contribution (of explaining the dependent variable) to the independent variable. Where $S(\hat{p}_i) \in [0,1]$, zero reflects no uncertainty while one reflects total uncertainty in the sense that P is uniformly distributed.

B. *Estimated Models.*

The basic time series data for the period 2002–2007 were taken from Eurostat (available at the web site http://epp.eurostat.ec.europa.eu):

<u>*Dependent variable y*:</u>

Electricity prices for household consumers: This indicator presents electricity prices charged to final consumers. Electricity prices for household consumers are defined as follows: Average national price in Euro per miles kWh without taxes applicable for the first semester of each year for medium size household consumers (Consumption Band Dc with annual consumption between 2500 and 5000 kWh).

<u>Independent variables, m:</u>

Electricity generated from renewable sources (% of gross electricity consumption): This indicator is the ratio between the electricity produced from renewable energy sources and the gross national electricity consumption for a given calendar year. It measures the contribution of electricity produced from renewable energy sources to the national electricity consumption. Electricity produced from renewable energy sources comprises the electricity generation from hydro plants (excluding pumping), wind, solar, geothermal and electricity from biomass/wastes. We also have information about *Electricity generated from wind power* and *Electricity generated from hydroelectricity* as % of gross electricity consumption)

Electricity generated from nuclear- % Total gross electricity generation

Electricity generated from natural gas- % Total gross electricity generation

Electricity generated from petroleum - % Total gross electricity generation

Electricity generated from hard coal- % Total gross electricity generation.

These two last variables are used also as proxy of the effects of GHG emissions on prices as electricity generated from petroleum or coal (thermal power stations) are the energy industries with more GHG emissions. We also have considered the variable *Greenhouse gas emissions by Energy industries* (-% total Greenhouse gas emissions).

As variables related to electricity market and its liberalization we consider:

Number of enterprises dedicated to generation of electricity

Number of enterprises dedicated to distribution and trade of electricity

GDP per capita (thousands of euros): Since in Spain there is a part of the electricity price paid by the government, it could be interesting to study the effect of the general economic activity on electricity price.

Moreover as Spain has a high level of *energy dependency* (around 80% levels) we also have into consideration this fact. The most imported energy is gas and petroleum so petroleum prices could have some impacts on electricity price. We use the *brent crude petroleum price* by barrel (in dollars).

For the solution of the optimization the GAMS program version 21.3 (*General Algebraic Modeling System*) is used. This is a programming language which allows diverse optimization problems to be solved.

We first deal with a general maximum entropy model with a reparametrized error. We establish an a priori range for the possible values that may be assumed by error u in the model, which may be employed to assume certain characteristics of its distribution: V. Since this decision is arbitrary, we have assigned a support vector for the errors: (-v, -v/2, 0, v/2, v) for v>0, which guarantees its symmetry around zero. The decision regarding the amplitude of the range of values which it may assume is arbitrary. According with [22] support vector v can be assessed if we perfect knew the variability presented on y and we can use the *three standard deviation rule* as estimation for v. In fact we follow the proposal of [23] who use the sample variance of y (5,38 miles de euros and then v=16.15) as an estimate for v. However, as a widening of the error bound by increasing v the estimated weights converge on the uniform distribution (the difference between the weights of the variables is reduced) we have used the most reduced v that makes the solution feasible (v=7).

Moreover, we establish an a priori range for the possible values that may be assumed by β in the model. Thus, we should choose the support space Z, and then use the data to estimate the P which in turn yields β. The restrictions imposed on the parameter space through Z should reflect our prior knowledge about the unknown parameters. However, such knowledge is not available as the estimated models are scarce, and we may want to entertain a variety plausible bound on β. However, we have considered a vector support symmetrical and centered on zero and in accordance with the value ranking that the independent variables may take. Moreover, as an initial approximation, we have calculated a covariate matrix finding negative values in β. So, we consider Z= (-z, 0, z) for z>0, which guarantees its symmetry around zero. We consider the same z for all coefficients (z=0.7). It implies that we have to be very cautious in the interpretation of the estimated $\hat{\beta}$. As we report $S(\hat{p}_i)$ and $S(\hat{p}_i) \cong 1$ implies $\hat{\beta}_i \cong 0$ a natural criterion for identification of the information content of a given x_i is just the normalized entropy.

