Data-Assisted Distributed Stabilization of Interconnected Linear Multiagent Systems without Persistency of Excitation

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***Abstract*—Stability of large-scale systems has been commonly achieved using centralized and decentralized control configurations. Several graph theoretic protocols have recently been developed for the distributed stabilization of partially unknown interconnected multiagent systems, when an upper bound on the unknown interconnection allocation matrices is provided to the control designer. The need for such an upper bound can be relaxed using adaptive control ideas. However, due to the inaccurate parameter estimation in the absence of the persistency of excitation for all agents' regressors, the use of traditional adaptive control ideas will only ensure the boundedness of all trajectories. We develop a data-assisted distributed protocol which operates under a new condition, collective finite excitation, in order to overcome this challenge. Despite the completely unknown interconnection allocation matrices, we prove the exponential estimation of all interconnection matrices and exponential convergence of all state trajectories of the interconnected multiagent systems to the origin.**

Keywords–Distributed Control; Decoupling Control; Finite Excitation; Interconnected Systems; Multiagent Systems.

I. INTRODUCTION

Along the advances in low-cost, low-dimension embedded sensing, computation, and communication systems, graph theoretic approaches have received significant attention in the consensus of Multi Agent Systems (MASs). Initial studies focused on the simple integrators or completely known agents [1]. Recently, the consensus for MASs with completely known interconnections among agent dynamics [2] and local or agent-level modeling uncertainties ([3] and [4]) has also been studied. In particular, [4] discusses that the consensus on zero is a nontrivial problem for MASs with modeling uncertainties.

Parallel to these efforts, the concept of multilayer control using graph theoretic approaches was proposed [5]. Egerstedt [6] considered the use of graph theory to capture the architectural aspect of cyber-physical systems for a completely known MAS of interconnected integrator agents. However, based on [7], we know that the cyber and physical layers of cyber-physical systems might be subject to various abnormalities.

With a focus on the modeling uncertainties over the physical (agent) layer, [8] proposed a graph theoretic decoupling framework in order to stabilize an interconnected MAS subject to the nonlinear modeling uncertainties. However, that result is based on the locally (interconnection-free) stable agents, and the control layer topology is identical to the completely known agent layer topology. Rezaei and Stefanovic [9] reformulated the solution approach in order to capture the architectural aspect of cyber-physical systems, yet the result was limited to a special agent layer interconnection topology. This issue was addressed in [10], however, it was based on a symmetric control layer topology. Rezaei et al. [11] designed several structurally nonsymmetric control layers for interconnected single and double integrator agents, and [12] developed two design procedures to build structurally nonsymmetric control layers for interconnected MASs subject to both matched and unmatched nonlinear modeling uncertainties. Nevertheless, all of these robust formulations require the knowledge about an upper bound on the norms of the unknown terms, and may end in a level of inherent conservatism (see section 4 in [12]).

Adaptive control ideas provide an appropriate framework to handle a wider range of modeling uncertainties in dynamical systems including networked systems. While the traditional adaptive control methods have been reported in several studies (e.g., see [13] for synchronization and [14] for consensus in MASs), the need for the Persistency of Excitation (PE) [15] might be problematic when the control signal depends on the estimation of the unknown parameters [16]. This might be serious challenge when dealing with an MAS. This is because the poor transient performance of each non-PE agent can easily propagate via the (networked) distributed controller which, consequently, degrades the performance of the entire MAS.

Motivated by the aforementioned observations, [17] proposed a cooperative PE condition to be satisfied by a group of agents (vs. each individual agent of an MAS). However, similar to the conventional PE condition, that cooperative PE condition must be satisfied in all (future) time windows. Yuan et al. [18] reported a cooperative Finite Excitation (FE) condition for the adaptive consensus in non-interconnected MASs. Further, [19] designed a cooperative FE condition for the stabilization of interconnected MASs. That method is based on a series of low-pass filtered signals and the control layer is symmetric. (See [19] for a more comprehensive survey of the literature.)

As discussed in [12], the use of an adaptive decoupling approach may reduce the conservatism that inherently comes with any robust control techniques. Therefore, motivated by the findings in [19] and inspired by [20] (for single dynamical systems), we develop a new data-assisted distributed protocol to stabilize a class of linear time-invariant interconnected MASs subject to the matched modeling uncertainties. The control layer is built by two sublayers: A decoupling sublayer to cancel the effect of the interconnections, and a cooperation layer to stabilize the interconnected MAS even when the non-interconnected agents are unstable or to shape the closed-loop time response. The new data-assisted approach relies on the use of a few data collection matrices to satisfy a collective

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FE condition which enables us to potentially improve the performance of the multilayer interconnected MAS using an appropriate criterion from the literature of matrix algebra. We theoretically prove that, under the proposed collective FE condition, all estimated interconnection allocation matrices exponentially converge to their actual values and, similarly, all state trajectories of the multilayer interconnected MAS converge to the origin. We also characterize the boundedness of all trajectories during the transient time.

We overview the required notation and definitions in Section II, provide the main theoretical developments in Section III, and summarize the paper in Section IV.

II. Notation and Definitions

$A \succ B$ ($\succcurlyeq$) means $A-B$ is a positive (semi) definite matrix. $\mathbf{0}$ denotes a matrix of all zeros, $\mathrm{diag}\{.\}$ a (block) diagonal matrix of the scalars (matrices) in $\{.\}$, $\mathrm{col}\{x_i\}$ an aggregated column vector, $\|.\|$ the (induced) 2-norm of a vector (matrix), and $\mathrm{vec}(A) \in \mathbb{R}^{mn}$ is a vectorization of $A \in \mathbb{R}^{m\times n}$.

