Stability Analysis of Cohen-Grossberg Neural Networks With Unbounded Delays

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***Abstract*—The asymptotic stability problem of Cohen-Grossberg neural networks with distributed delays is investigated in this paper. One new uniqueness theorem for the existence of the unique equilibrium of the class of neural networks is presented. Based on the new result, using the Lyapunov stability theory and linear matrix inequality (LMI) technique, and combining Cauchy's inequality, some new conditions for the asymptotic stability of Cohen-Grossberg neural networks with distributed delays are presented. In our results, we do not assume the signal propagation functions to be bounded, differentiable, strictly increasing, and even to satisfy the Lipschitz condition. Moreover, the symmetry of the connection matrix is not also necessary. Thus, we improve some previous works of other researchers. Some examples are also worked out to validate the advantages of our results.**

***Keywords*-Cohen-Grossberg neural networks; asymptotic stability; distributed delay.**

I. INTRODUCTION

In recent years, there has been increasing interest in the potential applications of neural networks in many areas. Many scientists established various types of conditions for the asymptotic stability, absolute stability, complete stability and exponential stability of Hopfield neural networks (HNN), cellular neural networks (CNN), bidirectional associative memory (BAM) neural networks and Cohen-Grossberg neural networks (CGNN) (see [1]–[3] and the references therein).

The Cohen-Grossberg neural network models, initially proposed and studied in Cohen and Grossberg [4], have attracted increasing interest. This class of networks has good application in associative memory, parallel computation and optimization problems, which has been an active area of research and has received much attention. Wang and Zou [5] presented some sufficient conditions for exponential stability of delayed CGNN with asymmetric connection matrix and gave an estimate of the convergence rate. In [6], several sufficient conditions were obtained to ensure a class of delayed CGNN to be asymptotically stable. In [7], based on Lyapunov stability theory and LMI, several sufficient conditions were obtained to ensure delayed CGNN to be robustly stable. Yuan and Cao [9] gave an analysis of global asymptotic stability for a delayed Cohen-Grossberg neural network via nonsmooth analysis. Lu and Chen [8] provided criteria for global stability and global exponential stability with consideration of signs of entries of the connection matrix by using the concept of Lyapunov diagonally stability (LDS) and LMI approach. All of these results above are based on the assumption that the signal propagation functions satisfy either the Lipschitz condition or the boundedness. However, in many evolutionary processes as well as optimal control models and flying object motions, there are many bounded monotone-nondecreasing signal functions which do not satisfy the Lipschitz condition [10]. Therefore, it is important and, in fact, necessary to study the issue of global stability of such a dynamical neural network with non-Lipschitzian activation functions.

Although the use of constant fixed delays in models of delayed feedback provides a good approximation in simple circuits composed of a small number of cells, neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths [2]. In these circumstances, the transmission of signal is no longer instantaneous and cannot be modelled with discrete delays. A more appropriate way is to incorporate distributed delays. Therefore, the studies of the model with distributed delays have more important significance than the ones of model with discrete delays and the distributed delay becomes a discrete delay when the delay kernel is a δ-function, at a certain time (see, Remark 4). However, to the best of our knowledge, few authors [11] have considered Cohen-Grossberg neural network model with distributed delays. Furthermore, the asymptotic stability analysis for CGNN with distributed delays via LMI technique has never been tackled.

Motivated by the above discussions, our objective in this paper is to study further the existence and uniqueness, and global asymptotic stability for the equilibrium point of CGNN with distributed delays, as in [11], but we drop the boundness, differentiability, monotonicity and the Lipschitz condition of the activation functions. Moreover, the symmetry of the connection matrix is not also necessary and the kernel functions need not satisfy the hypothesis $\int_0^\infty sK_j(s)ds < \infty$. Here, a new approach

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based on LMI technique combining Cauchy's inequality, is developed to obtain sufficient conditions, which guarantee the existence, uniqueness and global asymptotic stability for the equilibrium point of CGNN with distributed delays. The conditions are less conservative than those [11]. Therefore, our proposed results are practical and improve some previous works of other researchers.

