Fuzzified Clustering and Point Set Continuous Approximation in Prognosticating Gastric Cancer Surgery

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Abstract— **We discuss two computational techniques in the current paper. In the first part, we aim at employing FCM (fuzzy c-means) clustering to compute membership degrees of two clusters providing decisions to perform surgery or not for a testing set of 25 gastric cancer patients. The second part handles mathematical modelling of a common function approximating the information obtained from the c-means procedure. After constructing the equation of the function, we can make the decision about the surgery in the form of the surgery degree for an arbitrary gastric cancer patient. A centre, dealing with mathematical techniques concerning surgery prognoses, can quickly decide about surgery for the patient who lives in a remote place. A transmission of information among the centre and some hospitals, interested in adopting the centre services, can facilitate surgery decision-making. This trial can be treated as a contribution in the telemedicine domain.**

Keywords–c-means clustering; surgery degrees; clinical characteristic value; weights of importance; truncated π-functions.

I. INTRODUCTION

Multidisciplinary cancer conferences play a very important role in decision-making process in modern treatment of gastric cancer patients. The aim of the conference is to establish assessments and treatment decisions for particular patients. The most discussed method of gastric cancer treatment is the partial or the total resection of the stomach, which makes the surgery decision so important.

To support the surgery decision-making, we develop different mathematical models, in which the entry data consist of the values of clinical markers, sampled during the examinations of the patients.

We have been provided with the clinical data of 25 gastric cancer patients, randomly selected and treated as a testing set. When designing the mathematical apparatus, we first intend to adapt the c-means method, separating patients in two sets named “degree of surgery” and “degree of no surgery”. A multidimensional data point-vector, consisted of values of the decisive biological markers, is assigned to each patient. The patient’s age, the *crp*-value (the C reactive proteins value) [1] and the body weight play a significant role in the surgery diagnosis.

The differentiation of patients-vectors in two classes is well done by putting forward the fuzzified version of cluster analysis [2]. Among clustering approaches, the fuzzy c-means clustering is regarded as the most efficient [2][3][4]. The earlier trials, involving fuzzy c-means clustering in surgery decision-making, were discussed by us in [5][6].

For each tested patient, the cluster matrix will deliver degrees of surgery and degrees of no surgery belonging to interval [0, 1].

After running the c-means algorithm, some characteristic values will be assigned to all patients tested. The characteristic values will combine measurements of biological markers with importance weights of these markers. Further, we will determine a set of points containing pairs (patient characteristics, degree of surgery). After inserting the points into the two-dimensional coordinate system, in accordance with ascending order of patient characteristic values, we will make a trial of approximating this set of points by the truncated version of the π-function [7][8]. The equation of the truncated π function makes possible to evaluate the surgery degree for an arbitrary patient. The approximation of the point set and prognoses, made for casual gastric cancer patients, constitute the paper’s second part, where our earlier and new theoretical contributions are sampled.

We cannot compare our results with other mathematical trials testing the operation decisions, since we have not found any traces of such trials in literature. A confrontation of our results with the physicians’ decisions is the only way to validate a proposed mathematical system.

Our further intention is to implement centralized computer programs. By spreading the effects of the program actions, we count on awaking some interest in other centres. These are expected to communicate with the main transmission station in order to obtain the support in surgery decision making. This trial of starting the communication by computers in the matter of surgery decision can be regarded as a contribution in the telemedicine domain.

In Section II we list the steps of fuzzy c-means clustering. The technique of generating entries in the initial matrix is discussed in Section III. Section IV provides us with surgery prognoses made for 25 patients tested. Characteristic values of patients are introduced in Section V.

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The values will be later involved in the procedure of approximation of points (characteristics of patient, degree of surgery) by a continuous function in Section VI. We conclude in Section VII.

II. FUZZY C-MEANS CLUSTERING ALGORITHM

Let us recall the definition of a fuzzy set. The fuzzy set A is a collection $A = \{(x, y = \mu_A(x))\}, x \in A, \mu_A(x) \in [0,1]$. Each element x gets a membership degree $\mu_A(x)$, determined by the membership function μ_A.

