Blocking Equalization in the Erlang Multirate Loss Model for Elastic Traffic

Ioannis D. Moscholios
Dept. of Telecommunications Science and Technology,
University of Peloponnese,
22 100, Tripoli, Greece
idm@uop.gr

Vassilios G. Vassilakis, Michael D. Logothetis
WCL, Dept. of Electrical & Computer Engineering,
University of Patras,
26 504, Patras, Greece
vasilak@wcl.ee.upatras.gr
m-logo@wcl.ee.upatras.gr

Anthony C. Boucouvalas
Dept. of Telecommunications Science and Technology,
University of Peloponnese,
22 100, Tripoli, Greece
acb@uop.gr

***Abstract*—We consider a single-link loss system of fixed bandwidth capacity, which accommodates *K* service-classes of Poisson traffic with different bandwidth-per-call requirements. Depending on the occupied link bandwidth, in-service calls can tolerate bandwidth compression while increasing their service time (elastic calls). In this system, we study the effect of the bandwidth reservation (BR) policy on various performance measures and mainly on Call Blocking Probabilities (CBP). The BR policy can achieve CBP equalization among service-classes, or, alternatively, guarantee a certain quality of service for each service-class. We provide a recurrent formula for the calculation of the link occupancy distribution. Based on it we determine CBP, link utilization and average number of calls in the system. The accuracy of the proposed formula is verified by simulation and is found to be quite accurate.**

Keywords-loss system; blocking probability; reservation; elastic traffic; Markov chain.

I. Introduction

The classical Erlang Multi-rate Loss Model (EMLM) is used to analyze the call blocking behavior of a single-link loss system that accommodates *K* service-classes with different and fixed bandwidth-per-call requirements. Calls of each service-class arrive to the system according to a Poisson process and compete for the available link bandwidth under the complete sharing (CS) policy (calls of all service-classes compete for all available bandwidth resources). Calls are blocked and lost only if their required bandwidth is higher than the available link bandwidth. Otherwise, they are accepted in the system for a generally distributed service time [1]. Note that while in service, calls cannot alter their assigned bandwidth.

In the EMLM, exploiting the fact that the steady state distribution of the number of calls in the link has a product form solution (PFS) [2], an accurate recursive formula (known as Kaufman-Roberts formula, KR formula) has been separately proposed by Kaufman [1] and Roberts [3] which determines the link occupancy distribution and simplifies the determination of call blocking probabilities (CBP). This simplification resulted in a large amount of extensions of the EMLM and applications of the KR formula both in wired (e.g., [4]-[7]) and wireless networks (e.g., [8]-[11]). Among other EMLM extensions, Roberts proposed in [12] an approximate recursive formula for calculating CBP in the EMLM under the Bandwidth Reservation policy (EMLM/BR). The BR policy is used in order to achieve CBP equalization among service-classes, or guarantee a certain quality of service (QoS) for each service-class. Note that contrary to the CS policy where the stationary probabilities have a PFS, the BR policy cannot be analyzed by the use of a PFS. This is because one-way transitions appear in the state space, which destroy reversibility [13].

In this paper, we apply the BR policy and study its effects in another extension of the EMLM proposed in [14] by Stamatelos and Koukoulidis who incorporate elastic traffic in the EMLM. We name, herein, the model of [14] Extended EMLM (E-EMLM) and our proposed model E-EMLM/BR. In the E-EMLM, calls of each service-class arrive to the system according to a Poisson process with different and elastic bandwidth-per-call requirements. As long as the occupied link bandwidth does not exceed the capacity of the link, all in-service calls use their peak-bandwidth requirement. When a new call arrives and its required peak-bandwidth is higher than the available link bandwidth, the system accepts the new call (contrary to the EMLM where this call is blocked) by compressing not only the bandwidth of all in-service calls (of all service-classes) but also the initial peak-bandwidth of the new call. On the other hand when an in-service call, whose bandwidth is compressed, departs from the system then the remaining in-service calls (of all service-classes) expand their bandwidth. It is worth mentioning that the allocated bandwidth to elastic in-service calls alters (becomes compressed or expanded) in proportion to their peak-bandwidth requirement and that their service time is adjusted accordingly (becomes expanded or compressed) so that the product (service time) by (bandwidth per call) remains constant. A new call is blocked and lost when the compressed bandwidth should be less than a minimum proportion ($r_{\min}$) of its required peak-bandwidth. Note that $r_{\min}$ is common for all service-classes.