II. Model description

In this paper, we consider the following model

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -a_i(x_i(t))\Bigg[b_i(x_i(t)) - \sum_{j=1}^{n} w_{ij} f_j(x_j(t)) - J_i \\ &\quad - \sum_{j=1}^{n} w_{ij}^{\tau} \int_{-\infty}^{t} K_j(t-s) f_j(x_j(s)) ds\Bigg], \qquad (1)\end{aligned}$$

where $x_i(t)$ corresponds to the state of the ith unit at time t; J_i, $i = 1, 2, \cdots, n$, denote the constant inputs from outside of the system and w_{ij} represent the connection weights. $a_i(x_i(t))$ and $b_i(x_i(t))$, $i = 1, 2, \cdots, n$, are the amplification functions and the self-signal functions, respectively, while $f_j(x_j)$, $j = 1, 2, \cdots, n$, are the activation functions. $W = (w_{ij})_{n\times n}$ and $W = (w_{ij}^{\tau})_{n\times n}$ are the normal and the delayed connection weight matrix, respectively. The delay kernel K_j is a real value non-negative continuous function defined on $[0, \infty)$ and satisfies, for each j,

$$\int_0^{\infty} K_j(s) ds = 1.$$

Throughout the paper, we always assume that

• (H_1) $a_i(x)$ are continuous and positive, i.e., $a_i(x) > 0$, for all $x \in \mathbb{R}, i = 1, 2, \cdots, n$;

• (H_2) each function $b_i(x)$ is locally Lipschitz continuous and there exists $\gamma_i > 0$ such that

$$u[b_i(u + x) - b_i(x)] \geq \gamma_i u^2,$$

for all $x \in \mathbb{R}, i = 1, 2, \cdots, n$;

• (H_3) the functions f_i $(i = 1, 2, \cdots, n)$ satisfy $vf_i(v) > 0$ $(v \neq 0)$, and there exist positive constants μ_i $(i = 1, 2, \cdots, n)$ such that

$$\mu_i = \sup_{v\neq 0} \frac{f_i(v)}{v}, \ \ \forall v \in \mathbb{R}.$$

Remark 1. In [5]–[8], the activation function was required to be bounded, positive and continuous. However, the upper bound of amplification function in this paper is not required. In addition, assumption (H_3) in this paper is as same as that in [5], [9], the condition of differentiability of behaved function in [6]–[8] is not required.

Remark 2. Note that the assumption (H_3) is weaker than the locally and partially Lipschitz condition which is mostly used in literature [5]–[9]. The activation functions such as sigmoid type and piecewise linear type are also the special case of the function satisfying assumption (H_3). Further, if $f_j(\cdot)$ for each $j = 1, 2, \cdots, n$ is a Lipschitz function, then μ_j for each $j = 1, 2, \cdots, n$ can be replaced by the respective Lipschitz constant.

Remark 3. The kernel functions need not satisfy the hypothesis $\int_0^{\infty} sK_j(s)ds < \infty$ which is required in [11].

Let $\mathcal{C}[X, Y]$ be a continuous mapping set from the topological space X to the topological space Y, and $\mathbb{R}_+ = [0, \infty)$. Especially, $\mathcal{C} \triangleq \mathcal{C}[(-\infty, 0], \mathbb{R}^n]$. Denote A^T and A^{-1} to be the transpose and the inverse of any square matrix A. We use $A > 0$ $(A < 0)$ to denote a positive- (negative-) definite matrix A; and I is used to denote the $n \times n$ identity matrix.

Definition 1. $x(t) = x^* \in \mathbb{R}^n$ is called to be an equilibrium point of system (1), if the constant vector $x^* = (x_1^*, \cdots, x_n^*)^T$ satisfies

$$b_i(x_i^*) = \sum_{j=1}^{n} w_{ij} f_j(x_j^*) + \sum_{j=1}^{n} w_{ij}^{\tau} \int_{-\infty}^{t} K_j(t-s) f_j(x_j^*) ds + J_i$$

for $i = 1, 2, \cdots, n$.

Definition 2. The set $S \subset C$ is called to he a positive invariant set of the system (1) if for any initial value $\phi \in S$, we have the solution $x(t) \in S$, for $t \geq 0$.

III. Existence and uniqueness of the equilibrium point

In order to study the existence and uniqueness of the equilibrium point, we rewrite the system (1) as

$$\dot{X}(t) = F(X(t)), \qquad (2)$$

where
$X(t) = (x_1(t), \cdots, x_n(t))^T$,
$F(X(t)) = (\theta_1(t), \cdots, \theta_n(t))^T$ with

$$\begin{aligned}\theta_i(t) &= -a_i(x_i(t))\Bigg[b_i(x_i(t)) - \sum_{j=1}^{n} w_{ij} f_j(x_j(t)) \\ &\quad - \sum_{j=1}^{n} w_{ij}^{\tau} \int_{-\infty}^{t} K_j(t-s) f_j(x_j(s)) ds - J_i\Bigg]\end{aligned}$$

for $i = 1, 2, \cdots, n$.