Suppose that $X = \{x_1, \dots, x_n\}$ is a finite data set. Each data point $x_k = \left(x_{k_1}, \dots, x_{k_p}\right), k = 1, \dots, n,$ is a pattern vector in R^p. Fuzzy c-means algorithm partitions X in a collection of S_i subsets, $2 \le i \le c$, called fuzzy clusters. By running the algorithm repeatedly, a list of v_i cluster centres and a partition matrix U are returned.

The description of the c-means algorithm is performed in the following steps [2]:

1) Select c=2, initialize m=3 and the termination tolerance $\epsilon = 10^{-8}$.
2) Set l = 0.
3) Determine the initial values of degrees in partition matrix U^l.
4) Calculate cluster centres $\mathrm{v}_i^l, i = 1, \dots, c$, as

$$\mathrm{v}_i^l = \frac{\sum_{k=1}^{n}\left(\left(\mu_{ik}^l\right)^m \cdot x_k\right)}{\sum_{k=1}^{n}\left(\mu_{ik}^l\right)^m}. \quad (1)$$

5) Calculate the updated partition matrix U^{l+1} by

$$\mu_{ik}^{l+1} = \frac{\left(\frac{1}{d\left(x_k, \mathrm{v}_i^l\right)}\right)^{1/m-1}}{\sum_{j=1}^{c}\left(\frac{1}{d\left(x_k, \mathrm{v}_j^l\right)}\right)^{1/m-1}}. \quad (2)$$

6) If $\|U^{l+1} - U^l\| \ge \epsilon$, then set $l = l + 1$, and go to step 4. If $\|U^{l+1} - U^l\| \le \epsilon$, then stop the procedure. Matrix U^{l+1} is the optimal distribution of membership degrees of x_k in clusters S_i. The symbol $\|\ \|$ denotes a matrix norm.

The steps of the c-means algorithm [2] contain expressions, which are explained in turn as: n is a number of data points, c is a number of clusters, the value of $\mu_{S_i}(x_k)$ stands for the membership degree of x_k in cluster S_i, $d(\mathrm{v}_i, x_k)$ indicates the Euclidean distance between the cluster centre v_i and x_k, and constant $m > 1$ is a weighting exponent.

The Euclidean distance is proved to guarantee a fast convergence of the algorithm to final results. We state m = 3 as the curves, approximating clusters, are smoothest.

The prior determination of the membership degrees in U^0 plays a crucial role in the c-means algorithm, as their choice not only can affect the convergence speed, but also may have a direct impact on the results of the classification [2][9]. To avoid inaccuracy in final results, we will discuss our own technique of calculation of degrees in U^0 to avoid guessing at their values intuitively.

III. DEGREES IN THE INITIAL PARTITION MATRIX

To make appropriate evaluations of the membership of x_k in S_i, we adopt the s-class function [10]

$$s(z, \alpha, \beta, \gamma) = \begin{cases} 0 & \text{for} \quad z \le \alpha, \\ 2\left(\frac{z-\alpha}{\gamma-\alpha}\right)^2 & \text{for} \quad \alpha \le z \le \beta, \\ 1 - 2\left(\frac{z-\gamma}{\gamma-\alpha}\right)^2 & \text{for} \quad \beta \le z \le \gamma, \\ 1 & \text{for} \quad z \ge \gamma. \end{cases} \quad (3)$$

in further calculations.

Surgery prognoses usually can be expressed by "degree of surgery" contra "degree of no surgery", when basing on the age, the *crp*-values and the weight. The linguistic degrees of surgery, like, e.g., "little" or "large", can be proposed by a physician as terms of a list L.

Generally, let us suppose that $L = \{L_1, \dots, L_\omega\}$ is a linguistic list consisting of ω words, where ω is an odd integer. Each word is associated with a fuzzy set, also named L_s, s = 1,...,ω. Furthermore, let E be the length of a common reference set R, containing all fuzzy sets L_s. Let $z \in R$. For instance, R can be recognized as a density set between 0 and 100, in which densities about z = 20 belong to "little". We divide the linguistic terms into three groups named: a left group, a middle group and a right group.

