Performance Evaluation for DSRC Vehicular Safety Communication: A Semi-Markov Process Approach

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***Abstract*— In this paper, an analytic model is proposed for the performance evaluation of vehicular safety related services in the dedicated short range communications (DSRC) system on highways. The generation and service of safety messages in each vehicle is modeled by an M/G/1 queue. A semi-Markov process (SMP) model is developed to capture contention and backoff behavior in IEEE 802.11 broadcast ad hoc networks. Furthermore, this SMP interacts with the M/G/1 queue through fixed point iteration. Based on the fixed-point solution, performance indices including transmission delay and packet delivery ratio (PDR) are derived. Hidden terminal problem is taken into account for the PDR computation. Analytic-numeric results are verified through extensive simulations under various network parameters. Compared with the existing models, the proposed model is more general and accurate.**

Keywords-analytic model; DSRC; performance evaluation; safety message; SMP model; VANET.

I. INTRODUCTION

Inter-Vehicle Communication (IVC), as a vital part of Intelligent Transportation System (ITS) [1], has been extensively researched in recent years. In the vehicular ad hoc network (VANET), the transportation safety is one of the most crucial features needed to be addressed. Safety applications usually demand direct vehicle-to-vehicle ad hoc communication due to highly dynamic network topology and strict delay requirements. Such direct safety communication will involve a broadcast service because safety information can be beneficial to all vehicles around a sender. Broadcasting safety messages is one of the fundamental services in dedicated short range communications (DSRC) [1], which is adopted by IEEE and ASTM.

The performance of vehicular safety communication in DSRC system has been studied in [2][3][4]. However, the evaluations are mainly based on simulations. Recently, a few analytic models based on discrete time Markov chain (DTMC) are developed in [5][6][7] to analyze the performance of the broadcast service incorporating the backoff counter process, hidden terminals and message generation interval. Nevertheless, these cited papers conduct performance assessments in a discrete time fashion by synchronizing system behavior to unit time slot, which will lead to some approximations in the results. In addition, according to 802.11 DSRC MAC layer protocol, a vehicle can directly transmit a packet without undergoing backoff process. Such phenomena has been ignored in the previous work [5][6][7], which will result in further approximations.

In this paper, we develop more accurate analytic models using a semi-Markov process (SMP) [8][9] interacting with an M/G/1 queue for the performance evaluation of the broadcast service in DSRC safety communication system. Fixed point iteration [10] is applied to derive the converged solution in the steady state. New approaches to calculate the transmission delay of safety related messages and PDR utilizing features of SMP models are also developed in this paper. The analytic results are verified by simulations.

This paper is organized as follows. Section II briefly describes the system behavior in 802.11 MAC layer protocol and assumptions in the system to produce a simplified model. Section III presents the analytic models and the fixed point iteration. Performance indices including mean delay and PDR are derived in Section IV. The analytic and simulation results are compared in Section V. Conclusions are presented in the last section.

II. SYSTEM DESCRIPTION

A. Broadcast protocol

In the 802.11 MAC layer protocol [11], distributed coordination function (DCF) is the primary medium access control technique for broadcast services. This section briefly explains the basic access mechanism of DCF for broadcast.

Each vehicle in the network can occasionally generate safety related packets and compete for the channel resource to transmit the packet. For a newly generated packet in a vehicle, the vehicle senses the channel activity before it starts to transmit the packet. If the channel is sensed idle for a time period of distributed inter-frame space (DIFS), the packet can be directly transmitted. Otherwise, the vehicle continues to monitor the channel until channel is detected to be idle for DIFS time period. Subsequently, according to the collision avoidance feature of the protocol, the vehicle generates an initial random backoff counter and goes through the backoff process before transmitting the packet. Moreover, a vehicle must go through the backoff process between two consecutive packet transmissions even if the channel is sensed idle for the duration of DIFS time for the second packet. Therefore, a packet can directly transmit without undergoing the backoff process only when the following two conditions are satisfied:

- The packet is generated when the queue is empty;
- The channel is sensed idle for DIFS time starting from the time instant that the packet is generated;

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Regarding the backoff process for a packet transmission, the initial backoff counter is chosen randomly from a uniform density over the range (0, W_0-1). The backoff time counter is decreased by one if the channel is sensed idle for a time slot σ. The counter is *frozen* when channel is sensed busy and reactivated when the channel is sensed idle again for more than the DIFS duration. The packet is transmitted as soon as the backoff counter reaches zero.