We consider the initial value problem associated with the autonomous system (2), in which the initial functions are given by

$$x_i(t) = \phi_i(t), \ -\infty < t \leq 0, \ i = 1, 2, \cdots, n, \qquad (3)$$

where $\phi_i(t)$ $(i = 1, 2, \cdots, n)$ are assumed to be bounded and continuous functions on $(-\infty, 0]$. Let Ω be an open subset of $\mathbb{R}^n$. For any $\theta \in \mathbb{R}^n$, we define $\|\theta\| = \sum_{j=1}^{n} |\theta_j|$.

Theorem 1. Let $F : \Omega \to \mathbb{R}^n$ be continuous and satisfy the following condition: corresponding to each point $\theta \in \Omega$

and its neighborhood U, there exists a constant $k > 0$, and functions h_j and Ψ_l $(j, l = 1, 2, \cdots, n)$ such that

$$\|F(\vartheta) - F(\theta)\| \leq k\|\vartheta - \theta\| + k\sum_{l=1}^{n}\Big|\Psi_l(h_j(\vartheta)) - \Psi_l(h_j(\theta))\Big|$$

on U, where each $h_j : U \to \mathbb{R}$ is a continuously differentiable function in θ satisfying the relation

$$\sum_{j=1}^{n}\frac{\partial h_j(\theta)}{\partial \theta_i}F_i(\theta) \neq 0 \quad \text{on } U$$

and each $\Psi_l : \mathbb{R} \to \mathbb{R}$ is continuous and of bounded variation on bounded subintervals. Then, there exists a unique solution for the initial value problem Eq. (1) or Eq. (2) with (3).

IV. GLOBAL ASYMPTOTIC STABILITY OF THE EQUILIBRIUM POINT

In this section, we consider the global exponential stability for the system (1). Suppose $x^* = (x_1^*, \cdots, x_n^*)^T$ is any equilibrium point of the system (1).

Theorem 2. Suppose Theorem 1 hold for the functions f_j $(j = 1, 2, \cdots, n)$, and assumptions $(H_1) - (H_3)$ are satisfied. The equilibrium point x^* for the system (1) with (3) is globally asymptotically stable, if there exist a matrix $P > 0$, and two diagonal matrices $R > 0, Q > 0$, such that

$$\Omega = \begin{bmatrix} -2P\Gamma + R & PW & PW^{\tau} \\ W^T P & -RL^{-2} + Q & 0 \\ (W^{\tau})^T P & 0 & -Q \end{bmatrix} < 0, \quad (4)$$

where $\Gamma = \text{diag}[\gamma_1, \gamma_2, \cdots, \gamma_n]$, $L = \text{diag}[\mu_1, \mu_2, \cdots, \mu_n]$.

Theorem 3. Suppose Theorem 1 hold for the functions f_j $(j = 1, 2, \cdots, n)$, and assumptions $(H_1) - (H_3)$ are satisfied. The equilibrium point x^* of the system (1) with (3) is globally asymptotically stable if there exist a matrix $P > 0$, and two diagonal matrices $D > 0, Q > 0$, such that

$$\Theta = \begin{bmatrix} -P\Gamma - \Gamma P & PW & PW^{\tau} \\ W^T P & \Xi & DW^{\tau} \\ (W^{\tau})^T P & (W^{\tau})^T D & -Q \end{bmatrix} < 0 \quad (5)$$

where

$\Xi = -2D\Gamma L^{-1} + DW + W^T D + Q$,

$\Gamma = \text{diag}[\gamma_1, \gamma_2, \cdots, \gamma_n]$, $L = \text{diag}[\mu_1, \mu_2, \cdots, \mu_n]$.

Remark 4. If delay kernel functions $k_j(t)$ are of the form

$$k_j(t) = \delta(t - \tau_j), \quad j = 1, 2, \cdots, n, \quad (6)$$

then system (1) reduces to CGNN with discrete delays which has been lucubrated in many literatures. And many crucial results for dynamics of this class of neural networks have been obtained. Therefore the discrete delays can be included in our models by choosing suitable kernel functions.