Table I shows our estimated weights for the electricity price (β) under the reparameterized system. We report the estimated coefficients for the model with highest R obtained. The results are those obtained under the narrowest V vector.

The estimated information index R=0.722 indicates a reduction of the uncertainty by using the maximization entropy approach, however, the findings yield that the variable *Electricity generated from renewable energies (RES)* does not have sense to explain electricity prices as $S(\hat{p}_i) \cong 1$.

Therefore, we tried to study separately the renewable energy sources. In Spain, the largest part of the electricity generation by RES is devoted to wind power and hydro, so we estimate the model with *Electricity generated from wind power* and *Electricity generated from hydroelectricity* variables. Table II shows our new estimated weights for the electricity price (β).

TABLE I. HOUSEHOLD ELECTRICITY PRICE. A MAXIMUM ENTROPY ECONOMETRIC ESTIMATED MODEL.

Variables	$\hat{\beta}_i$	$S(\hat{p}_i)$
Electricity generated from RES	0.193	0.833
Electricity generated from nuclear	0.388	0.127
Electricity generated from natural gas	-0.398	0.021
Electricity generated from petroleum	-0.396	0.048
Electricity generated from hard coal	0.399	0.443
Nº enterprises electricity generation	-0.007	1.000
Nº enterprises distribution and trade of electricity	0.399	0.014
GDP per capita	-0.307	0.537
Energy dependency	0.399	0.014
Petroleum price	0.331	0.113
Support vector for the errors (-v, -v/2, 0, v/2, v)	v= 7	
Support space for coefficients (-z, 0, z)	z= 0.7	
Estimated information index	R= 0.722	

TABLE II. HOUSEHOLD ELECTRICITY PRICE . A MAXIMUM ENTROPY ECONOMETRIC ESTIMATED MODEL.

Variables	$\hat{\beta}_i$	$S(\hat{p}_i)$
Electricity generated from wind	-0.391	0.095
Electricity generated from hydro	0.116	0.942
Electricity generated from nuclear	0.380	0.183
Electricity generated from natural gas	-0.372	0.234
Electricity generated from petroleum	-0.306	0.540
Electricity generated from hard coal	0.397	0.035
Nº enterprises electricity generation	-0.006	1
Nº enterprises distribution and trade of electricity	0.361	0.298
GDP per capita	0.062	0.984
Energy dependency	0.399	0.014
Petroleum price	0.361	0.299
Support vector for the errors (-v, -v/2, 0, v/2, v)	v= 7	
Support space for coefficients (-z, 0, z)	z= 0.7	
Estimated information index	R= 0.58	

 ISBN: 978-1-61208-138-0

The results give us additional information with respect to the first model about the direction in which the RES affect the dependent variable. *Electricity generated from wind* is a very important variable ($S(\hat{p}_i) = 0.095$) which contributes to reduce electricity prices. Moreover, when *electricity generated from hard coal* increases the electricity prices, these increases could be in part due by the cost of the GHG emissions which have to be paid by hard coal based energy industries. Energy dependence has also an important effect.

Regarding to the electricity market liberalization variables, the *number of enterprises dedicated to electricity generation* does not have sense to explain electricity prices. However, the *Number of enterprises dedicated to distribution and trade of electricity* gives more information to explain *y* as $S(\hat{p}_i) = 0.298$, nevertheless it has a positive effect on electricity prices. Electricity market opening does not necessarily imply effective competition and competitive prices. Achieving competitive prices depends on the number of the players and the nature of consumer demand as consumers could be resistant to switching.

IV. MAIN FINDINGS AND CONCLUDING REMARKS

We have analyzed the role of some economic and environmental variables on electricity price in Spain for the time period 2002-2007, applying Theory Information Approach due the small sample size.

By applying the General Maximum Entropy Estimation Approach, it becomes possible to estimate a model bearing in mind all, and only, the limited information available which would not be possible with more traditional estimation methods.

Regarding the price effects of renewable energies, since the majority of renewable energy technologies are not profitable at current energy prices, they are expected to increase energy costs.

In addition, environmental costs related to CO_2 emissions in electricity generation in general have a significant negative effect on energy costs. Moreover, the regulatory reforms in the Spanish Electricity Market also could have its effect on electricity prices.

