Energy Efficiency in MC-DS/CDMA Cooperative Networks: Centralized and Distributed Solutions

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***Abstract*—The energy efficiency (EE) maximization on the uplink of multi-carrier direct sequence code division multiple access (MC-DS/CDMA) cooperative wireless networks is an NP-hard optimization problem of great interest for future networks systems. This paper presents three centralized solutions and two distributed ones: the centralized solutions include an iterative solution based on the Dinkelbach method and two heuristic approaches, *i.e.*, the Firefly algorithm (FA) and the Particle Swarm Optimization (PSO) algorithm. The distributed solutions are based on a game theory framework and employ two different algorithms for resource allocation: the classical waterfilling algorithm adapted to the discussed problem and the Verhulst distributed power control algorithm (V-DPCA). Simulations were conducted in order to establish which technique has the best EE, spectral efficiency (SE) and complexity tradeoff.**

Keywords–Cooperative Networks; MC-DS/CDMA; Energy Efficiency; Spectral Efficiency; Game Theory; Dinkelbach.

I. Introduction

Energy-efficient telecommunications systems is of great importance in terms of design and implementation due to carbon emission reduction and cost. Besides, in wireless communications the transmission power usually is not linearly proportional to the transmitter-receiver distance, which makes cooperative networks often more energy-efficient than non-cooperative networks. In such networks each mobile terminal (MT) communicates with one or multiple relay stations (RS) that forward the MT message to its respective base station (BS). Besides the increase in EE, this sort of networks may also be used to increase spectral efficiency (SE), system throughput or decrease the average transmission power.

MC-DS/CDMA networks are characterized by the division of the total available spectrum into uncorrelated CDMA non-selective sub-channels. This characteristic improves granularity which usually enhances system throughput, capacity, SE and EE as well as lower the average transmission power (ATP). Each one of the sub-channels is interference limited, thus better interference cancellation techniques may be implemented in order to further improve the system. In addition, MC-DS/CDMA networks may carry out adaptive modulation or multiprocessing gain techniques to support different multimedia services increasing the system flexibility from voice service to high data rate services.

1) Related Work: Many studies have been conducted recently aiming to find implementable resource allocation (RA) algorithms for cooperative networks, among them [1]–[6]. RA procedures are directly related to user satisfaction, company profits and environment issues. The work in [1] presents a game theoretic approach for power control and receiver design in cooperative direct sequence (DS)/CDMA networks. A performance analysis for wireless cooperative networks using amplify and forward (AF) protocol is presented in [2] while a method for energy efficiency maximization with minimum transmission rate requeriments is presented in [3]. Meanwhile, in [4] a concave fractional programming approach for EE maximization in orthogonal frequency division multiple access (OFDMA) cooperative networks is presented while in [5] an analysis on EE-SE tradeoff for DS/CDMA networks is offered. Furthermore, in [6] three different distributed solution methods for the EE maximization problem in MC-DS/CDMA cooperative networks are discussed.

2) Contributions: The main contribution in this work is to analyse different algorithms, methods and approaches that can be deployed in resource allocation of the uplink (UL) cooperative MC-DS/CDMA networks. In order to find a good solution/algorithm for power allocation and EE in MC-DS/CDMA cooperative networks, which compromises both performance and computational complexity, this paper extends the work presented in [6] and incorporates two centralized heuristic algorithms to solve the RA problem. All five algorithms/methods are briefly presented and analyzed under the same scenario which consists of a single cell with one fixed relay station, directional antennas and cooperative communications aiming to expand the coverage area without expensive costs for the telecommunications company.

This paper is organized as follows. Section II presents the system model and description; EE and SE, as well as the problem formulation are presented in Section III. Furthermore, Section IV discusses centralized solutions while Section V presents the game theoretic approach. Numerical examples and results are offered in Section VI. Conclusions and future work are addressed in Section VII.