The membership functions, assigned to the leftmost terms, are parametric functions, which are yielded by (4) as [11][12]

$$\mu_{L_t}(z) = \begin{cases} 1 \text{ for } z \le \frac{E(\omega-1)}{2(\omega+1)}\delta(t), \\ 1 - 2\left(\frac{z - \frac{E(\omega-1)}{2(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 \\ \text{for } \frac{E(\omega-1)}{2(\omega+1)}\delta(t) \le z \le \frac{E(\omega-1)}{2\omega}\delta(t), \\ 2\left(\frac{z - \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t)}{\frac{E(\omega-1)}{\omega(\omega+1)}\delta(t)}\right)^2 \\ \text{for } \frac{E(\omega-1)}{2\omega}\delta(t) \le z \le \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \\ 0 \text{ for } z \ge \frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\delta(t), \end{cases} \quad (4)$$

where $\delta(t) = \frac{2t}{\omega-1}, t = 1, \dots, \frac{\omega-1}{2}$ is a parametric function, depending on left function number t.

The membership function in the middle has the form of a bell. It is designed by (5) in the form of [11][12]

$$\mu_{L_{\frac{\omega+1}{2}}}(z)=\begin{cases}0 & \text{for} \quad z\le \frac{E(\omega-2)}{2\omega},\\ 2\left(\frac{z-\frac{E(\omega-2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega-2)}{2\omega}\le z\le \frac{E(\omega-1)}{2\omega},\\ 1-2\left(\frac{z-\frac{E}{2}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega-1)}{2\omega}\le z\le \frac{E}{2},\\ 1-2\left(\frac{z-\frac{E}{2}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E}{2}\le z\le \frac{E(\omega+1)}{2\omega},\\ 2\left(\frac{z-\frac{E(\omega+2)}{2\omega}}{\frac{E}{\omega}}\right)^2 & \text{for} \quad \frac{E(\omega+1)}{2\omega}\le z\le \frac{E(\omega+2)}{2\omega},\\ 0 & \text{for} \quad z\ge \frac{E(\omega+2)}{2\omega}.\end{cases} \tag{5}$$

Finally, the membership functions on the right-hand side are expressed by (6) as

$$\mu_{L_{\frac{\omega+3}{2}+t-1}}(z)$$
$$=\begin{cases}0 \text{ for } z\le E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\varepsilon(t),\\ 2\left(\frac{z-\left(E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\varepsilon(t)}\right)^2\\ \text{for } E-\frac{E(\omega-1)(\omega+2)}{2\omega(\omega+1)}\varepsilon(t)\le z\le E-\frac{E(\omega-1)}{2\omega}\varepsilon(t),\\ 1-2\left(\frac{z-\left(E-\frac{E(\omega-1)}{2(\omega+1)}\varepsilon(t)\right)}{\frac{E(\omega-1)}{\omega(\omega+1)}\varepsilon(t)}\right)^2\\ \text{for } E-\frac{E(\omega-1)}{2\omega}\varepsilon(t)\le z\le E-\frac{E(\omega-1)}{2(\omega+1)}\varepsilon(t),\\ 1 \text{ for } z\ge E-\frac{E(\omega-1)}{2(\omega+1)}\varepsilon(t).\end{cases} \tag{6}$$

A new function $\varepsilon(t)=1-\frac{2(t-1)}{\omega-1}, t=1,\dots,\frac{\omega-1}{2}$ allows generating all rightmost functions one by one, when setting t-values in (6).

IV. THE SURGERY DECISION FOR 25 PATIENTS

To make a decision about surgery, concerning an individual patient in accordance with his/her biological markers' values, we must involve the medical experience in the decisive process. In order to facilitate a conversation with a physician, we have prepared a list named "The primary medical linguistic judgment of surgery grade"= L = $\{L_1$ = "none", L_2 = "little", L_3 = "medium", L_4 = "large", L_5 = "total"$\}$. The evaluation of no surgery will be an inverted surgery term with respect to L.

The excerpt of the data set, shown in TABLE I, consists of the patients' clinical records and primary linguistic estimations of surgery grades. The judgments are made by the medical expert. The total medical report contains 25 gastric cancer patients, randomly selected.