Remark 5. We can see that the LMI criterion (4) is similar to condition (27) of Corollary 1 in Ref. [7]. However, it should be noted that our result contain that in Ref. [7], because the discrete delays can be included in our models by choosing suitable kernel functions as said in Remark 4. Moreover, the signal propagation functions need not to be bounded and satisfy the Lipschitz condition in this paper, while the assumptions are required in Ref. [7].

Remark 6. For system (1), when $a_i(x_i(t)) = 1$, $b_i(x_i(t)) = b_i(t)x_i(t)$ (in which $b_i(t)$ is not only differentiable but also bounded on interval $(-\infty, +\infty)$, and its maximal lower bound is denoted as $\gamma_i > 0$) and let $W \equiv 0$, the system (1) reduces to a class of pure-delay models with distributed delays which has been studied in [12], but the results derived in this paper are less conservative than those in [12] because of the loose restrictions on the activation functions; when the delay kernel is a δ-function based on the case above, i.e., the distributed delay becomes a discrete delay, system (1) has been briefly indicated in [2].

Remark 7. If the activation functions are bounded and satisfy the Lipschitz condition, Theorem 2 is equivalent to Corollary 1 in Ref. [7]; Theorems 2-3 extend and improve Theorem 3 in Ref. [6].

If the model (1) is simplified to cellular neural networks with time delay, that is, let $a_i(x) = 1$, $b_i(x) = x$, $f_i(x) = 0.5(|x+1| - |x-1|)$, then we have $\Gamma = I$, $L = I$. We can have the following corollaries.

Corollary 1. The equilibrium point x^* of the system (1) with (3) is globally asymptotically stable if there exist a matrix $P > 0$, and two diagonal matrices $R > 0, Q > 0$, such that

$$\Omega = \begin{bmatrix} -2P + R & PW & PW^{\tau} \\ W^T P & -R + Q & 0 \\ (W^{\tau})^T P & 0 & -Q \end{bmatrix} < 0. \quad (7)$$

Corollary 2. The equilibrium point x^* of the system (1) with (3) is globally asymptotically stable if there exist a matrix $P > 0$, and two diagonal matrices $D > 0, Q > 0$, such that

$$\Theta = \begin{bmatrix} -2P & PW & PW^{\tau} \\ W^T P & -2D + DW + W^T D + Q & DW^{\tau} \\ (W^{\tau})^T P & (W^{\tau})^T D & -Q \end{bmatrix} < 0. \quad (8)$$

Remark 8. The common feature for the asymptotic stability of CGNN with distributed delays is that the conditions are expressed in terms of some nonlinear inequalities, which involve the tuning of some scalar parameters. Although the suitability of these criteria is improved due to these adaptable parameters, it is not easy to check the availability of the scalars since we have no a systematic tuning procedure by now. The criteria in Theorems 2-3 are LMI conditions, which do not require the tuning of parameters.

V. NUMERICAL SIMULATIONS

In the previous sections, some new sufficient criteria for the global asymptotic stability of the Cohen-Grossberg neural networks with distributed delays have been derived.

In the following examples, for simplicity, some Cohen-Grossberg models with only two neurons are simulated and analyzed.

Example 1. Consider the following CGNN with distributed delays:

$$\begin{cases} \frac{dx_1(t)}{dt} = -(2+\sin(x_1(t))\Big[2x_1(t) \\ \quad -\frac{1}{4}|x_1(t)| - \frac{1}{4}|x_2(t)| \\ \quad -\frac{1}{4}\int_{-\infty}^{t} K(t-s)|x_1(s)|ds \\ \quad -\frac{1}{4}\int_{-\infty}^{t} K(t-s)|x_2(s)|ds + 2\Big], \\ \frac{dx_2(t)}{dt} = -(3+\cos(x_2(t))\Big[2x_2(t) \\ \quad -\frac{1}{6}|x_1(t)| - \frac{1}{3}|x_2(t)| \\ \quad -\frac{1}{3}\int_{-\infty}^{t} K(t-s)|x_1(s)|ds \\ \quad -\frac{2}{3}\int_{-\infty}^{t} K(t-s)|x_2(s)|ds - 2\Big], \end{cases} \tag{9}$$

with initial values

$$\begin{cases} \phi_1(s) = 0.8, s \in (-\infty, 0], \\ \phi_2(s) = 0.5, s \in (-\infty, 0]. \end{cases} \tag{10}$$