We consider three graph topologies: $\mathscr{G}_a$ to represent the physical interaction of agents' dynamics over the agent layer, $\mathscr{G}_d$ a communication topology to compensate for the effect of interconnections on agents' dynamics using a data-assisted strategy over a decoupling layer, and $\mathscr{G}_c$ a communication topology for the controllers' communication over a cooperation layer. We allow the existence of selfloops over all layers where, by a selfloop, we refer to an edge outgoing from and returning to the same node without passing though any other nodes. Since the standard definitions of adjacency and Laplacian matrices do not admit selfloops [21], we redefine them in the rest of this section.

An agent layer digraph $\mathscr{G}_a$ with N nodes is characterized by an adjacency matrix $\mathscr{A}_a = [a^a_{ij}] \in \mathbb{R}^{N\times N}$ where $a^a_{ij} \neq 0$ if the i^{th} agent is affected by the j^{th} agent's dynamics for $i,j \in \{1,2,...,N\}$, and $a^a_{ij} = 0$ otherwise. Unlike the standard definition, $j = i$ is acceptable and each a^a_{ij} is a real valued scalar with either positive or negative sign. $\mathscr{N}_i^a$ represents the set of i^{th} agent's neighbors over $\mathscr{G}_a$ which may include the number i as well (selfloop). *We introduce $\mathscr{S}_a$ as the set of nodes affected by some neighbor agents over the agent layer (including themselves through the selfloops).*

The control layer is divided into two separate (sub-) graphs with N nodes: $\mathscr{G}_c$ and $\mathscr{G}_d$. We do not discuss the communication topology $\mathscr{G}_d$ for the data-assisted decoupling control, because it is the same as $\mathscr{G}_a$. A cooperation layer digraph $\mathscr{G}_c$ with N nodes is characterized by a modified Laplacian matrix $\mathscr{H}_c = \mathscr{L}_c + \mathscr{S}_c \in \mathbb{R}^{N\times N}$. $\mathscr{L}_c \in \mathbb{R}^{N\times N}$ is a standard Laplacian matrix for a digraph $\mathscr{G}_c'$, with non-negative edge weights a^c_{ij}, to be obtained by removing all selfloops from $\mathscr{G}_c$: $\mathscr{L}^c_{ij} = -a^c_{ij}$, $\mathscr{L}^c_{ii} = \sum_{j\in\mathscr{N}_i^c} a^c_{ij}$, and $\mathscr{N}_i^c$ characterizes the i^{th} agent's (controller) neighbors over $\mathscr{G}_c'$. Also, $\mathscr{S}_c = \mathrm{diag}\{s^c_i\} \in \mathbb{R}^{N\times N}$ is a diagonal matrix to represent selfloops: $s^c_i > 0$ when there is a selfloop around the i^{th} controller, and $s^c_i = 0$ otherwise. These selfloops and directed one-way communications between the control nodes create a *structurally nonsymmetric control layer*.

A typical three-layer (closed-loop) interconnected MAS is depicted in Figure 1. Both $\mathscr{G}_a$ (thus, $\mathscr{G}_d$) and $\mathscr{G}_c$ can be disconnected; however, all control nodes in each connected component of $\mathscr{G}_c$ must have access (direct path) to at least one node with a selfloop.

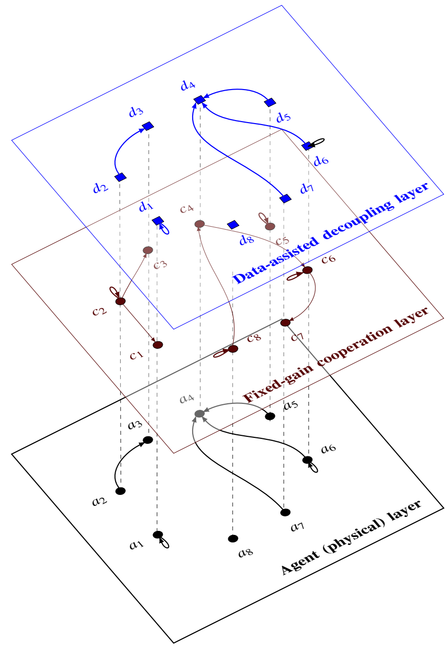

Figure 1. Example of a closed-loop distributed stabilization framework with separate agent and control layers. Top two layers form the control layer. Legend: a_i are agent nodes, c_i are fixed-gain cooperation nodes, d_i are data-collection (data-assisted control) nodes.

III. Main Results

A. Problem statement

We consider a MAS of N agents with the following interconnected dynamics:

$$\dot{x}_i(t) = Ax_i(t) + B\big(u_i(t) + C_i \sum_{j\in\mathscr{N}_i^a} a^a_{ij} x_j(t)\big) \tag{1}$$

where $x_i \in \mathbb{R}^{n_x}$ denotes the i^{th} agent's state variable, $u_i \in \mathbb{R}^{n_u}$ control input, $A \in \mathbb{R}^{n_x\times n_x}$ and $B \in \mathbb{R}^{n_x\times n_u}$ system matrices, $C_i \in \mathbb{R}^{n_u\times n_x}$ interconnection matrix, and $\mathscr{N}_i^a$ the set of i^{th} agent's neighbors over the agent (or physical) layer graph $\mathscr{G}_a$. We assume the pair (A,B) is stabilizable and the control allocation matrix B is full column rank (This stabilizability assumption is milder than the associated controllability one and, according to assumption 3.1.2 in [22], is required in order to have a solvable problem). For all affected agents $i \in \mathscr{S}_a$, the interconnection matrices C_i are completely unknown to the control layer designer which means the fixed-gain distributed stabilization ideas of [10] and [11] are no longer applicable for the distributed stabilization problem of this paper:

$$\lim_{t\to\infty} x_i(t) = \mathbf{0} \qquad \forall i \in \{1,2,...,N\} \tag{2}$$

which must be achieved (exponentially) by all agents.