In broadcast services, the transmitting vehicle does not receive any feedback from the receivers and will not retransmit a packet. The detailed descriptions for IEEE 802.11 standard can be found in [11].

B. System assumptions

Several assumptions are made in the broadcast system to produce a simplified yet a high fidelity model.

- The vehicular ad hoc network is considered to be one-dimensional. The number of vehicles in a lane is Poisson distributed with parameter β (vehicle density), i.e., the probability $P(i, l)$ of finding i vehicles in a lane of length of l is given by:

$$P(i,l) = \frac{(\beta l)^i}{i!} \cdot e^{-\beta l} \tag{1}$$

- All vehicles have the same transmission range as well as receiving range R.
- Each vehicle is assumed to generate packets as a Poisson stream with rate λ (in packets per second).
- Each vehicle has an infinite queue to store the packets at the MAC layer. Hence, each vehicle can be modeled as an M/G/1 queue, which has exponentially distributed packet inter-arrival time (represented by M), an arbitrary distribution for service time (represented by G) and one server.

Due to the contention medium, the overall problem can be seen as a set of interacting M/G/1 queues. We simplify the problem by developing an SMP model for the tagged vehicle that does not directly keep track of the queued requests but captures the channel contention and backoff behavior. This SMP model interacts with the M/G/1 queue hence we need to use fixed-point iteration to solve the overall model.

III. ANALYTIC MODELS

A. SMP model

The behavior of a tagged vehicle for packet transmission can be characterized using the SMP model in Figure 1.

The tagged vehicle is in *idle* state if there is no packet in its queue. After a packet is generated, the vehicle senses channel activity for DIFS time period. If channel is detected not busy during this period (with probability 1-q_b), the vehicle goes from *idle* state to *XMT* state, which means that a packet is transmitting. Otherwise, the vehicle will randomly choose a backoff counter in the range (0, W_0-1). The backoff counter will be decreased by one if channel is detected to be idle for a time slot σ (with probability 1-p_b), which is captured by the transition from state W_0-i to state W_0-i-1. If the channel is busy during a backoff time slot σ (i.e., another vehicle is transmitting a packet), the backoff counter of the tagged vehicle will be suspended and deferred for the duration of packet transmission time T, which presents the transition from state W_0-i to D_{W_0-i-1} with probability p_b. When the backoff counter reaches zero, the packet will directly be transmitted (an SMP transition occurs from state *0* to state *XMT* with probability one). In *XMT* state, a packet is transmitting. After the packet transmission, if there is no packet left in the queue of the tagged vehicle (with probability 1-ρ), the vehicle will go from *XMT* to *idle* state and wait for a new incoming packet. If there are packets left in the queue after a packet transmission (with probability ρ),

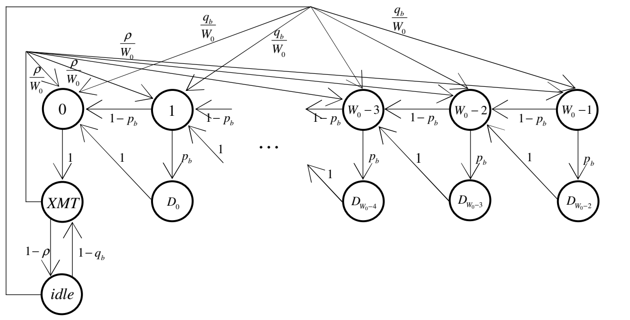

Figure 1. SMP model for IEEE 802.11 broadcast

the vehicle will sense the channel again for DIFS time and then randomly choose a backoff counter before transmitting the next packet.