The compression/expansion of bandwidth destroys reversibility in the E-EMLM and therefore no PFS exists. However, in [14] an approximate recursive formula is proposed which determines the link occupancy distribution. Before we proceed to the E-EMLM/BR note that extensions of the E-EMLM which study the co-existence of elastic and adaptive traffic (in-service calls can alter their bandwidth but not their service time) can be found in [15], [16]. Potential applications of the E-EMLM (and the E-EMLM/BR) are mainly in emerging wireless networks supporting elastic traffic (e.g., [17], [18]).

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Since the proposed E-EMLM/BR does not have a PFS we provide an approximate recursive formula for the calculation of the link occupancy distribution that simplifies the determination of various performance measures including: a) CBP, b) link utilization, c) average number of calls of each service-class in the system and d) delay of calls due to their bandwidth fluctuation.

The remainder of this paper is as follows: In Section II, we review the EMLM, the EMLM/BR and the E-EMLM. In Section III, we propose the E-EMLM/BR. In Section IV, we present numerical results where the new model is compared to the existing models and evaluated through simulation results. We conclude in Section V.

II. Review of The EMLM, EMLM/BR and E-EMLM

A. Review of the EMLM

Consider a link of capacity C bandwidth units (b.u.) that accommodates calls of K service-classes. A call of service-class k (k =1,…,K) arrives in the system according to a Poisson process with rate λ_k, requests b_k b.u. and if these b.u. are available it remains in the system for an exponentially distributed service time with mean μ_k^{-1}. While in service, the call cannot alter its assigned bandwidth. If the b_k b.u. are not available the call is blocked and lost. Let j be the occupied link bandwidth (j=0,…,C) then the link occupancy distribution, $G(j)$, is given by the accurate and recursive KR formula [1] ,[3]:

$$G(j) = \left\langle \begin{array}{c} 1 \quad for\ j=0 \\ \frac{1}{j}\sum_{k=1}^{K} a_k\, b_k\, G(j-b_k)\ for\ j=1,...,C \\ 0 \quad otherwise \end{array} \right\rangle \tag{1}$$

where: $\alpha_k = \lambda_k \mu_k^{-1}$ is the offered traffic load of service-class k calls.

The proof of (1) is based on the fact that the steady state distribution of the number of calls in the link has a PFS. If n_k is the number of calls of service-class k in the steady state and $\boldsymbol{n}$=(n_1, n_2,…,n_k,…, n_K) then the steady state distribution, $P(\boldsymbol{n})$, is given by [2]:

$$P(\boldsymbol{n}) = G^{-1}\left(\prod_{k=1}^{K}\frac{a_k^{n_k}}{n_k!}\right) \tag{2}$$

where: G is the normalization constant given by $G \equiv G(\boldsymbol{\Omega})$ $= \sum_{\boldsymbol{n}\in\boldsymbol{\Omega}}\left(\prod_{k=1}^{K}\frac{a_k^{n_k}}{n_k!}\right)$ and $\boldsymbol{\Omega}=\{\boldsymbol{n}:0 \le \boldsymbol{nb} \le C\}$ is the state space with $\boldsymbol{b}$=(b_1, b_2,…,b_k,…, b_K) and $j = \boldsymbol{nb} = \sum_{k=1}^{K} n_k b_k$.

Having determined the values of $G(j)$'s we can calculate various performance measures, including:

1) The CBP of service-class k, denoted as B_k, is calculated by the formula:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1}G(j) \tag{3}$$

where $G = \sum_{j=0}^{C} G(j)$ is the normalization constant.

2) the link utilization, denoted as U:

$$U = \sum_{j=1}^{C} jG(j) \tag{4}$$

3) The average number of service k calls in the system, denoted as $\bar{n}_k$:

$$\bar{n}_k = \sum_{j=1}^{C} y_k(j)G(j) \tag{5}$$

where $y_k(j)$ is the average number of service-class k calls given that the system state is j, and can be determined by (proof is similar to [15] and thus is omitted):

$$y_k(j) = \frac{1}{jG(j)}\left[a_k b_k G(j-b_k)(1+y_k(j-b_k))\right] + \frac{1}{jG(j)}\sum_{\substack{i=1\\ i\neq k}}^{K} a_i b_i G(j-b_i)\, y_k(j-b_i) \tag{6}$$

where j = 1,…,C while $y_k(x)$ = 0 for $x \le 0$ and k = 1, …, K.