B. Design foundation

We propose a distributed stabilization protocol with two components:

$$u_i(t) = u_{ci}(t) + u_{di}(t) \qquad \forall i \in \{1,2,...,N\} \tag{3}$$

where $u_{ci} \in \mathbb{R}^{n_u}$ denotes the i^{th} agent's cooperation input signal over a cooperation layer $\mathscr{G}_c$, to be developed in Subsection III-C, and $u_{di} \in \mathbb{R}^{n_u}$ data-assisted decoupling signal over a decoupling layer $\mathscr{G}_d$ whose topology is indeed the same as

$\mathscr{G}_a$, to be designed in Subsection III-D. Note that $u_{di} = \mathbf{0}$ if $i \notin \mathscr{S}_a$. We rewrite the agent model (1) as follows:

$$\dot{x}_i = Ax_i + B\big(u_{ci} + u_{di}\big) + B \sum_{j \in \mathscr{N}_i^a} a_{ij}^a \big(I_{n_u} \otimes x_j^T\big)\theta_i^\star \tag{4}$$

where $\theta_i^\star = \text{vec}(C_i^T) \in \mathbb{R}^{n_u n_x}$ denote the unknown constant parameters for all $i \in \mathscr{S}_a$. We propose the following fixed-gain cooperation and data-assisted decoupling protocols:

$$\begin{aligned} u_{ci} &= K\Big(\textstyle\sum_{j \in \mathscr{N}_i^c} a_{ij}^c (x_i - x_j) + s_i^c x_i\Big) \ \forall i \in \{1,2,...,N\} \\ u_{di} &= -\textstyle\sum_{j \in \mathscr{N}_i^a} a_{ij}^a \big(I_{n_u} \otimes x_j^T\big)\hat{\theta}_i \qquad\quad \forall i \in \mathscr{S}_a \end{aligned} \tag{5}$$

in which $\hat{\theta}_i(t) = \text{vec}(\hat{C}_i^T(t)) \in \mathbb{R}^{n_u n_x}$ denote the data-assisted time-varying estimates of the unknown constant parameters $\theta_i^\star$, and $\hat{C}_i(t)$ the estimated matrices of C_i in (1) for all $i \in \mathscr{S}_a$. Accordingly, in addition to the main objective (2), we must address a side objective for the exact parameter estimation to be used in u_{di} of (5). We re-state the control objectives as follows:

$$\lim_{t \to \infty} x_i(t) = \mathbf{0} \qquad \text{and} \qquad \lim_{t \to \infty} \hat{\theta}_i(t) = \theta_i^\star \tag{6}$$

To facilitate the theoretical derivations of this paper, we go one step further and find the dynamics of the closed-loop agents (4) with the cooperation and decoupling protocols (5):

$$\begin{aligned} \dot{x}_i = Ax_i &+ B\textstyle\sum_{j \in \mathscr{N}_i^a} \big(I_{n_u} \otimes x_j^T\big)\theta_i^\star \\ &+ BK\big(\textstyle\sum_{j \in \mathscr{N}_i^c} a_{ij}^c (x_i - x_j) + s_i^c x_i\big) \\ &- B\textstyle\sum_{j \in \mathscr{N}_i^a} a_{ij}^a \big(I_{n_u} \otimes x_j^T\big)\hat{\theta}_i \end{aligned} \tag{7}$$

to model all agent layer (first row), cooperation layer (second row), and data-assisted decoupling layer (third row). We further partition this multilayer interconnected MAS:

$$\begin{aligned} \dot{x}_i = Ax_i \quad &+ BK\big(\textstyle\sum_{j \in \mathscr{N}_i^c} a_{ij}^c (x_i - x_j) + s_i^c x_i\big) \\ &- B\textstyle\sum_{j \in \mathscr{N}_i^a} a_{ij}^a \big(I_{n_u} \otimes x_j^T\big)\tilde{\theta}_i \end{aligned} \tag{8}$$

where $\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i^\star$ represents the the i^{th} agent's parameter estimation error.

Remark 1: (Cyber-physical systems) We note that (7) represents a three-layer interconnected MAS with a data-assited control layer $\mathscr{G}_d$ whose topology is identical to the agent layer topology $\mathscr{G}_a$, and a cooperation layer $\mathscr{G}_c$ whose topology can be completely different from the agent layer topology $\mathscr{G}_a$. This framework enables us to distinguish the cyber (communication) malfunctions from the abnormalities of the physical components.

Definition 1: (Collective FE) A collection of bounded signals $x_j \in \mathbb{R}^n$, $j \in \mathscr{N}_i^a$, satisfies the collective finite excitation (FE) condition if there exist finite scalars $n_{Di} \in \mathbb{N}^+$, $t_l^s > 0$ and $t_l^e > 0$ for $l \in \{1,2,...,n_{Di}\}$, and $\gamma_i > 0$ such that:

$$\sum_{l=1}^{n_{Di}} \int_{t_l^s}^{t_l^e} \sum_{j \in \mathscr{N}_i^a} a_{ij}^a x_j(\tau) d\tau \int_{t_l^s}^{t_l^e} \sum_{j \in \mathscr{N}_i^a} a_{ij}^a x_j^T(\tau) d\tau \geq \gamma_i I_{n_x} > \mathbf{0}$$

where n_{Di} denotes the number of possibly discontinuous integration intervals based on the data collected by the i^{th} agent. Also, $t_l^s \geq 0$ and $t_l^e > t_l^s$ refer to the *finite* start and end points of the l^{th} integration time interval, respectively.