TABLE I. THE DATA SET OF 25 GASTRIC CANCER PATIENTS

Patient x_k	**Attribute-vectors and surgery judgments**		
	Attribute-vectors (Age, weight, crp)	***Surgery cluster S_1***	***No Surgery cluster S_2***
x_1	(71, 85, 1)	Total	None
x_2	(81, 70, 9)	Medium	Medium
...	...	...	...
x_{25}	(54, 49, 36)	None	Total

Each verbal expression, being the term of L, is associated with a fuzzy set. L_1 and L_2 represent two left fuzzy sets. L_3 is the fuzzy set in the middle, whereas L_4 and L_5 constitute two rightmost fuzzy sets. Unfortunately, these linguistic items do not provide us with any information about degrees, expected in matrix U^0. To estimate degrees of surgery in cluster S_1 and degrees of no surgery in cluster S_2, we have initiated the following enumeration technique.

By employing (4), (5) and (6) for E = 100 (a typical reference set in medical investigations of densities) and ω = 5, we derive the membership functions of L_s, s = 1,…,5. Functions L_s are sketched in Figure 1.

After setting $\alpha=0, \beta=50$ and $\gamma=100$ in a new s-function, impacted over set R, we determine

$$\mu_R(z) = s(z,0,50,100), \tag{7}$$

whose graph is added to Figure 1.

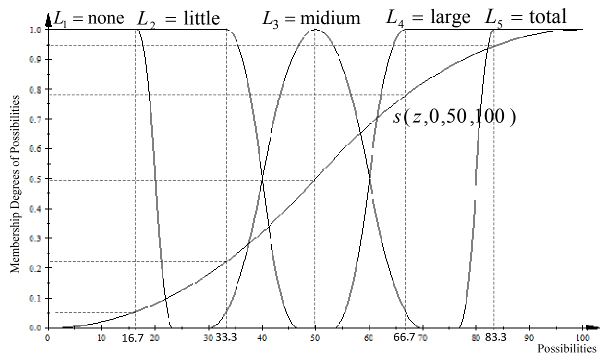

Figure 1. The collection of membership functions generated by (4), (5), (6) and (7)

Figure 1 helps us to evaluate the degrees taking place in the first partition matrix U^0. The second coordinates of the intersection points between $\mu_R(z)$ and $\mu_{L_s}(z)$, s = 1,…,5, will substitute the linguistic structures, filling L. Therefore, 0.056 is assigned to "none", 0.22 to "little", 0.5 to "medium", 0.78 to "large" and 0.944 to "total". The list L can be extended by adding other verbal expressions to it.

After the arrangements of numerical compensations of the terms from L, the words in TABLE I are replaced by values put in TABLE II.

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TABLE II. DATA SET WITH INITIAL DEGREES

Patient x_k	Attribute-vectors and surgery degrees		
	Attribute-vectors (Age, weight, crp)	$\mu_{S_1}(x_k)$	$\mu_{S_2}(x_k)$
x_1	(71, 85, 1)	0.944	0.056
x_2	(81, 70, 9)	0.5	0.5
...	...	...	...
x_{25}	(54, 49, 36)	0.056	0.944

The entries of the initial partition matrix $U^0_{2\times 25}$ contain the values coming from the last two columns in TABLE II.

If we go back to the c-means clustering algorithm and involve Matlab in calculations, then the cluster centres will become stable after 31 iterations.

The last partition matrix has a pattern

$$U^{31} = \begin{matrix} & x_1 & \cdots & x_{25} \\ S_1 & 0.79 & \cdots & 0.31 \\ S_2 & 0.21 & \dots & 0.69 \end{matrix}_{2x25}.$$

The relations referring to magnitudes of the final degrees for 25 patients, classified for surgery in S_1 and for no surgery in S_2, have confirmed the primary hypotheses. The final verdicts are softer, e.g., "total" is reduced to "large" for surgery prognosis, whereas "none" gets a status of "little" for no surgery.

V. PATIENT CHARACTERISTIC VALUES

We now generate a set of points $(x_k,\ \mu_{S_1}(x_k)), k = 1,\dots,n$. Let us note that the first coordinates x_k do not belong to any real-valued x_k-axis, since x_k have not been characterized by any quantity. We thus want to assign a characteristic value to each patient x_k [12] to be able to place x_k's characteristic values in an ascending order.

The patients' *crp*-values, the ages and the body weights play a significant role in the surgery diagnosis. Let us denote the space of the *crp*-values by CRP = [0, 85], the space of ages by A = [0, 100] and express the body weight space by BW = [40, 120].