B. Review of the EMLM/BR

If we apply the BR policy to the EMLM according to Roberts [12], then the formula for the approximate calculation of $G(j)$ takes the form:

$$G(j) = \left\langle \begin{array}{c} 1 \quad for\ j=0 \\ \frac{1}{j}\sum_{k=1}^{K} a_k\, D_k(j-b_k) G(j-b_k)\ for\ j=1,...,C \\ 0 \quad otherwise \end{array} \right\rangle \tag{7}$$

where: $D_k(j-b_k) = \left\langle \begin{array}{l} b_k \ \ for\ j \le C-t(k) \\ 0 \ \ for\ j > C-t(k) \end{array} \right\rangle$ (8)

and $t(k)$ is the reserved bandwidth (BR parameter) for service-class k calls.

Note that (7) is recursive, although the EMLM/BR is a non PFS model. This feature is based on the assumption (approximation) that calls of service-class k do not exist (are negligible) in states $j > C-t(k)$ and is incorporated in (7) by the variable $D_k(j-b_k)$ of (8). The BR policy is used to attain

CBP equalization among different service-classes that share a link by a proper selection of the BR parameters. If, for example, CBP equalization is required between calls of two service-classes with b_1=1 and b_2=10 b.u., respectively, then $t(1)$ = 9 b.u and $t(2)$ = 0 b.u. so that $b_1 + t(1) = b_2 + t(2)$. Note that $t(1)$ = 9 b.u means that 9 b.u. are reserved to benefit calls of the 2nd service-class. The CBP of service-class k, B_k, in the EMLM/BR is given by:

$$B_k = \sum_{j=C-b_k-t(k)+1}^{C} G^{-1}G(j) \qquad (9)$$

If $t(k) = 0$ for all k (k=1,…,K) then the EMLM results.

Having obtained the values of $G(j)$'s according to (7) we calculate the link utilization and the average number of service-class k calls in the system according to (4) and (5), respectively.

C. Review of the E-EMLM

Consider again a link of capacity C b.u. that accommodates calls of K service-classes. A call of service-class k (k =1,…,K) arrives in the system according to a Poisson process with arrival rate λ_k and requests b_k b.u. (peak-bandwidth requirement). If $j + b_k \leq C$, the call is accepted in the system with its peak-bandwidth requirement and remains in the system for an exponentially distributed service time with mean μ_k^{-1}. If $T \geq j + b_k > C$ the call is accepted in the system by compressing not only its peak-bandwidth requirement but also the assigned bandwidth of all in-service calls. The compressed bandwidth of the new service-class k call is:

$$b_k^{'} = rb_k = \frac{C}{j^{'}} b_k \qquad (10)$$

where $r \equiv r(\boldsymbol{n}) = C/j'$, $j^{'} = j + b_k = \boldsymbol{nb} + b_k$ and T is the limit (in b.u.) up to which bandwidth compression is permitted.

Similarly, the bandwidth of all in-service calls will be compressed and become equal to $b_i^{'} = \frac{C}{j^{'}} b_i$ for i = 1,…,K. After the compression of both the new call and the in-service calls the state of the system is $j = C$. The minimum bandwidth that a call of service-class k (either new or in-service) can tolerate is given by the expression:

$$b_{k,\min}^{'} = r_{\min} b_k = \frac{C}{T} b_k \qquad (11)$$

where $r_{\min} = C/T$ is the minimum proportion of the required peak-bandwidth and is common for all service-classes.

This means that if upon arrival of a service-class k call, with peak-bandwidth requirement b_k b.u., we have $j' = j + b_k > T$ (or equivalently, $j^{'} > T$ or $C/j' < r_{\min}$) then the call is blocked and lost without further affecting the system.

After the bandwidth compression, calls increase their service time so that the product (service time) by (bandwidth per call) remains constant. Thus, due to bandwidth compression calls of service-class k may remain in the system more than μ_k^{-1} time units. Increasing the value of T, decreases $r_{\min}$ and increases the delay of calls of service-class k (compared to the initial service time μ_k^{-1}). Therefore the value of T can be chosen so that this delay remains within acceptable levels.