Remark 2: (Collective FE vs. PE, FE, and collective PE) The exact parameter convergence condition in (6) plays a key role in relaxing the need for an upper-bound on unknown parameters of the interconnected MAS (vs. [10] and [11]). This requires a persistency of excitation (PE) condition [15] to be satisfied by all regressors across the interconnected MAS. A bounded signal $x_j \in \mathbb{R}^n$ satisfies a PE condition, if there exist constant scalars $T_j, \gamma_j > 0$ such that the $\int_t^{t+T_j} x_j(\tau) x_j^T(\tau) d\tau \geq \gamma_j I_{n_x} > \mathbf{0}$ holds for all $x_j \neq \mathbf{0}$ and for all $t \geq 0$. We consider the following *modified PE condition* which is comparable to Definition 1:

$$\int_t^{t+T_j} x_j(\tau) d\tau \int_t^{t+T_j} x_j^T(\tau) d\tau \geq \gamma_j I_{n_x} > \mathbf{0}$$

Using this modified definition, we note that a bounded signal $x_j \in \mathbb{R}^n$, $j \in \mathscr{N}_i^a$, satisfies an *FE condition*, if there exist finite scalars $n_{Dj} \in \mathbb{N}^+$, $t_l^s > 0$ and $t_l^e > 0$ for $l \in \{1,2,...,n_{Dj}\}$, $\gamma_j > 0$, such that the following inequality holds:

$$\sum_{l=1}^{n_{Dj}} \int_{t_l^s}^{t_l^e} x_j(\tau) d\tau \int_{t_l^s}^{t_l^e} x_j^T(\tau) d\tau \geq \gamma_j I_{n_x} > \mathbf{0}$$

where n_{Dj} denotes the number of possibly discontinuous integration intervals for the j^{th} agent. We find that a collection of bounded signals $x_j \in \mathbb{R}^n$ for all $j \in \mathscr{N}_i^d$ satisfies a *collective (modified) PE condition*, if there exist constant scalars $T_i, \gamma_i > 0$ such that the following inequality holds for all $t \geq 0$:

$$\int_t^{t+T_i} \sum_{j \in \mathscr{N}_i^a} a_{ij}^a x_j(\tau) d\tau \int_t^{t+T_i} \sum_{j \in \mathscr{N}_i^a} a_{ij}^a x_j^T(\tau) d\tau \geq \gamma_i I_{n_x} > \mathbf{0}.$$

C. Fixed-gain cooperation protocol

Now, we focus on the cooperation protocol u_{ci} in (5). We rewrite the fixed part of (8) as follows:

$$\dot{x}_i = Ax_i + \kappa B v_i + BK\big(\textstyle\sum_{j \in \mathscr{N}_i^c} a_{ij}^c (x_i - x_j) + (s_i^c - \kappa) x_i\big) \tag{9}$$

where $\kappa > 0$ is a design scalar to be discussed in Design Procedure 1, and $v_i = Kx_i$ is a virtual (decoupled) control signal with the same gain K as in the actual (coupled) cooperation protocol u_{ci} in (5). We name the first two terms a "networked nominal dynamics" and the third term a "fictitious modeling uncertainty," and proceed with a set of decoupled dynamics:

$$\dot{x}_i'(t) = Ax_i'(t) + \kappa B v_i'(t) \qquad \forall i \in \{1,2,...,N\} \tag{10}$$

which, indeed, are the same as the networked nominal dynamics in (9) with the new variables $x_i' \in \mathbb{R}^{n_x}$ and $v_i'(t) = Kx_i'(t) \in \mathbb{R}^{n_u}$. Note that (10) is a stabilizable system because (A,B) represents stabilizable dynamics (by assumption) and $\kappa > 0$. For the design purpose, we recommend using a sufficiently large κ to avoid any singularity (poor controllability) in solving the design problem of this subsection. It can be easily done noting the fact that, unlike the agent layer interconnection topology $\mathscr{G}_a$, the cooperation communication topology $\mathscr{G}_c$ (thus, $\mathscr{H}_c$ and κ) is a design degree of freedom.

Let $\mathscr{U}_{vi}$ be the set of all admissible, static linear state feedback, stabilizing signals v_i' for the networked nominal dynamics (10). The following fact holds for any valid control layer topology $\mathscr{G}_c$ as defined in Section II.

Fact 1: [23] There exists a positive definite matrix $\Delta = \text{diag}\{\delta_i\} \in \mathbb{R}^{N \times N}$ such that $\Delta \mathscr{H}_c + \mathscr{H}_c^T \Delta \succ \mathbf{0}$, where $\delta_i = \frac{\delta_i^n}{\delta_i^d} > 0$ with $\text{col}\{\delta_i^n\} = (\mathscr{H}_c^{-1})^T \mathbf{1}_N$ and $\text{col}\{\delta_i^d\} = \mathscr{H}_c^{-1} \mathbf{1}_N$.

Design Procedure 1: The candidate $\mathscr{G}_c$ and K of the distributed cooperation protocol (5) are designed as follows:

1) Choose a nonsymmetric control layer topology $\mathscr{G}_c$ with $\mathscr{H}_c$ as its modified Laplacian matrix. Let $\kappa > 0$ be a real-valued scalar such that $\Delta \mathscr{H}_c + \mathscr{H}_c^T \Delta \succcurlyeq 2\kappa\Delta$. Let the state weighting matrix $W_x \in \mathbb{R}^{n_x \times n_x}$ and the

control input weighting matrix $W_v \in \mathbb{R}^{n_u \times n_u}$ be positive definite design matrices.

2) Find the solution $v_i' = Kx_i'$ of the following Linear Quadratic Regulator (LQR) problem. Then, K gives a candidate stabilization gain to be used in the cooperation protocol u_{ci} in (5).

$$V(x_i'(0)) = \min_{v_i' \in \mathscr{U}_{vi}} \int_0^\infty (x_i'^T(\tau) W_x x_i'(\tau) + v_i'^T(\tau) W_v v_i'(\tau)) d\tau \quad \text{subject to} \quad (10)$$

This design procedure for structurally nonsymmetric control layer is modified from [12] such that, now, the state weighting matrix W_x of the LQR problem is chosen completely arbitrary. We note that the candidate stabilization gain K is characterized as follows [24]:

$$K = -\kappa W_v^{-1} B^T P \tag{11}$$

in which the positive definite matrix $P \in \mathbb{R}^{n_x \times n_x}$ is the unique stabilizing solution of the Algebraic Riccati Equation (ARE):

$$A^T P + PA + W_x - \kappa^2 PBW_v^{-1} B^T P = \mathbf{0} \tag{12}$$

The existence and uniqueness of P follow from the stabilizability and observability of $(W_x^{1/2}, A, \kappa B])$.