For x_k, a characteristic value $f_{x_k}(\ crp^c\ a^c\ bw^c) = w_{crp}crp^c + w_a a^c + w_{bw} bw^c$, $a^c \in A$, $crp^c \in CRP$ and $bw^c \in BW$, k = 1,...,n, [12]. The multipliers w_{crp}, w_a and w_{bw} are the importance weights, emphasizing the decisive power of each biological parameter for the surgery decision.

To find values of importance weights w_{crp}, w_a and w_{bw}, we present our own procedure, sketched below.

Generally, we compare p parameters to assign importance weights to them. A sequence $p_1 > p_2 > \dots > p_p$ will be thus arranged due to the expert's opinion, provided that ">" is interpreted as "more important than". We wish the sum of all weights w_q, q = 1,...,p, to be 1 in accordance with

$$p \cdot r + (p-1) \cdot r + \dots + 2 \cdot r + 1 \cdot r = 1 \qquad (8)$$

where r is a quotient depending on p. Hence, $w_q = (p - q + 1) \cdot r$, for q = 1,...,p.

In the gastric cancer example, the physician determines the sequence *crp*>age>weight, which lets us evaluate $w_{crp} = 0.498$, $w_a = 0.333$ and w_{bw} =0.166.

Example 1

The eighty-one year old man x_k, weighing 90 kg and revealing *crp*=16, is given by $f_{x_k}(crp^c\ a^c\ \ bw^c) = 49.88$.

VI. THE CURVE FITTING FOR THE POINT SETS

We wish to find a curve, which approximates the set of pairs $(f_{x_k}(crp^c\ a^c\ \ bw^c), \mu^{U^{31}}_{S_1}(x_k))$, symbolically denoted by $\left(f_{x_k}, \mu(x_k)\right)$. In the set, the pairs are arranged in ascending order of characteristic values f_{x_k}.

In accordance with [7][8] (our earlier procedures), we utilize the equation of the truncated π function in the process of approximation of point sets, which build the pattern of a bell. The classical π-function is limited by $s(z,\alpha_1,\beta_1,\gamma_1{=}\alpha_2)$ in the left part and $1{-}s(z,\alpha_2{=}\gamma_1,\beta_2,\gamma_2)$ in the right part, respectively [10]. Its truncated version has no intersection points with the z-axis. Without discussing the details, which are available in [7][8], we only mention that we need three characteristic points to start with the approximation. When remembering that $f_{x_k}, k = 1,\dots,n$, are ordered in the ascending sequence, we select: ($f_1 = \min_{k=1,\dots,n}(f_{x_k})$, μ_1), (f_2, $\mu_2 = \max_{k=1,\dots,n}(\mu(x_k))$) and ($f_3 = \max_{k=1,\dots,n}(f_{x_k})$, μ_3).

The coordinates of the points are included into four general equations of the truncated π:

a)

$$\pi_{left\,slope}(f_{x_k}) = \mu_{"surgery"}(f_{x_k}) = \begin{cases} 0 & \text{for } f_{x_k} < f_1, \\ 2\mu_2\left(\frac{f_{x_k}-\alpha}{f_2-\alpha}\right)^2 & \text{for } f_1 \le f_{x_k} \le \beta_1, \\ \mu_2\left(1-2\left(\frac{f_{x_k}-f_2}{f_2-\alpha}\right)^2\right) & \text{for } \beta_1 \le f_{x_k} \le f_2, \end{cases} \qquad (9)$$

for $\alpha = \frac{f_1 - f_2\sqrt{\frac{\mu_1}{2\mu_2}}}{1-\sqrt{\frac{\mu_1}{2\mu_2}}}$, $\mu_1 < \frac{\mu_2}{2}$ and $\beta_1 = \frac{\alpha + f_2}{2}$,

b)

$$\pi_{left\,slope}(f_{x_k}) = \mu_{"surgery"}(f_{x_k}) = \begin{cases} 0 & \text{for } f_{x_k} < f_1, \\ \mu_2\left(1-2\left(\frac{f_{x_k}-f_2}{f_2-\alpha}\right)^2\right) & \text{for } f_1 \le f_{x_k} \le f_2, \end{cases} \qquad (10)$$

where $\alpha = f_2 - \frac{f_2 - f_1}{\sqrt{\frac{\mu_2-\mu_1}{2\cdot\mu_2}}}$ for $\mu_1 \ge \frac{\mu_2}{2}$.