Define the sojourn time in state j as T_j. The mean and variance of T_j in the SMP model are:

$$E\left[T_j\right] = \tau_j = \begin{cases} \sigma & j = 0,1,2,\cdots,W_0-1 \\ T & j = D_0, D_1, \cdots, D_{W_0-2} \\ T & j = XMT \\ \dfrac{1}{\lambda} + DIFS & j = idle \end{cases} \tag{2}$$

$$Var\left[T_j\right] = \theta_j^2 = \begin{cases} 0 & j = 0,1,2,\cdots,W_0-1 \\ Var[PA] & j = D_0, D_1, \cdots, D_{W_0-2} \\ Var[PA] & j = XMT \\ 1/\lambda^2 & j = idle \end{cases} \tag{3}$$

where T=$E[PA]$+T_H+$DIFS$+δ. The mean and variance of the packet length are $E[PA]$ and $Var[PA]$ respectively. T_H presents the packet header including physical layer header and MAC layer header. δ is the propagation delay. The parameters above are transferred to time unit for sojourn time calculation.

For the model in Figure 1, the embedded DTMC is first solved for its steady-state probabilities:

$$\begin{cases} v_j = (W_0 - j) \cdot v_{W_0-1} & j = 0,1,\cdots W_0-1 \\ v_{D_j} = (W_0 - j - 1) \cdot p_b \cdot v_{W_0-1} & j = 0,1,\cdots W_0-2 \\ v_{XMT} = \dfrac{W_0}{\rho + q_b(1-\rho)} \cdot v_{W_0-1} & \\ v_{idle} = \dfrac{(1-\rho)W_0}{\rho + q_b(1-\rho)} \cdot v_{W_0-1} & \end{cases} \tag{4}$$

$$v_{W_0-1}=\frac{2\left[\rho+q_b(1-\rho)\right]}{\left[W_0+1+p_b(W_0-1)\right]\left[\rho+q_b(1-\rho)\right]\cdot W_0+2(2-\rho)W_0} \tag{5}$$

Taking account of the mean sojourn time in each state, the steady-state probabilities of the SMP are given by:

$$\pi_i=\frac{v_i\tau_i}{\sum_j v_j\tau_j} \tag{6}$$

Therefore, the steady-state probability that a vehicle is in the *XMT* state is given by:

$$\pi_{XMT}=\frac{2T}{\left[\rho+q_b(1-\rho)\right]\left[(\sigma+p_b\cdot T)W_0+(\sigma-p_b\cdot T)\right]+2T+2(1-\rho)(1/\lambda+DIFS)} \tag{7}$$

Although the sojourn time in *XMT* state is T, the real packet transmission only occupies a portion of this sojourn time, which is $E[PA]+T_H+\delta=T-DIFS$. Hence, the probability that a vehicle transmits in steady state is $\pi_{XMT}(T-DIFS)/T$.

In Equation (7), three unknown parameters are:

- ρ: the probability that there are packets in the queue of the tagged vehicle.
- p_b: the probability that the channel is detected busy in one time slot by the tagged vehicle.
- q_b: the probability that the channel is detected busy in DIFS time by the tagged vehicle.

In Section III.C, we will see that p_b and q_b are functions of ρ, whereas ρ depends on the mean service time to transmit a packet. Therefore, the service time is derived first in the next subsection. Section III.C subsequently illustrates the relationships between these parameters and fixed point iteration algorithm is utilized to compute the numerical results for these parameters as well as the service time.

B. Service time computation

As mentioned above, each vehicle in the network can be modeled as an M/G/1 queue. The MAC layer service time is defined as the time interval from the time instant when a packet becomes the head of the queue and starts to contend for transmission, to the time instant when the packet is received.

The SMP model in Section III.A describes the behavior of a tagged vehicle continuously transmitting packets in its queue. In this section, the service time for any one packet in the queue needs to be derived. Therefore, the SMP model in Section III.A can be modified to contain an absorbing state as shown in Figure 2. By properly allocating the initial probability, the time to reach the absorbing state will be the service time for a packet transmission.