The results showed that electricity generated from wind contributes to reduce electricity prices. Moreover, when electricity generated from hard coal increases the electricity prices, these increases could be in part due by the cost of the GHG emissions which have to be paid by hard coal based energy industries. Energy dependence has also an important effect on electricity prices in Spain.

The liberalization of electricity industry, in retail activities, has a positive effect on electricity prices. Electricity market opening does not necessarily imply effective competition and competitive prices. Achieving competitive prices depends on the number of the players and the nature of consumer demand as consumers could be resistant to switching.

REFERENCES

[1] P.M. Connor, "UK renewable energy policy: a Review", Renewable and Sustainable Energy Reviews, nº7, 2003, pp. 65-82.

[2] B. Hillebrand, H.G. Buttermann, J.M. Behringer, J.M. and M. Bleuel, "The expansion of renewable energies and employment effects in Germany", Energy Policy, vol. 34, 2006, pp. 3484-3494.

[3] B. Moreno and A.J. López, "The effect of renewable energy on employment: The case of Asturias (Spain)", Renewable and Sustainable Energy Reviews, vol. 12, 2008, pp. 732-751

[4] M. Rathmann, "Do support systems for RES-E reduce EU-ETS-driven electricity prices?", Energy Policy , vol. 35, 2007, pp. 342–349.

[5] S. Jensen and K. Skytte, "Simultaneous attainment of energy goals by means of green certificates and emission permits", Energy Policy, vol. 31, 2003, pp. 63-71.

[6] D.J. Swider, L. Beurskens, S. Davidson, J. Twidell, J. Pyrko, W. Prüggler, H. Auer, K. Vertin and R, Skema, "Conditions and costs for renewables electricity grid connection: Examples in Europe", Renewable Energy, vol. 33, 2008, pp. 1832-1842.

[7] L. Dale, D. Milborrow, R. Slark and G. Strbac, "Total cost estimates for large-scale wind scenarios in UK," Energy Policy, vol. 32, 2004, pp. 1949-1956.

[8] H. Nagayama, "Effects of regulatory reforms in the electricity supply industry on electricity prices in developing countries", Energy Policy vol. 35, 2007, pp. 3440-3462.

[9] H. Nagayama, "Electric power sector reform liberalization models and electric power prices in developing countries. An empirical analysis using international panel data", Energy Economics, vol. 31, 2009, pp. 463-472.

[10] A. Golan, "Information and entropy econometrics –Editor´s view", Journal of Econometrics, vol. 107, 2002, pp. 1-15.

[11] C.E. Shannon, "A Mathematical Theory of Communication", Bell System Technology Journal, vol. 27, pp. 379-423.

[12] E.T. Jaynes, "Information Theory and Statistical Mechanics I", Physics Review, vol. 196, 1957, pp. 620-630.

[13] E.T. Jaynes, "Information Theory and Statistical Mechanics II", Physics Review, vol. 108, 1957 , pp.171-190.

[14] A. Golan, G. Judge and D. Miller, Maximum Entropy Econometrics: robust estimation with limited data. John Wiley & Sons Ltd: London, 1996.

[15] Law 54/1997, of 27 of November, of Electricity Industry.

[16] Directive 2001/77/EC of the European Parliament and of the Council of 27 September 2001on the promotion of electricity produced from renewable energy sources in the internal electricity.

[17] Directive 2009/28/EC of the European Parliament and of the Council of 23 April 2009on the promotion of the use of energy from renewable sources and amending and subsequently repealing Directives 2001/77/EC and 2003/30/EC.

[18] Royal Decree 661/2007, BOE 178, 26/07/2007.

[19] A. Golan, "A multi-variable stochastic theory size distribution of firms with empirical evidence", Advances in Econometrics, vol. 10, 1994, pp.1-46.

[20] E.S. Soofi, "A generalizable formulation of conditional logit with diagnostics", Journal of the American Statistical Association, vol. 89, 1992, pp.1243-1254.

[21] E.S. Soofi, "Capturing the intangible concept of information", Journal of the American Statistical Association, vol. 89, 1994, pp. 1243-1254.

[22] F. Pukelsheim, "The three sigma rule", The American Statician, vol. 48, nº 2, 1994, pp. 88-91

[23] A. Golan, G. Judge and D. Perloff, "Estimation and inference with censored and ordered multinomial response data", Journal of Econometrics, vol. 79, 1997, pp. 23-51.

 ISBN: 978-1-61208-138-0