We use (11) and (12), and find the following equalities:

$$\begin{aligned} \Delta \otimes (A^T P + PA + W_x - \kappa^2 PBW_v^{-1} B^T P) &= \mathbf{0} \\ \Delta \otimes (K + \kappa W_v^{-1} B^T P) &= \mathbf{0} \end{aligned} \tag{13}$$

Fact 2: The following MAS-level equalities hold in an MAS of networked nominal dynamics and the candidate gains K and G of Design Procedure 1:

$$\begin{aligned} x^T \bar{W}_x x + v^T \bar{W}_v v + \bar{V}_x^T (\bar{A}x + \kappa \bar{B} v) &= 0 \\ 2v^T \bar{W}_v + \kappa \bar{V}_x^T \bar{B} &= \mathbf{0} \end{aligned} \tag{14}$$

where $x = [x_1^T, x_2^T, ..., x_N^T]^T$, $v = \operatorname{col}\{v_i\} = \bar{K}x = (I_N \otimes K)x$, $V_x^T = \frac{\partial \bar{V}}{\partial x}$, $\bar{V} = x^T(\Delta \otimes P)x$, $\bar{W}_x = \Delta \otimes W_x$, and $\bar{W}_v = \Delta \otimes W_v$. (See [24] for single dynamical systems.)

Remark 3: The results of this section remain valid if we use a symmetric cooperation layer. In particular, $\mathscr{H}_c$ is a (symmetric) positive definite modified Laplacian matrix with strictly positive eigenvalues to be sorted as $0 < \mu_{c1} \le \mu_{c2} \le ... \le \mu_{cN}$ [10]. Then, Fact 1 holds with $\Delta = I_N$. Further, Step 1 of Design Procedure 1 is satisfied with $\kappa = \mu_{c1}$.

D. Data-assisted decoupling protocol

We start the design of u_{di} in (5) for agents $i \in \mathscr{S}_a$ by choosing an integration window length $\delta t_i > 0$ for each $i \in \mathscr{S}_a$, and integrating both sides of (4) from $t - \delta t_i$ to t as follows:

$$\Delta_{gi}(t) := x_i(t) - x_i(t - \delta t_i) - s_{gi}(t) = BR_{gi}(t)\theta_i^\star \tag{15}$$

where the subscript "g" distinguishes the integrated variables from the non-integrated ones, and the system-related signals $s_{gi} \in \mathbb{R}^{n_x}$ and regressor-related matrices $R_{gi} \in \mathbb{R}^{n_u \times n_x n_u}$ are defined as follows for $i \in \mathscr{S}_a$:

$$s_{gi}(t) = Ax_{gi}(t) + Bu_{gi}(t) \quad \text{and} \quad R_{gi}(t) = \sum_{j \in \mathscr{N}_i^a} a_{ij}^a \big(I_{n_u} \otimes x_{gj}^T(t)\big)$$

$$x_{gi}(t) = \int_{\max(t-\delta t_i, 0)}^t x_i(\tau) d\tau \quad \text{and} \quad u_{gi}(t) = \int_{\max(t-\delta t_i, 0)}^t u_i(\tau) d\tau$$

We define data-collection matrices $D_i \in \mathbb{R}^{n_x n_u \times n_x n_u}$:

$$D_i(n_{Di}) = \sum_{l=1}^{n_{Di}} R_{gi}^T(t_l) B^T B R_{gi}(t_l) \tag{16}$$

to be updated at the time $t_{n_{Di}}$ of agent i if it is an "acceptable" excitation sample instance n_{Di}. This is defined as the sample $n_{Di} \in [0, n]$ that results in the following inequality:

$$\lambda_{min}\big(D_i(n_{Di})\big) > \lambda_{min}\big(D_i(n_{Di} - 1)\big) \tag{17}$$

when $n \in \mathbb{N}^+$ increases in time, as the integration window moves forward.

For each agent $i \in \mathscr{S}_a$, we use (15)-(17) and propose the following update rule for the unknown parameter estimation:

$$\begin{aligned} \dot{\hat{\theta}}_i(t) = \Gamma_i \Big(R_i^T(t) B^T P x_i \quad & - \gamma_{Di} \big(D_i(t_{n_{Di}}) \hat{\theta}_i(t) \\ & - \textstyle\sum_{l=1}^{n_{Di}} R_{gi}^T(t_l) B^T \Delta_{gi}(t_l)\big)\Big) \end{aligned} \tag{18}$$

where $R_i = \sum_{j \in \mathscr{N}_i^a} a_{ij}^a \big(I_{n_u} \otimes x_j^T\big)$, and $\Gamma_i \in \mathbb{R}^{n_x n_u \times n_x n_u}$ and γ_{Di} are two design sets of positive-definite matrices and positive scalars, respectively, and $t_{n_{Di}}$ is the time associated to the sample instance n_{Di}.

The update law (18) can be rewritten as $\dot{\hat{\theta}}_i(t) = \Gamma_i R_i^T(t) B^T P x_i - \gamma_{Di} \Gamma_i D_i(t_{n_{Di}}) \tilde{\theta}_i(t)$ which justifies the selection of the criterion (17) to characterize an acceptable excitation time. While the first part is similar to the traditional adaptive control laws, the second is a data-assisted one to obviate the need for the PE condition in parameter estimation.