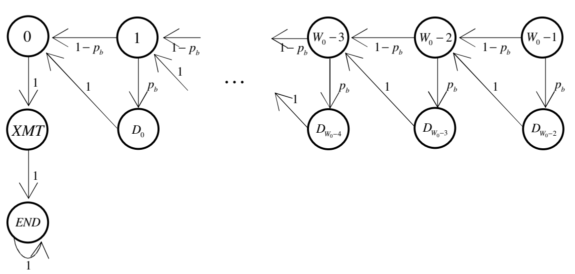

Figure 2. SMP model for the service time computation

For a newly generated packet in the tagged vehicle, if there is no packet in the queue and the channel is sensed idle for DIFS time with probability $(1-\rho)(1-q_b)$, the packet will be directly transmitted. In other words, the initial probability that a packet starts its transmission in *XMT* state is $(1-\rho)(1-q_b)$. Otherwise, the vehicle will randomly choose a backoff counter before the packet transmission. Therefore, the initial probability that a packet starts its transmission in state i ($i=0,1,\dots,W_0-1$) is $[1-(1-\rho)(1-q_b)]/W_0$. Hence, the initial probabilities for all states are:

$$q_i=\begin{cases}\left[1-(1-\rho)(1-q_b)\right]/W_0 & for\quad i=0,1,\dots,W_0-1\\ (1-\rho)(1-q_b) & for\quad i=XMT\\ 0 & for\quad i=D_0,D_1,\dots,D_{W_0-2}\end{cases} \tag{8}$$

Since the Markov chain contains an absorbing state, the transition probability matrix can be partitioned so that:

$$P=\begin{bmatrix}Q & C\\ 0 & 1\end{bmatrix} \tag{9}$$

where Q is a $2W_0$ by $2W_0$ sub-stochastic matrix describing the probabilities of transitions only among the transient states. The fundamental matrix is:

$$M=(I-Q)^{-1} \tag{10}$$

Let X_{ij} be the random variable denoting the visit counts to state j before entering the absorbing state, given that embedded DTMC started in state i. The expected number of visits to state j starting from state i before absorption is given by the $(i,j)^{th}$ element of the fundamental matrix M. Hence,

$$E\left[X_{ij}\right]=m_{ij} \tag{11}$$

Due to the acyclic nature of the DTMC model in Figure 2, the fundamental matrix can be easily obtained through the definition of X_{ij} instead of computing (10).

$$M=\left[m_{ij}\right]=\begin{array}{c c}
 & \begin{matrix}0 & 1 & 2 & \cdots & W_0-2 & W_0-1 & D_0 & D_1 & D_2 & \cdots & D_{W_0-2} & XMT\end{matrix}\\
\begin{matrix}0\\1\\2\\\vdots\\W_0-2\\W_0-1\\D_0\\D_1\\D_2\\\vdots\\D_{W_0-2}\\XMT\end{matrix} &
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1\\
1 & 1 & 0 & \cdots & 0 & 0 & p_b & 0 & 0 & \cdots & 0 & 1\\
1 & 1 & 1 & \cdots & 0 & 0 & p_b & p_b & 0 & \cdots & 0 & 1\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
1 & 1 & 1 & \cdots & 1 & 0 & p_b & p_b & p_b & \cdots & 0 & 1\\
1 & 1 & 1 & \cdots & 1 & 1 & p_b & p_b & p_b & \cdots & p_b & 1\\
1 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 1\\
1 & 1 & 0 & \cdots & 0 & 0 & p_b & 1 & 0 & \cdots & 0 & 1\\
1 & 1 & 1 & \cdots & 0 & 0 & p_b & p_b & 1 & \cdots & 0 & 1\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
1 & 1 & 1 & \cdots & 1 & 0 & p_b & p_b & p_b & \cdots & 1 & 1\\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1
\end{bmatrix}\end{array} \tag{12}$$

Furthermore, the variance of the number of visits can also be derived using the fundamental matrix. Define $M_D=[md_{ij}]$ by:

$$md_{ij}=\begin{cases}m_{ij} & if\quad i=j\\ 0 & otherwise\end{cases} \tag{13}$$

Define $M_2=[m_{ij}^2]$. Hence, the variance of the visit counts is:

$$\sigma^2=M\left(2M_D-I\right)-M_2 \tag{14}$$

The service time for a packet transmission starting from state i is given by:

$$S_i=\sum_j T_j\cdot X_{ij} \tag{15}$$

$$E[S_i]=E\left[\sum_j T_j\cdot X_{ij}\right]=\sum_j E\left[T_j\cdot X_{ij}\right]=\sum_j E\left[T_j\right]\cdot E\left[X_{ij}\right]=\sum_j \tau_j\cdot m_{ij}$$
$$=\begin{cases}(i+1)\sigma+i\cdot p_b\cdot T+T & for \quad i=0,1,\cdots,W_0-1\\ T & for \quad i=XMT\end{cases} \tag{16}$$