To illustrate the previous compression mechanism consider the following simple example. Let C = 3 b.u., T = 5. b.u., K = 2 service-classes, $\alpha_1 = \alpha_2$ = 1 erl, b_1 = 1 b.u., b_2 = 2 b.u and $\mu_1^{-1} = \mu_2^{-1} = 1$ time unit. The permissible states $\boldsymbol{n} = (n_1, n_2)$ of the system are 12 and are presented in Table I together with the occupied link bandwidth, $j=n_1b_1+n_2b_2$, before and after compression has been applied. Note that compression is applied if $T \geq j > C$ (bold values of the 3rd column of Table I). After compression has been applied, we have that $j = C$ (bold values of the 4th column of Table I). For example, assume that a new 2nd service-class call arrives while the system is in state (n_1, n_2) = (1, 1) and $j = C$ = 3 b.u. The new call is accepted in the system, since $j^{'} = j + b_2 = T$ = 5 b.u., after bandwidth compression has been applied to all calls (new and in-service calls). The new state of the system is now (n_1, n_2) = (1, 2). In this state, and based on (11), calls of the 1st and 2nd service-class compress their bandwidth to:

$b_{1,\min}^{'} = r_{\min} b_1 = \frac{3}{5} b_1 = 0.6$, $b_{2,\min}^{'} = r_{\min} b_2 = \frac{3}{5} b_2 = 1.2$ so that $j = n_1 b_{1,\min}^{'} + n_2 b_{2,\min}^{'} = 0.6 + 2.4 = 3 = C$. Similarly, the values of μ_1^{-1}, μ_2^{-1} become $\frac{\mu_1^{-1}}{r_{\min}}, \frac{\mu_2^{-1}}{r_{\min}}$ so that $b_1\mu_1^{-1}$ and $b_2\mu_2^{-1}$ remain constant.

Consider now that the system is in state (n_1, n_2) = (1, 2) and a 2nd service-class call departs from the system. Then, its assigned bandwidth $b_{2,\min}^{'} = 1.2$ is shared to the remaining calls in proportion to their peak-bandwidth requirement. Thus, in the new state (n_1, n_2) = (1, 1) the 1st service-class call expands its bandwidth to $b_1^{'} = b_1 = 1$ b.u. and the 2nd service-class call to $b_2^{'} = b_2 = 2$ b.u. Thus, $j = n_1b_1+n_2b_2 = C$ = 3 b.u. Furthermore, the service times of both calls are decreased to their initial values $\mu_1^{-1} = \mu_2^{-1} = 1$ time unit.

The compression/expansion of bandwidth destroys reversibility in the E-EMLM and therefore no PFS exists. However, in [14] an approximate recursive formula is proposed which determines $G(j)$'s:

$$G(j) = \left\langle \begin{array}{ll} 1 & for \;\; j = 0 \\ \frac{1}{\min(j, C)} \sum_{k=1}^{K} a_k b_k G(j - b_k) & for \;\; j = 1,..., T \\ 0 & otherwise \end{array} \right\rangle \qquad (12)$$

Equation (12) is based on a reversible Markov chain which approximates the bandwidth compression/expansion mechanism of the E-EMLM, described above. The local balance equations of this Markov chain are of the form [14]:

$$\lambda_k P(\boldsymbol{n}_k^-) = n_k \mu_k \phi_k(\boldsymbol{n}) P(\boldsymbol{n}) \quad (13)$$

where $P(\boldsymbol{n}) = (n_1, n_2, \ldots, n_k, \ldots, n_K)$, $P(\boldsymbol{n}_k^-) = (n_1, n_2, \ldots, n_{k-1}, n_k-1, n_{k+1} \ldots, n_K)$ and $\varphi_k(\boldsymbol{n})$ is a state dependent factor which describes: i) the compression factor of bandwidth and ii) the increase factor of service time of service-class k calls in state $\boldsymbol{n}$, so that (service time) by (bandwidth per call) remains constant. In other words, $\varphi_k(\boldsymbol{n})$ has the same role with $r(\boldsymbol{n})$ in (10) or $r_{\min}$ in (11) but it may be different for each service-class. It is apparent now why the model of (12) approximates the E-EMLM. The values of $\varphi_k(\boldsymbol{n})$ are given by:

$$\phi_k(\boldsymbol{n}) = \begin{cases} 1 & , \textit{ when } \boldsymbol{nb} \le C \textit{ and } \boldsymbol{n} \textit{ in } \boldsymbol{\Omega} \\ \dfrac{x(\boldsymbol{n}_k^-)}{x(\boldsymbol{n})}, & \textit{ when } C < \boldsymbol{nb} \le T \textit{ and } \boldsymbol{n} \textit{ in } \boldsymbol{\Omega} \\ 0 & , \textit{ otherwise} \end{cases} \quad (14)$$

where $\boldsymbol{\Omega} = \{\boldsymbol{n}: 0 \le \boldsymbol{nb} \le T\}$ and $\boldsymbol{nb} = \sum_{k=1}^{K} n_k b_k$.