Assumption 1: For each agent $i \in \mathscr{S}_a$, there exists a finite $n_{Di} \in \mathbb{N}^+$ such that the collective FE condition in Definition 1 is gradually satisfied over a finite time interval $[t_{si}^{start}, t_{si}]$.

The emphasis of the above assumption is on the existence of a "finite" time interval rather than its start point. Thus, while t_{si} is chosen as noted below, the start point t_{si}^{start} can be any number equal to or greater than zero. Indeed, a $t_{si}^{start} > 0$ may refer to the time when an external probing signal is turned on in order to sufficiently excite the i^{th} agent.

E. Theoretical analysis

We follow similar steps as those of [19] for the analyses of this subsection.

Properties 1: The data-collection matrix (16) has the following guaranteed properties for all $i \in \mathscr{S}_a$:

1) $D_i(n_{Di}) \succeq \mathbf{0}$ for all $t \ge 0$,
2) $D_i(n_{Di}) \succ \mathbf{0}$ for all $t \ge t_{si}$ where $t_{si} > \delta t_i$
3) $\lambda_{min}\big(D_i(n_i')\big) \ge \lambda_{min}\big(D_i(n_{Di})\big)$ using each new sample $n_i' > n_{Di}$.

Proof: To prove the *first property*, we note that for any vector $z \in \mathbb{R}^{n_x n_u}$, $z^T D_i(n_{Di}) z = \sum_{l=1}^{n_{Di}} z^T R_{gi}^T(t_l) B^T B R_{gi}(t_l) z = \sum_{l=1}^{n_{Di}} \|BR_{gi}(t_l) z\|^2 \ge 0$ which means either $D_i(n_{Di})$ or each new arrived matrix $R_{gi}^T(t_l) B^T B R_{gi}(t_l)$ is a positive semidefinite matrix, even if the collective FE condition is not satisfied.

Regarding the *second property*, we start with

$$D_i(n_{Di}) = \sum_{l=1}^{n_{Di}} R_{gi}^T(t_l) B^T B R_{gi}(t_l) \ge \lambda_{min}(B^T B) \sum_{l=1}^{n_{Di}} R_{gi}^T(t_l) R_{gi}(t_l)$$

where $\lambda_{min}(B^T B) > 0$ because B is a full column rank matrix, and $\sum_{l=1}^{n_{Di}} R_{gi}^T(t_l) R_{gi}(t_l)$ is equal to

$$I_{n_u} \otimes \sum_{l=1}^{n_{Di}} \int_{t_l - \delta t_i}^{t_l} \sum_{j \in \mathscr{N}_i^a} a_{ij}^a x_j(\tau) d\tau \int_{t_l - \delta t_i}^{t_l} \sum_{j \in \mathscr{N}_i^a} a_{ij}^a x_j^T(\tau) d\tau$$

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which, when the collective FE Assumption 1 is satisfied, means $\sum_{l=1}^{n_{Di}} R_{gi}^T(t_l)R_{gi}(t_l) \succ \mathbf{0}$.

The *third property* is guaranteed by the definition of acceptable excitation time (17) which acts as a criterion to decide whether we should consider new data in the update law (18) or not. Therefore, the proof is immediate. We note that at least one update exists for each data collection matrix if the collective FE conditional is met. This is because we start from $D_i = \mathbf{0}$ and $D_i(n_{Di}) \succ \mathbf{0}$ when the i^{th} agent is sufficiently and collectively excited at time t_{si}. ■

To facilitate the analysis of the main theorem, we aggregate the cooperation signal u_{ci} of (5), and find

$$u_c = (\mathscr{H}_c \otimes K)x$$

where $u_c = \mathrm{col}\{u_{ci}\}$ for all $i \in \{1,2,...,N\}$, and $\mathscr{H}_c$ is the modified Laplacian matrix associated to the nonsymmetric cooperation layer $\mathscr{G}_c$. We further find the following representation of the three-layer (closed-loop) interconnected MAS:

$$\dot{x} = \bar{A}x + \kappa\bar{B}v + \kappa\bar{B}\bar{E}_c v + \bar{B}u_a + \bar{B}\tilde{C}\bar{\mathscr{A}}_a x \tag{19}$$

where $\bar{E}_c = \big((\frac{\mathscr{H}_c}{\kappa} - I_N)\otimes I_{n_u}\big)$, $\bar{\mathscr{A}}_a = \mathscr{A}_a \otimes I_{n_x}$, $\bar{C} = \mathrm{diag}\{C_i\}$, and $u_a = \mathrm{col}\{u_{ai}\}$ for all $i \in \{1,2,...,N\}$ (if $i \notin \mathscr{S}_a$, we consider zero for the associated interconnection allocation matrix C_i and data-assisted decoupling signal u_{di}). We also define:

$$\Omega_b(t) = x^T(t)\bar{P}x(t) + \frac{1}{\lambda_{min}(\Gamma)}\|\tilde{\theta}(t)\|^2 \tag{20}$$

$$\rho_\omega = \min\big(\frac{\lambda_{min}(W_{xK})}{\lambda_{max}(P)}, 2\gamma_D^{min}\lambda_{min}(\Gamma)\lambda_{min}(D(n_D))\big) \tag{21}$$

where $\tilde{\theta} = \mathrm{col}\{\tilde{\theta}_i\}$ and $\Gamma = \mathrm{diag}\{\Gamma_i\}$ for all $i \in \mathscr{S}_a$, and $W_{xK} = W_x + K^T W_v K \succ \mathbf{0}$, $\gamma_D^{min} = \min_i\{\gamma_{Di}\}$ $\forall i \in \mathscr{S}_a$, and $\lambda_{min}(D(n_D)) = \min_i\{\lambda_{min}(D_i(n_{Di}))\}$.