Since the sojourn time in state *0* is zero in the protocol instead of σ as specified in the model, we adjust the mean of S_i starting from i=0,1,..., W_0-1 by decreasing σ in the results. Hence, we obtain:

$$E[S_i]=\begin{cases}i\cdot\sigma+i\cdot p_b\cdot T+T & for \quad i=0,1,\cdots,W_0-1\\ T & for \quad i=XMT\end{cases} \tag{17}$$

The variance of S_i is given by (18).

$$Var[S_i]=Var\left[\sum_j T_j\cdot X_{ij}\right]=\sum_j Var\left[T_j\cdot X_{ij}\right]$$
$$=\sum_j\left\{Var\left[T_j\right]\cdot E\left[X_{ij}\right]+\left(E\left[T_j\right]\right)^2\cdot Var\left[X_{ij}\right]\right\}$$
$$=\sum_j\left(\theta_j^2\cdot m_{ij}+\tau_j^2\cdot\sigma_{ij}^2\right) \tag{18}$$
$$=\begin{cases}i\left\{Var[PA]\cdot p_b+T^2\cdot p_b\cdot(1-p_b)\right\}+Var[PA] & for \quad i=0,1,\cdots,W_0-1\\ Var[PA] & for \quad i=XMT\end{cases}$$

The service time for a packet transmission without conditioning on the starting state is presented as follows.

$$S=\begin{cases}S_0 & with \quad probability \quad \left[1-(1-\rho)(1-q_b)\right]/W_0\\ S_1 & with \quad probability \quad \left[1-(1-\rho)(1-q_b)\right]/W_0\\ \quad\vdots & \\ S_{W_0-1} & with \quad probability \quad \left[1-(1-\rho)(1-q_b)\right]/W_0\\ S_{XMT} & with \quad probability \quad (1-\rho)(1-q_b)\end{cases} \tag{19}$$

Therefore, the mean and variance of the service time are given by (20) and (21), respectively.

$$E[S]=\sum_i E[S_i]\cdot q_i=\frac{(\sigma+p_b\cdot T)\left[1-(1-\rho)(1-q_b)\right](W_0-1)}{2}+T \tag{20}$$

$$Var[S]=E\left[S^2\right]-\left(E[S]\right)^2=\sum_i E\left[S_i^2\right]\cdot q_i-\left(E[S]\right)^2$$
$$=\sum_i\left\{Var[S_i]+\left(E[S_i]\right)^2\right\}\cdot q_i-\left(E[S]\right)^2 \tag{21}$$

C. *Fixed point iteration*

In the previous section, the mean service time is shown to depend on three unknown parameters ρ, p_b and q_b, whereas ρ depends on the mean service time according to the M/G/1 queue behavior. Therefore, relationships between ρ, p_b and q_b are determined first in this section, and then the fixed point iteration algorithm is used to obtain the final solution.

Let N_{cs} denote the average number of vehicles in carrier sensing range of the tagged vehicle, whereas N_{tr} denote the average number of vehicles in transmission range of the tagged vehicle. Hence, without loss of generality, we have

$$N_{cs}=N_{tr}=2\beta R \tag{22}$$

The average number of vehicles in potential hidden area is:

$$N_{ph}=4\beta R-N_{cs}=2\beta R \tag{23}$$

From the tagged vehicle's point of view, p_b is the probability that it senses channel busy during one time slot in the backoff process. Since the channel is detected busy if there is at least one neighbor (i.e., a vehicle in the transmission range of the tagged vehicle) transmitting in a backoff time slot of the tagged vehicle, we have

$$p_b=1-\sum_{i=0}^{\infty}\left(1-P_{XMT}\right)^i\frac{\left(N_{tr}\right)^i}{i!}e^{-N_{tr}}=1-e^{-N_{tr}\cdot P_{XMT}} \tag{24}$$

where P_{XMT} is the probability that a neighbor is transmitting in a backoff time slot of the tagged vehicle, to be derived next.

Equation (7) shows that the probability that a vehicle transmits a packet in steady state is π_{XMT}(*T-DIFS*)/*T*. In addition, the time to transmit a packet is *T-DIFS*. Therefore, we can abstractly define the total time to be T_{total} as shown in Figure 3. Hence, π_{XMT}(*T-DIFS*)/*T* =(*T-DIFS*)/ T_{total}.