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c)

$$\pi_{right\ slope}(f_{x_k}) = \mu_{"surgery"}(f_{x_k}) = \begin{cases} \mu_2\left(1-2\left(\frac{f_{x_k}-f_2}{\gamma-f_2}\right)^2\right) & \text{for} \quad f_2 \le f_{x_k} < \beta_2, \\ 2\mu_2\left(\frac{f_{x_k}-\gamma}{\gamma-f_2}\right)^2 & \text{for} \quad \beta_2 \le f_{x_k} \le f_3, \\ 0 & \text{for} \quad f_{x_k} > f_3, \end{cases} \tag{11}$$

for $\gamma = \frac{f_3 - f_2\sqrt{\frac{\mu_3}{2\cdot\mu_2}}}{1-\sqrt{\frac{\mu_3}{2\cdot\mu_2}}}$, when $\mu_3 < \frac{\mu_2}{2}$ and $\beta_2 = \frac{f_2+\gamma}{2}$

and

d)

$$\pi_{right\ slope}(f_{x_k}) = \mu_{"surgery"}(f_{x_k}) = \begin{cases} \mu_2\left(1-2\left(\frac{f_{x_k}-f_2}{\gamma-f_2}\right)^2\right) & \text{for} \quad f_2 \le f_{x_k} < f_3, \\ 0 & \text{for} \quad f_{x_k} > f_3, \end{cases} \tag{12}$$

in which $\gamma = f_2 + \frac{f_3-f_2}{\sqrt{\frac{\mu_2-\mu_3}{2\mu_2}}}$ for $\mu_3 \ge \frac{\mu_2}{2}$.

Example 2

The data of 25 patients x_k, k = 1,...,25, are rearranged in ascending order due to $f_{x_k}(crp^c\ a^c\ bw^c) = 0.498crp^c + 0.333a^c + 0.166bw^c$. For $(f_{x_k}(crp^c\ a^c\ bw^c), \mu_{S_1}^{U^{31}}(x_k))$, we select ($f_1$, μ_1) = (32.23, 0.682), (f_2, μ_2) = (36.428, 0.824) and (f_3, μ_3) = (112.60, 0.374). Figure 2 plots all points, assisting the patients' clinical data and surgery degrees.

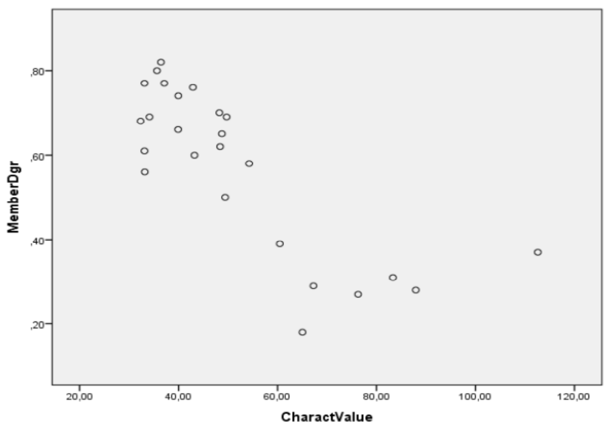

Figure 2. The set of points $(f_{x_k}(crp^c\ a^c\ bw^c), \mu_{S_1}^{U^{31}}(x_k)), k = 1, \ldots, 25$

The left part of the truncated π, when $\mu_1 > \frac{\mu_2}{2}$ (0.682>0.412), has the equation

$$\pi_{left\ slope}(f_{x_k}) = \mu_{"surgery"}(f_{x_k}) = \begin{cases} 0 & \text{for } f_{x_k} < 32.23, \\ 0.824\left(1-2\left(\frac{f_{x_k}-36.428}{36.428-22.127}\right)^2\right) & \text{for } 32.23 \le f_{x_k} \le 36.428, \end{cases}$$

for restored $\alpha = 36.428 - \frac{36.428-32.23}{\sqrt{\frac{0.824-0.682}{2\cdot 0.824}}} = 22.127$, due to b).