One can check $(H_1)-(H_3)$ are satisfied. In this example,

$$W = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}, \; W^{\tau} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix},$$

$$J = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \; \Gamma = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

For numerical simulation, we choose the delay kernel as $K(r) = e^{-r}$. Applying our Theorem 2, by solving the LMI (4) using the Matlab LMI Toolbox, a feasible solution is

$$P = \begin{bmatrix} 0.9813 & -0.0423 \\ -0.0423 & 0.9311 \end{bmatrix} > 0,$$

$$Q = \begin{bmatrix} 1.2750 & 0 \\ 0 & 1.2750 \end{bmatrix} > 0,$$

$$R = \begin{bmatrix} 2.5499 & 0 \\ 0 & 2.5499 \end{bmatrix} > 0.$$

Therefore, the conditions of Theorem 2 in this paper are satisfied, which implies system (9) has a unique equilibrium point, which is asymptotically stable. Figure 1 shows the time responses of the state variables $x_1(t)$ and $x_2(t)$ with 10 initial states. They have confirmed that by fulfilling the proposed conditions, the existence of a unique equilibrium point $x^* = [-0.5218, 1.9131]^T$, and the global asymptotic stability of system (9) are guaranteed.

Since $f_1(x) = f_2(x) = |x|$ here, we can easily verify that the assumptions of boundedness, monotonicity, and differentiability for the activation functions is not available, so the results in [11] and the references cited therein can not be applicable to system (9).

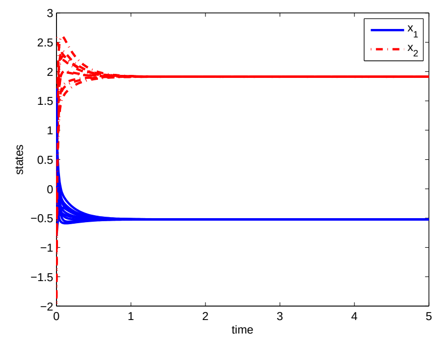

Figure 1. Transient response of state variables $x_1(t)$ and $x_2(t)$ for Example 1.

Example 2. To illustrate Theorem 3, we consider the following Cohen-Grossberg model with distributed delays:

$$\begin{cases} \frac{dx_1(t)}{dt} = -(2+\sin(x_1(t))\Big[2x_1(t) \\ \quad -\frac{1}{4}f_1(x_1(t)) - \frac{1}{4}f_2(x_2(t)) \\ \quad -\frac{1}{4}\int_{-\infty}^{t} K(t-s)f_1(x_1(s))ds \\ \quad -\frac{1}{4}\int_{-\infty}^{t} K(t-s)f_2(x_2(s))ds + 1\Big], \\ \frac{dx_2(t)}{dt} = -(3+\cos(x_2(t))\Big[2x_2(t) \\ \quad -\frac{1}{6}f_1(x_1(t)) - \frac{1}{3}f_2(x_2(t)) \\ \quad -\frac{1}{3}\int_{-\infty}^{t} K(t-s)f_1(x_1(s))ds \\ \quad -\frac{2}{3}\int_{-\infty}^{t} K(t-s)f_2(x_2(s))ds - 1\Big], \end{cases} \tag{11}$$

with initial values

$$\begin{cases} \phi_1(s) = -0.5, s \in (-\infty, 0], \\ \phi_2(s) = 0.5, s \in (-\infty, 0], \end{cases} \tag{12}$$

where f_1 and f_2 are exponentially weighted time averages of the sampled pulse

$$f_j(x_j(s)) = \int_{-\infty}^{s} x_j(\theta)e^{\theta - s}d\theta, \; j = 1, 2, \tag{13}$$

the functions x_1 and x_2 equal one when a pulse arrives at time s and zero when no pulse arrives. Obviously, f_j satisfy (H_3) with μ_j but it does not satisfy the Lipschitz condition.