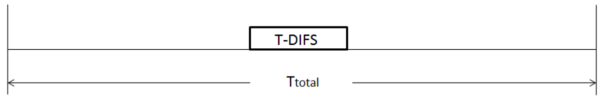

Figure 3. Abstraction of the packet transmission time

Suppose a neighbor of the tagged vehicle transmits a packet as shown in Figure 3 in time duration T_{total}, a backoff time slot of the tagged vehicle can occupy any one time slot within T_{total}.

For the first backoff time slot of the tagged vehicle, the time duration that can capture the transmission of the neighbor is *T-DIFS*+2σ. One extra time slot σ is the one just before transmission and another is the one just after transmission, which can capture the starting time instant and ending time instant of the packet transmission. Therefore, the probability that a neighbor's transmission is detected in the first backoff time slot of the tagged vehicle is π_{XMT}(*T-DIFS*+2σ)/*T*.

For a backoff time slot that is not the first backoff time slot of the tagged vehicle, the time duration that captures the transmission of the neighbor is 2σ, which captures the starting time instant of the transmission. This is because when the neighbor's transmission is detected in the first backoff time slot by the tagged vehicle, the backoff counter will suspend and wait until the end of this transmission for further decrement. Therefore, if the first backoff time slot detects the transmission, there is no chance for the later backoff time slots to detect the same transmission. As a result, the non-first backoff time slot can only detect the transmission when the starting point of the transmission falls within this time slot. Therefore, the probability that a neighbor's transmission is detected in non-first backoff time slot of the tagged vehicle is $\pi_{XMT}\cdot 2\sigma/T$.

Since the probability that a backoff time slot is the first backoff time slot is 1/W_0 and non-first backoff time slot is (1-1/W_0), the probability that a neighbor's transmission is detected by a backoff time slot of the tagged vehicle is:

$$P_{XMT}=\frac{1}{W_0}\cdot\frac{T-DIFS+2\sigma}{T}\pi_{XMT}+\left(1-\frac{1}{W_0}\right)\cdot\frac{2\sigma}{T}\pi_{XMT} \tag{25}$$

Next, q_b denotes the probability that channel is detected busy by the tagged vehicle in the DIFS duration. Therefore, we can similarly define P_{XMT}' to be the probability that a neighbor's transmission is detected in the DIFS duration by the tagged vehicle.

$$P_{XMT}' = \frac{T - DIFS + 2DIFS}{T}\pi_{XMT} = \frac{T + DIFS}{T}\pi_{XMT} \qquad (26)$$

Hence, q_b is given by (27).

$$q_b = 1 - \sum_{i=0}^{\infty}(1 - P_{XMT}')^i \frac{(N_{tr})^i}{i!}e^{-N_{tr}} = 1 - e^{-N_{tr}\cdot P_{XMT}'} \qquad (27)$$

Based on Equations (24)-(27), q_b is expressed in terms of p_b to simplify the iteration algorithm:

$$q_b = 1 - (1 - p_b)^{\frac{(T+DIFS)W_0}{T-DIFS+2\sigma W_0}} \qquad (28)$$

From the above analysis of the relationship between two parameters ρ and p_b (q_b can be expressed in terms of p_b), we notice that p_b depends on ρ and p_b itself. Hence, we denote $p_b=g(\rho,p_b)$ and the reciprocal of mean service time for M/G/1 queue to be $\mu=h(\rho,p_b)$. The fixed point iteration algorithm is outlined as follows according to the import graph in Figure 4.

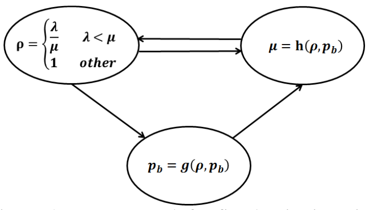

Figure 4. Import graph for fixed point iteration

Fixed point iteration algorithm:

Step 1: Initialize ρ=1, which is the saturation condition;

Step 2: With ρ, solve p_b according to (24)(25)(7)(28);

Step 3: With ρ and p_b, calculate service rate μ=1/E[S] according to (20);

Step 4: If $\lambda<\mu$, $\rho=\lambda/\mu$; otherwise, ρ=1;

Step 5: If ρ converges to the previous value, then stop the iteration algorithm; otherwise, go to step 2 with the updated ρ.