In (14), $x(\boldsymbol{n})$ is a state multiplier, associated with state $\boldsymbol{n}$, whose values, are chosen so that (13) holds, [14]:

$$x(\boldsymbol{n}) = \begin{cases} 1 & , \textit{ when } \boldsymbol{nb} \le C, \ \boldsymbol{n} \textit{ in } \boldsymbol{\Omega} \\ \dfrac{1}{C} \sum_{\boldsymbol{k}=1}^{\boldsymbol{K}} n_k b_k x(\boldsymbol{n}_k^-), & \textit{ when } C < \boldsymbol{nb} \le T, \ \boldsymbol{n} \textit{ in } \boldsymbol{\Omega} \\ 0 & , \textit{ otherwise} \end{cases} \quad (15)$$

Table II shows, for our simple example, the values of $r(\boldsymbol{n})$ (common for both service-classes), $\varphi_1(\boldsymbol{n})$ and $\varphi_2(\boldsymbol{n})$.

TABLE I. STATE SPACE AND OCCUPIED LINK BANDWIDTH

n_1	n_2	j (before compression) $0 \le j \le T$	j (after compression) $0 \le j \le C$
0	0	0	0
0	1	2	2
0	2	**4**	**3**
1	0	1	1
1	1	3	3
1	2	**5**	**3**
2	0	2	2
2	1	**4**	**3**
3	0	3	3
3	1	**5**	**3**
4	0	**4**	**3**
5	0	**5**	**3**

TABLE II. VALUES OF STATE DEPENDENT FACTORS

n_1	n_2	$r(\boldsymbol{n})$	$\varphi_1(\boldsymbol{n})$	$\varphi_2(\boldsymbol{n})$
0	0	1.00	1.00	1.00
0	1	1.00	1.00	1.00
0	2	0.75	0.00	0.75
1	0	1.00	1.00	1.00
1	1	1.00	1.00	1.00
1	2	0.60	0.75	0.5625
2	0	1.00	1.00	1.00
2	1	0.75	0.75	0.75
3	0	1.00	1.00	1.00
3	1	0.60	0.67	0.50
4	0	0.75	0.75	0.00
5	0	0.60	0.60	0.00

Having determined the values of $G(j)$'s we can calculate various performance measures, including:

1) The CBP of service-class k, B_k :

$$B_k = \sum_{j=T-b_k+1}^{T} G^{-1} G(j) \quad (16)$$

where $G = \sum_{j=0}^{T} G(j)$ is the normalization constant.

2) the link utilization, denoted as U :

$$U = \sum_{j=1}^{C} jG(j) + \sum_{j=\boldsymbol{C}+1}^{T} CG(j) \quad (17)$$

3) The average number of service-class k calls in the system, $\bar{n}_k$:

$$\bar{n}_k = \sum_{j=1}^{T} y_k(j) G(j) \quad (18)$$

where $y_k(j)$ is the average number of service-class k calls given that the system state is j, and is given by [15]:

$$y_k(j) = \frac{1}{min(j,C)G(j)} \left[a_k b_k G(j-b_k)(1 + y_k(j-b_k)) \right] + \frac{1}{min(j,C)G(j)} \sum_{\substack{i=1 \\ i \ne k}}^{K} a_i b_i G(j-b_i) y_k(j-b_i) \quad (19)$$

where $j = 1, \ldots, T$ while $y_k(x) = 0$ for $x \le 0$ and $k = 1, \ldots, K$.