In this example,

$$W = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}, \; W^{\tau} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix},$$

$$J = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \; \Gamma = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \; L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

By taking $K_j(r) = \frac{2}{\pi(1+r^2)}, j = 1, 2$. Then we have $\int_0^{\infty} K_j(s)ds = 1$. Clearly, the kernel functions K_j $(j = 1, 2)$ do not satisfy the hypothesis $\int_0^{\infty} sK_j(s)ds < \infty$. Applying our Theorem 3, by solving (5) using the Matlab LMI Toolbox, a feasible solution is

$$P = \begin{bmatrix} 0.4038 & -0.0174 \\ -0.0174 & 0.3832 \end{bmatrix} > 0,$$

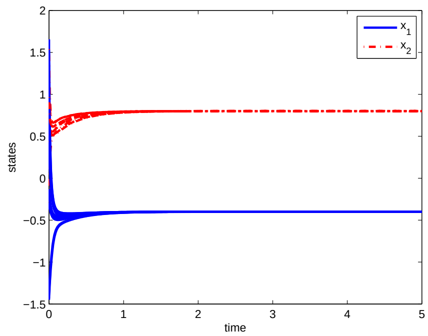

Figure 2. Transient response of state variables $x_1(t)$ and $x_2(t)$ for Example 2.

$$Q = \begin{bmatrix} 1.4488 & 0 \\ 0 & 1.4488 \end{bmatrix} > 0,$$

$$D = \begin{bmatrix} 0.8481 & 0 \\ 0 & 0.8481 \end{bmatrix} > 0.$$

Therefore, the conditions of Theorem 3 in this paper are satisfied, which implies system (11) has a unique equilibrium point. It is easy to verify that $x^* = [-0.4000, 0.8001]^T$ is the unique equilibrium point which is asymptotically stable. Figure 2 shows the time responses of the state variables $x_1(t)$ and $x_2(t)$ with 10 initial states. However, it is very difficult to obtain the result by using the technique in [11] for system (11) with the non-Lipschitzian activation functions.

VI. Conclusions

In this paper, using the Lyapunov stability theory and LMI technique, and combining Cauchy's inequality, we have derived some new sufficient conditions in term of LMI for the existence and uniqueness, and global asymptotic stability for the equilibrium point of CGNN model with distributed delays. The results presented here are more general and easier to check than those given in the related literature because the restrictions of sufficient conditions are less restrictive than those in [5]–[9]. Two examples are provided to illustrate our results.

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References

[1] K. Gopalsamy and X. He, "Stability in asymmetric Hopfield nets with transmission delays," *Physica D* 76 (1994) 344-358.

[2] Y. T. Li and C. B. Yang, "Global exponential stability analysis on impulsive BAM neural networks with distributed delays," *J. Math. Anal. App.* 324 (2006) 1125-1139.

[3] O. Faydasicok and S. Arik, "Equilibrium and stability analysis of delayed neural networks under parameter uncertainties," *Applied Mathematics and Computation* 218 (2012) 6716-6726.

[4] M. A. Cohen and S. Grossberg, Absolute stability of global pattern formation and parallel memory storage by competitive neural networks," *IEEE Trans. Syst., Man, Cybern.* 13(5) (1983) 815-826.

[5] L. Wang and X. F. Zou, "Exponential stability of Cohen-Grossberg neural networks," *Neural Networks* 15 (2002) 415-422.

[6] T. P. Chen and L. B. Rong, "Delay-independent stability analysis of Cohen-Grossberg neural networks," *Phys. Lett. A* 317 (2003) 436-449.

[7] L. B. Rong, "LMI-based criteria for robust stability of Cohen-Grossberg neural networks with delay," *Phys. Lett. A* 339 (2005) 63-73.

[8] W. L. Lu and T. P. Chen, "New conditions on global stability of Cohen-Grossberg neural networks," *Neural Comput.* 15 (2003) 1173-1189.

[9] K. Yuan and J. D. Cao, "An analysis of global asymptotic stability of delayed Cohen-Grossberg neural networks via nonsmooth analysis," *IEEE Trans. Circuits Syst. I* 52 (2005) 1854-1861.

[10] B. Kosko, *Neural Networks and Fuzzy System-A Dynamical Systems Approach to Machine Intelligence*, New Delhi, India: Prentice-Hall of India, 1994.

[11] X. F. Liao, C. G. Li and K. W. Wong, "Criteria for exponential stability of Cohen-Grossberg neural networks," *Neural Networks* 17 (2004) 1401-1414.

[12] Q. Zhang, X. P. Wei and J. Xu, "Global exponential stability of Hopfield neural networks with continuously distributed delays," *Phys. Lett. A* 315 (2003) 431-436.