By utilizing the fixed point iteration algorithm, the parameters ρ, p_b, q_b, π_{XMT} as well as the mean and the variance of the service time can be determined, which are used for performance indices computation in the next section.

IV. PERFORMANCE INDICES

A. Transmission delay

The packet transmission delay is defined as the average delay a packet experiences from the time the packet is generated, until the time the packet is successfully received by all neighbors of the vehicle that generates the packet. The transmission delay $E[D]$ includes the queuing delay and medium service time (due to backoff, packet transmission, and propagation delay, etc.)

The expected queuing delay can be obtained from the *Pollaczek-Khinchin* mean value formula of M/G/1 queue:

$$E\left[D_q\right] = \frac{\lambda\left(Var[S] + \left(E[S]\right)^2\right)}{2\left(1 - \lambda E[S]\right)} \qquad (29)$$

The average packet transmission delay is then calculated as:

$$E[D] = E\left[D_q\right] + E[S] \qquad (30)$$

B. Packet Delivery Ratio

The PDR is defined as: given a broadcast packet sent by the tagged vehicle, the probability that all vehicles in its transmission range receive the packet successfully. Taking account of hidden terminals, we have

$$PDR = P(N_{cs})P(N_{ph}) \qquad (31)$$

where $P(N_{cs})$ is the probability that no vehicles in the transmission range of the tagged vehicle transmits when the tagged vehicle starts transmission, and $P(N_{ph})$ is the probability that no transmissions from the vehicles in the potential hidden terminal area collide with the broadcast packet from the tagged vehicle.

$P(N_{cs})$ can also be interpreted as the no-concurrent transmission probability, i.e., two packets do not start transmission at the same time. Since DCF employs a discrete-time backoff scheme, if the backoff process is involved, a vehicle is only allowed to transmit at the beginning of each slot time after an idle DIFS. Therefore, if the tagged vehicle has not gone through the backoff process before transmitting the packet (with probability (1-ρ)(1-q_b)), the concurrent transmission will not occur. Otherwise, the packet transmission is synchronized to the beginning of a slot time, and concurrent transmission may occur if other vehicles' transmission is also synchronized by the backoff process. From the model, we know that the probability that a neighbor starts to transmit a packet at the beginning of the same time slot with the tagged vehicle is $\pi_0= \pi_{XMT}\cdot\sigma/T$. This is because the sojourn time in state *0* is one time slot σ as shown in the SMP model, hence, π_0 is the probability that a vehicle starts to transmit in the beginning of a time slot immediately after the backoff process. Hence, $P(N_{cs})$ is:

$$\begin{aligned} P(N_{cs}) &= \left[1-(1-\rho)(1-q_b)\right]\cdot\left[\sum_{i=0}^{\infty}(1-\pi_0)^i\frac{(N_{cs}-1)^i}{i!}e^{-(N_{cs}-1)}\right] \\ &\quad + (1-\rho)(1-q_b) \\ &= \left[1-(1-\rho)(1-q_b)\right]\cdot e^{-(N_{cs}-1)\pi_0} + (1-\rho)(1-q_b) \end{aligned} \qquad (32)$$

Since the transmission time for a packet is T-$DIFS$=E[PA]+T_H+δ, the event that transmission from hidden terminals collides with the tagged vehicle's transmission only happens when hidden terminals start to transmit during the vulnerable period 2(T-$DIFS$)=2(E[PA]+T_H+δ) [6]. Using $\pi_{XMT}=T/T_{total}$ as an abstraction of the steady state behavior shown in Figure 3, the probability that a vehicle starts to transmit during the vulnerable period is:

$$\frac{2(T - DIFS)}{Ttotal} = \pi_{XMT}\frac{2(T - DIFS)}{T} \qquad (33)$$

Therefore,

$$\begin{aligned} P(N_{ph}) &= \sum_{i=0}^{\infty}\left(1 - \pi_{XMT}\cdot\frac{2(T-DIFS)}{T}\right)^i\frac{(N_{ph})^i}{i!}e^{-N_{ph}} \\ &= e^{\frac{-2(T-DIFS)\cdot N_{ph}\cdot\pi_{XMT}}{T}} \end{aligned} \qquad (34)$$