To derive the right part of π, we study c), as $\mu_2 < \frac{\mu_3}{2}$. Equation (11) provides us with the formula

$$\mu_{"right\ slope"}(f_{x_k}) = \mu_{"surgery"}(f_{x_k}) = \begin{cases} 0.824\left(1-2\left(\frac{f_{x_k}-36.428}{181.9-36.428}\right)^2\right) & \text{for } 36.428 \le f_{x_k} < 109.164, \\ 0.824\left(2\left(\frac{f_{x_k}-181.9}{181.9-36.428}\right)^2\right) & \text{for } 109{,}164 \le f_{x_k} \le 112.6, \\ 0 & \text{for } f_{x_k} > 112.6, \end{cases}$$

where γ = 181.9.

The graph of both branches of $\mu_{"surgery"}(f_{x_k})$ is drawn in Figure 3. The function, created for degrees of no surgery, has a formula $1-\mu_{"surgery"}(f_{x_k})$.

The formulas, expressed in Example 2, allow making surgery prognoses for an arbitrary patient, whose characteristic value lies in interval [32.23, 112.6]. If we face more extreme quantities, then we should construct another partition matrix, adapted to a new collection of clinical data.

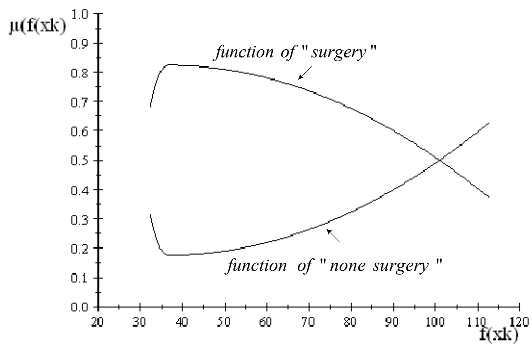

Figure 3. The membership functions of "surgery" and "none surgery"

Example 3

Evaluate a degree of surgery for the patient from Example 1. The patient characteristic value has equalled 49.88. Hence,

$$\mu_{"surgery"}(49.88) = 0.824\left(1-2\left(\frac{49.88-36.428}{181.9-36.428}\right)^2\right) = 0.809.$$

VII. Conclusion and Future Work

We have applied fuzzy 2-means clustering analysis to partition a patient data set, containing clinical records of 25 gastric cancer patients, in two fuzzy clusters. These reveal the numerical decision of states: “surgery” and “no surgery”.

We notice that the patients’ original clinical marker values lead to higher membership degrees in the initial partition matrix, when comparing them to the lower values in the final matrix. This phenomenon can be explained by the fact that the decision for an individual patient has been made by the assistance of all data filling the data set. This means that the medical knowledge provided in the form of the collective information, reset numerically, could decide “softer” decisions. The obtained results converge to the surgery judgments made by physicians from Blekinge County Hospital, Karlskrona, Sweden.

In the second part of the study, we have started with the constructions of characteristic values. The values are mixtures of clinical measurements and importance weights of markers examined. Then the points, characterized by coordinates equal to the patient characteristics and degrees of surgery, have been surrounded by the curve. The equation of this curve may be used to prognosticate a degree of surgery for any gastric cancer patient. The approximation by the truncated π cumulates a little error for point shapes, similar to parts of a bell. The placement of minimal and maximal degrees in the graph of the curve, connected to “degree of surgery”, agrees with the medical knowledge on recommendations of surgery in the cases of gastric cancer patients.

The idea of applying fuzzy set theory to the surgery decision is a pioneer in the field of medical applications of mathematics. Therefore, we cannot compare our effects to similar contributions, made in this domain. In spite of that, the physicians, cooperating with us, have confirmed the reliability of mathematical models.

Apart from applications of ready-made algorithms, like the c-means method, we have introduced our own earlier and newer mathematical models to this medical example. The membership function families, exploiting to determine the initial membership degrees in the partition matrix, have been an efficient tool in the algorithm. The functions, furnished with parameters, allow constructing arbitrary linguistic lists containing many verbal judgments. The weights of importance have been computed by the action of a simple algorithm, specially constructed for this purpose. Lastly, the procedure of approximation of point sets, resembling the shape of a bell, remains our own substantial contribution. Without the equation of the approximating curve, we could not make any surgery prognoses for casual patients, who do not exist in the testing set of patients.

Future challenges are also planned. We want to test larger samples of patients to open a database of truncated π equations, covering the most cases of patient clinical data. This may give us a chance to establish the information computer centre, making surgery prognoses.

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