Theorem 1: In a closed-loop interconnected MAS of agents (1), the two-layer data-assisted distributed stabilization protocol (3) and (5), and the update rule (18), the following is guaranteed under the collective FE Assumption 1:

1) All trajectories $x_i(t)$ and $\tilde{\theta}_i(t)$ are bounded $\forall t \geq 0$,
2) All trajectories $x_i(t)$ and $\tilde{\theta}_i(t)$ exponentially converge to the origin for $t \geq t_s = \max_i\{t_{si}\}$ where t_{si} are defined in Assumption 1,
3) All trajectories $x_i(t)$ and $\tilde{\theta}_i(t)$ are upper-bounded as in (22) and (23), respectively, where $\Omega_b(t_s) \leq \Omega_b(0)$.

$$\|x_i(t)\| \leq \begin{cases} \sqrt{\frac{1}{\lambda_{min}(P)}\Omega_b(0)}, & \forall 0 \leq t < t_s \\ \sqrt{\frac{\exp^{-\rho_\omega(t-t_s)}}{\lambda_{min}(P)}\Omega_b(t_s)}, & \forall t \geq t_s \end{cases} \tag{22}$$

$$\|\tilde{\theta}_i(t)\| \leq \begin{cases} \sqrt{\lambda_{max}(\Gamma)\Omega_b(0)}, & \forall 0 \leq t < t_s \\ \sqrt{\lambda_{max}(\Gamma)\exp^{-\rho_\omega(t-t_s)}\Omega_b(t_s)}, & \forall t \geq t_s \end{cases} \tag{23}$$

Proof: Step 1) We propose a Lyapunov function:

$$\Omega(x,\tilde{\theta}) = \bar{V}(x) + \sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i \succ \mathbf{0}$$

where $\bar{V}(x) = x^T\bar{P}x$ was introduced in Fact 2 and $\gamma_i \succ \mathbf{0}$ in the update law (18). Along the unknown trajectories of the three-layer interconnected MAS (19), we find:

$$\dot{\Omega} = \dot{\bar{V}} + 2\sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T\Gamma_i^{-1}\dot{\tilde{\theta}}_i = \bar{V}_x^T\dot{x} + 2\sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T\Gamma_i^{-1}\dot{\tilde{\theta}}_i$$

The first part of $\dot{\Omega}$ results in the following inequality:

$$\bar{V}_x^T\dot{x} = \bar{V}_x^T\left(\bar{A}x + \kappa\bar{B}v\right) + \kappa\bar{V}_x^T\bar{B}\bar{E}_c v - \bar{V}_x^T\bar{B}\tilde{C}\bar{\mathscr{A}}_a x$$

in which $\tilde{C} = \bar{\hat{C}} - \bar{C}$, $\bar{\hat{C}} = \mathrm{diag}\{\hat{C}_i\}$, and $\bar{C} = \mathrm{diag}\{C_i\}$. Using the second equality in Fact 2, we know $\kappa\bar{V}_x^T\bar{B}\bar{E}_c v = -2v^T\big((\frac{\Delta\mathscr{H}_c + \mathscr{H}_c^T\Delta}{2\kappa} - \Delta)\otimes W_v\big)v \preccurlyeq \mathbf{0}$ where the negative semi definiteness is immediate by definition of κ and W_v in Step 1, Design Procedure 1.

Using $v = -\kappa\bar{W}_v^{-1}\bar{B}^T\bar{P}x$ and $\bar{W}_{xK} = I_N \otimes W_{xK}$, and based on the second equality in Fact 2:

$$\begin{aligned}\dot{\bar{V}} &\leq -x^T\bar{W}_x x - v^T\bar{W}_v v + \frac{2}{\kappa}v^T\bar{W}_v\tilde{C}\bar{\mathscr{A}}_a x \\ &\leq -x^T\bar{W}_{xK}x - 2\sum_{i\in\mathscr{S}_a}x_i^T PB\tilde{C}_i\sum_{j\in\mathscr{N}_i^a}a_{ij}^a x_j \\ &\leq -x^T\bar{W}_{xK}x - 2\sum_{i\in\mathscr{S}_a}x_i^T PB\sum_{j\in\mathscr{N}_i^a}a_{ij}^a(I_{n_u}\otimes x_j^T)\tilde{\theta}_i \\ &\leq -x^T\bar{W}_{xK}x - 2\sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T\sum_{j\in\mathscr{N}_i^a}a_{ij}^a(I_{n_u}\otimes x_j)B^T P x_i\end{aligned}$$

The second part of $\dot{\Omega}$ leads to the following inequality:

$$\begin{aligned}2\sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T\Gamma_i^{-1}\dot{\tilde{\theta}}_i =& 2\sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T\sum_{j\in\mathscr{N}_i^a}a_{ij}^a(I_{n_u}\otimes x_j)B^T P x_i \\ & -2\sum_{i\in\mathscr{S}_a}\gamma_{Di}\tilde{\theta}_i^T D_i(n_{Di})\tilde{\theta}_i\end{aligned}$$

Consequently, for the candidate Lyapunov function and closed-loop trajectories, we find

$$\dot{\Omega} \leq -x^T\bar{W}_{xK}x - 2\sum_{i\in\mathscr{S}_a}\gamma_{Di}\tilde{\theta}_i^T D_i(t_{n_{Di}})\tilde{\theta}_i \preccurlyeq \mathbf{0}$$

where the negative semidefiniteness is concluded because $D_i(n_{Di}) \succcurlyeq \mathbf{0}$ for all $t > 0$ (see the first item in Properties 1). Thus, all state trajectories and estimated parameter values remain bounded even in the transient time, when collective FE Assumption 1 is not satisfied [25]. (See Step 3 for the transient bounds.)