V. NUMERICAL AND SIMULATION RESULTS

The computation for analytic models and corresponding simulations are conducted in Matlab. Table 1 shows the parameters used in this paper, which reflect typical DSRC

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network settings in [1]. Figures 5 and 6 present the mean transmission delay and PDR, respectively, vs. the vehicle density β (# vehicles per meter), data rate R_d (Mbps), packet arrival rate λ (packets per second) and average packet length $E[PA]$ (bytes).

Table 1. Parameters

Parameters	Values	Parameters	Values
Tx range R	500 m	Propagation delay δ	0 μs
Average Packet Length $E[PA]$	variable	Variance of Packet Length $Var[PA]$	0
PHY preamble	40 μs	PLCP header	4 μs
MAC header	272 *bits*	CWMin W_0-1	15
Packet arrival rate λ	variable	Vehicle density β	variable
Slot time σ	16 μs	DIFS	64 μs

Figures 5 and 6 show that the analytic results from the model have better match with the simulation results than those from the model in [6]. The 95% confidence intervals are shown in the figures.

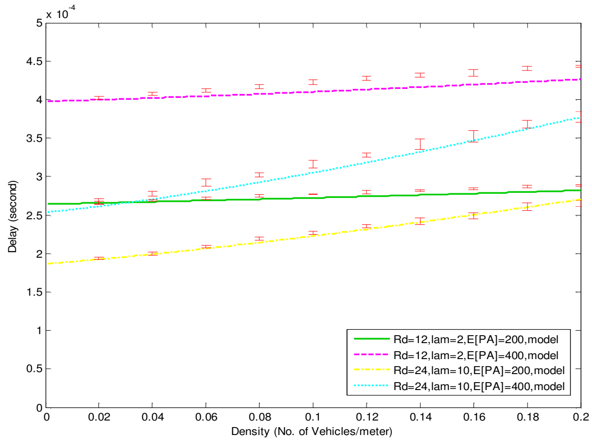

Figure 5. Delay of DSRC Highway safety messaging

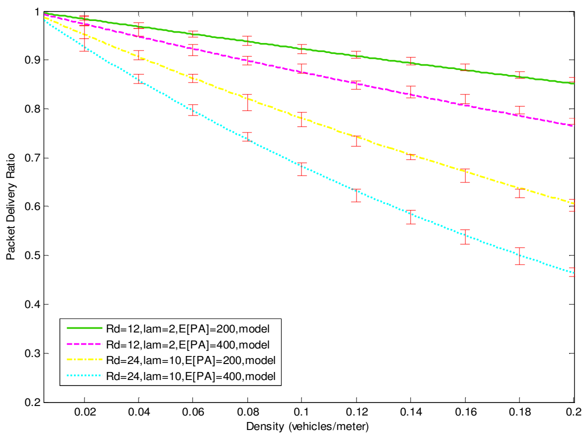

Figure 6. PDR of DSRC Highway safety messaging

Since the SMP model considers the fact that a packet can be directly transmitted without undergoing backoff process, the delay is lower compared with [6]. Another observation in Figure 5 is that high data rate and shorter packet length facilitate the decrease of the transmission delay.

The PDR decreases fast as the density β increases as shown in Figure 6. Similar to the delay, PDR also benefits from high data rate and short packet length.

VI. CONCLUSION AND FUTURE WORK

In this paper, a more general and accurate analytic model using SMP has been developed to characterize the behavior of DSRC for highway safety communications. The model is cross validated against simulations. Moreover, the performance with different input parameters is analyzed to suggest better parameter settings that will improve the performance by decreasing the delay and increasing PDR. In future, performance optimization will be conducted for more parameters including W_0. The tradeoff between delay and PDR will be evaluated based on optimization results. In addition, the SMP model will be extended to incorporate different packet arrival processes such as Markov modulated Poisson process (MMPP), Markov arrival process (MAP) instead of Poisson arrival. Besides one-hop direct broadcast transmission strategy, multi-hop and multi-cycle transmission strategy will also be considered in future.

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