4) The average delay of service-class k calls, denoted by D_k, given by Little's formula, [19]:

$$D_k = \frac{\bar{n}_k}{\lambda_k (1 - B_k)} \quad (20)$$

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As T increases, B_k decreases and D_k increases. Therefore, the choice of T can be a trade-off between B_k and D_k. Before we proceed to the application of the BR policy in the E-EMLM we give the accurate and approximate CBP results for our simple example in the E-EMLM, and the corresponding CBP results for the EMLM, when C = 3:

E-EMLM
Accurate CBP: B_1 = 17.48%, B_2 = 35.74%
Approx. CBP (based on (12), (16)): B_1=17.00%, B_2=36.04%
EMLM
Accurate CBP (based on (1), (3)): B_1= 25.00%, B_2 = 57.14%

In the E-EMLM, the accurate CBP results are based on the numerical calculation of the irreversible Markov chain. The comparison shows that even in a small example, the approximation of [14] is quite well. Furthermore, compared to the EMLM we see a substantial CBP decrease due to the existence of a compression/expansion mechanism.

III. THE E-EMLM UNDER THE BR POLICY

If we apply the BR policy to the E-EMLM (E-EMLM/BR) according to [12], then (12) takes the form:

$$G(j) = \left\langle \begin{array}{c} 1 \quad for\ j=0 \\ \frac{1}{\min(j,C)} \sum_{k=1}^{K} a_k D(j-b_k) G(j-b_k) \ for\ j=1,...,T \\ 0 \quad otherwise \end{array} \right\rangle \quad (21)$$

where: $$D_k(j-b_k) = \left\langle \begin{array}{l} b_k \ \ for\ j \le T-t(k) \\ 0 \ \ for\ j > T-t(k) \end{array} \right\rangle \quad (22)$$

and $t(k)$ is the reserved bandwidth (BR parameter) for service-class k calls.

The CBP of service-class k, B_k, in the E-EMLM/BR is given by:

$$B_k = \sum_{j=T-b_k-t(k)+1}^{T} G^{-1} G(j) \quad (23)$$

If $t(k)$ = 0 for all k (k=1,…,K) then the E-EMLM results. Having obtained the values of $G(j)$'s according to (22) we can calculate the link utilization, the average number of service-class k calls in the system and their average delay according to (18), (19) and (20), respectively. Note that in (19), $y_k(j)$ = 0 if $j>T-t(k)$ as (22) implies. The accurate and approximate equalized CBP results for our simple example in the E-EMLM/BR, and the corresponding equalized CBP results for the EMLM/BR, when C = 3 are:

E-EMLM/BR (t(1) = 1, t(2)=0)
Accurate CBP: $B_1 = B_2$ =32.34%
Approx. CBP (based on (21)-(23)): $B_1 = B_2$ = 31.71%
EMLM/BR (t(1) = 1, t(2)=0)
Accurate CBP: $B_1 = B_2$ = 54.5%
Approx. CBP (based on (7)-(9)): $B_1 = B_2$ = 52.0%

In the E-EMLM/BR and the EMLM/BR, the accurate CBP results are based on the numerical calculation of the corresponding irreversible Markov chains. In the E-EMLM/BR, the comparison between the accurate and approximate CBP results shows that even in this small example, the proposed formulas ((21)-(23)) are valid.

IV. EVALUATION

In this section, we compare the analytical CBP and link utilization results obtained by the EMLM, EMLM/BR, E-EMLM and E-EMLM/BR via a numerical example. Due to space limitations we present simulation CBP results (mean values of 7 runs) only for the E-EMLM and the E-EMLM/BR. Simulation is based on Simscript II.5 [20].

We consider a single link of capacity C = 60 b.u. that accommodates calls of two service-classes, with the following traffic characteristics:
1st service-class: α_1 = 24 erl, b_1 = 1 b.u.
2nd service-class: α_2 = 6 erl, b_2 = 4 b.u.

The value of T = 70, and $r_{\min} = C/T$=6/7 is the minimum proportion of the required peak-bandwidth. In the case of the BR policy, we choose t(1)=3 and t(2)=0 in order to achieve CBP equalization between the two service-classes since: $b_1 + t(1) = b_2 + t(2)$. In the x-axis of all figures, α_1 increases in steps of 1 erl while α_2 is constant. So Point 1 is (α_1, α_2) = (24.0, 6.0) while Point 8 is (α_1, α_2) = (31.0, 6.0). In Fig. 1 and 2, we present the analytical and the simulation CBP results of the 1st and the 2nd service-class calls, respectively, in the case of the E-EMLM. For comparison, we give the corresponding analytical CBP results of the EMLM. In Fig. 3, we present the analytical and simulation CBP results (equalized CBP) in the case of the E-EMLM/BR policy. For comparison, we give the corresponding analytical results for the EMLM/BR. All figures show that: i) analytical and simulation CBP results are very close and ii) the compression/expansion mechanism of the E-EMLM and the E-EMLM/BR, reduces the CBP compared to those obtained by the EMLM and the EMLM/BR, respectively. Finally in Fig.4, we present the link utilization (analytical results) for all models. The compression/expansion mechanism increases the link utilization since it decreases CBP.