Step 2: When $t \geq t_s$, the latter inequality on $\dot{\Omega}$ can be rewritten as follows:

$$\dot{\Omega} \leq -\frac{\lambda_{min}(W_{xK})}{\lambda_{max}(P)}V(x) - 2\alpha\sum_{i\in\mathscr{S}_a}\tilde{\theta}_i\Gamma_i^{-1}\tilde{\theta}_i$$

where $\alpha = \gamma_D^{min}\lambda_{min}(\Gamma)\lambda_{min}(D(n_D)) > 0$ because $D_i(n_{Di}) \succ \mathbf{0}$ for all $t \geq t_s$ (see the second item in Properties 1). In fact, we know that the minimum $\lambda_{min}(D_i(n_{Di}))$ of each agent i occurs when at time t_{si} when it has been just sufficiently excited, and the future updates will either keep it constant or increase it (see the third item in Properties 1). Using the definition of $\rho_\omega > 0$, given prior to the main statement of this theorem, we find $\dot{\Omega} \leq -\rho_\omega\Omega$ which, based on the comparison lemma [25], indicates $\Omega(t) \leq \exp^{-\rho_\omega(t-t_s)}\Omega(t_s)$ where $\Omega(t) \triangleq \Omega(x(t),\tilde{\theta}(t))$. Thus, we find $\lim_{t\to\infty}\Omega(t) = 0$ which implies that all state and estimation error trajectories of the multilayer interconnected MAS exponentially converge to the origin ((6)).

Step 3) The Lyapunov function Ω of this theorem satisfies

$$\lambda_{min}(P)\|x(t)\|^2 \leq V(t) \leq \Omega(t)$$

$$\frac{1}{\lambda_{max}(\Gamma)}\|\tilde{\theta}(t)\|^2 \leq \sum_{i\in\mathscr{S}_a}\tilde{\theta}_i^T(t)\Gamma_i^{-1}\tilde{\theta}_i(t) \leq \Omega(t)$$

where $\tilde{\theta} = \text{col}\{\tilde{\theta}_i\}$ for $i \in \mathscr{S}_a$. Thus, $\|x(t)\| \leq \sqrt{\frac{1}{\lambda_{min}(P)}\Omega(t)}$ and $\|\tilde{\theta}(t)\| \leq \sqrt{\lambda_{max}(\Gamma)\Omega(t)}$.

Based on the first step of this proof, $\dot{\Omega} \preccurlyeq 0$ holds for all time including $0 \leq t \leq t_s$. Thus,

$$\Omega(t) \leq \Omega(0) = \bar{V}(0) + \sum_{i \in \mathscr{S}_a} \tilde{\theta}_i^T(0)\Gamma_i^{-1}\tilde{\theta}_i(0) \leq \Omega_b(0)$$

where $\Omega_b(0)$ can be found from (20). Consequently, for all $0 \leq t \leq t_s$:

$$\|x(t)\| \leq \sqrt{\frac{1}{\lambda_{min}(P)}\left(x^T(0)\bar{P}x(0) + \frac{\|\tilde{\theta}(0)\|^2}{\lambda_{min}(\Gamma)}\right)}$$

$$\|\tilde{\theta}(t)\| \leq \sqrt{\lambda_{max}(\Gamma)\left(x^T(0)\bar{P}x(0) + \frac{\|\tilde{\theta}(0)\|^2}{\lambda_{min}(\Gamma)}\right)}$$

Based on the second step of this proof, we know $\Omega(t) \leq \exp^{-\rho\omega(t-t_s)}\Omega(t_s)$ holds for all $t \geq t_s$. Thus, $\Omega(t) \leq \exp^{-\rho\omega(t-t_s)}\left(x^T(t_s)\bar{P}x(t_s) + \sum_{i \in \mathscr{S}_a}\tilde{\theta}_i^T(t_s)\Gamma_i^{-1}\tilde{\theta}_i(t_s)\right) \leq \exp^{-\rho\omega(t-t_s)}\Omega_b(t_s)$ where $\Omega_b(t_s)$ can be found from (20). For all $t > t_s$, we have $\|x(t)\| \leq \sqrt{\frac{\exp^{-\rho\omega(t-t_s)}}{\lambda_{min}(P)}\left(x^T(t_s)\bar{P}x(t_s) + \frac{\|\tilde{\theta}(t_s)\|^2}{\lambda_{min}(\Gamma)}\right)}$ and $\|\tilde{\theta}(t)\| \leq \sqrt{\lambda_{max}(\Gamma)\exp^{-\rho\omega(t-t_s)}\left(x^T(t_s)\bar{P}x(t_s) + \frac{\|\tilde{\theta}(t_s)\|^2}{\lambda_{min}(\Gamma)}\right)}$. Recalling that $\|x_i\| \leq \|x\|$ and $\|\tilde{\theta}_i\| \leq \|\tilde{\theta}\|$, the bounds in (22) and (23) can be derived. It is also evident that $\Omega_b(t_s) \leq \Omega_b(0)$ based on the first step of this proof. Thus, these inequalities can be further upper-bounded using $\Omega_b(0)$ instead of $\Omega_b(t_s)$. ■

IV. Concluding Remarks

We develop a distributed protocol to guarantee the exponential convergence of state trajectories to the origin in an interconnected MAS with completely unknown interconnection allocation matrices. A nonsymmetric cooperation layer whose topology is completely independent of the agent and data-assisted decoupling layer is designed using a matrix algebraic approach. A data-assisted decoupling protocol compensates for the effect of unknown interconnections among agents' dynamics. A new finite excitation condition is proposed to relax the need for either persistency of excitation or the excitation of all agents in the interconnected multiagent system. We prove that all interconnection matrices are exponentially estimated under the proposed collective FE condition. Consequently, exponential convergence of the state trajectories of the interconnected multiagent system to the origin is also guaranteed. Extension of the collective FE-based idea to the output feedback problem is an interesting future topic [26].

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