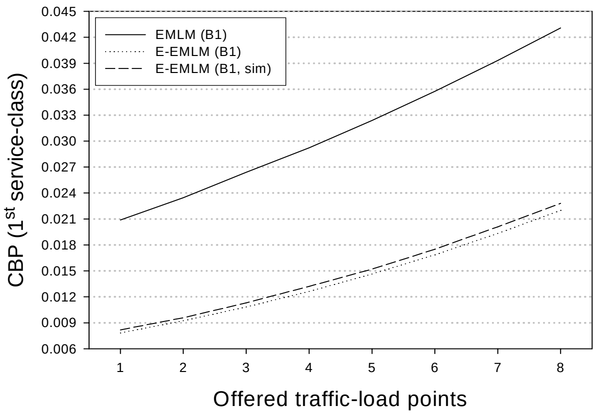

Figure 1. CBP of the 1st service-class (EMLM, E-EMLM).

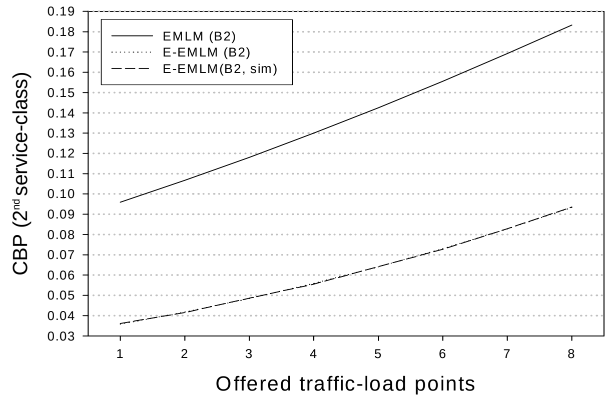

Figure 2. CBP of the 2nd service-class (EMLM, E-EMLM).

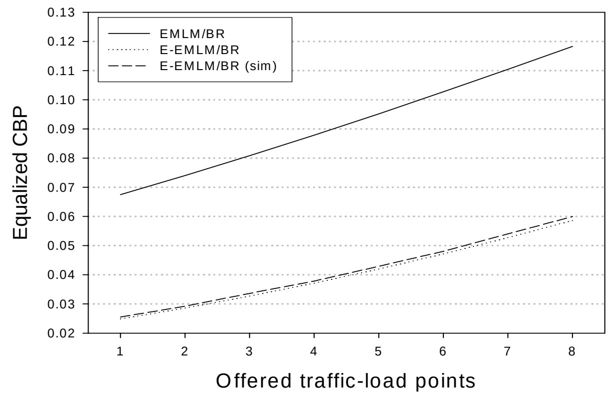

Figure 3. Equalized CBP (EMLM/BR, E-EMLM/BR).

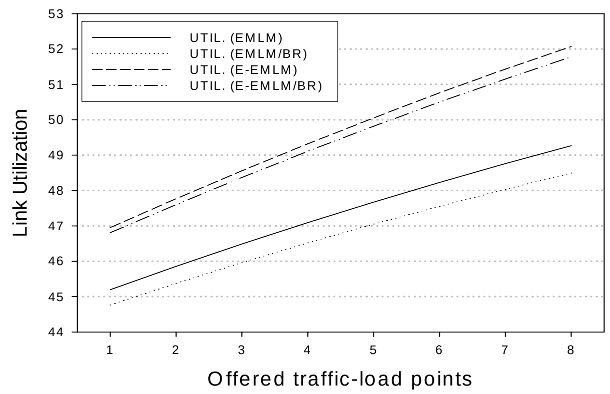

Figure 4. Link utilization for all models.

V. CONCLUSION

We propose an analytical model for the recursive calculation of various performance measures in the E-EMLM/BR. The BR policy guarantees a certain QoS among elastic calls of different service-classes. Simulation results verify the analytical results and prove the accuracy and the consistency of the proposed model. Potential applications of the proposed model are in emerging wireless networks that support elastic traffic. A future extension, is the application of our model in such networks based on the reduced load approximation method which has been extensively used for the CBP calculation in multirate loss networks.

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