II. System Description

On the UL of MC-DS/CDMA cooperative systems each MT communicates with the BS using a RS to forward its message. In this paper we consider a single cell environment with one fixed RS (FRS) as shown in Fig. 1. The complex channel gain for the MT-FRS link is described as $h_i(k)$ where i and k are the MT and subcarrier indexer, respectively. Equivalently, the complex channel gain for the FRS-BS channel is denoted as $g(k)$. Therefore the *power channel gain* vector for the MT-FRS path is:

$$\mathbf{h}(k) = [h_1(k)\, h_2(k)\, \cdots h_U(k)]^\top \qquad k = 1, \dots N \tag{1}$$

where U is the system loading and N the number of parallel non-overlaping subcarriers. The *power channel gain* vector for the FRS-BS path is:

$$\mathbf{g} = [g(1)\, g(2)\, \cdots\, g(k)\, \cdots\, g(N)] \tag{2}$$

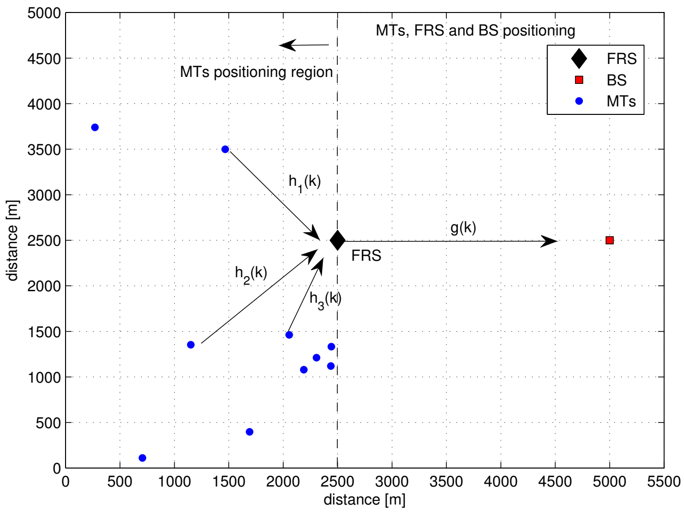

Figure 1. MTs, FRS and BS positioning in a uplink MC-DS/CDMA system with a single fixed-RS.

In cooperative scenarios, the signal received at the relay may go through different procedures before it is forwarded to the next destination. The relay may use amplify-and-forward (AF) protocol, decode-and-forward (DF) protocol or compress-and-forward (CF) protocol. We only consider the AF protocol. Hence, the signal received at the FRS is first normalized by the square root of the average received power and afterwards amplified by the $U \times U$ amplification matrix $\mathbf{A}$ constrained by $\mathrm{tr}(\mathbf{A}\mathbf{A}^H) \leq p_R$ where p_R is the available power at the FRS.

In interference limited multiple access networks an important QoS measure is the signal-interference-plus-noise ratio (SINR) since all users transmit over the same channel at the same time causing multiple access interference (MAI). In a MC-DS/CDMA system, the post-detection SINR, considering the adoption of linear receivers, may be generically expressed for the ith user, kth sub-carrier as [7]:

$$\delta_i(k) = F_i(k)\frac{p_i(k)\mathrm{h}_i(k)\mathrm{g}(k)|\mathbf{d}_i^H\mathbf{A}\mathbf{s}_i(k)|^2}{\mathcal{I}_i(k) + \mathcal{N}_{\mathrm{T}}(k) + \sigma^2\mathrm{g}(k)||\mathbf{A}^H\mathbf{d}_i||^2} \quad (3)$$

where $\mathrm{h}_i(k) = |h_i(k)|^2$ is the *channel power gain* between user i and the single FRS and $\mathrm{g}(k) = |g(k)|^2$ is the *channel power gain* from single FRS to BS, and $\mathbf{d}_i$ is the linear filter at receiver, such as single-user matched filter (MF) or multi-user Decorrelator, minimum mean square error (MMSE) filter and so forth; following the results of [5] only the Decorrelator will be considered herein since it had the best results in terms of complexity-performance trade off. Hence, the Decorrelator filter may be mathematically expressed as:

$$\mathbf{d}_{\mathrm{DEC}} = [\mathbf{d}_1, \ldots, \mathbf{d}_i, \ldots, \mathbf{d}_U] = \mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1} = \mathbf{S}\mathbf{R}^{-1} \quad (4)$$

where $\mathbf{d}_i$ is the linear filter for the i-th user, $\mathbf{S}$ is the spreading sequence matrix with each column representing an user spreading code and $\mathbf{R}$ is the spreading sequence correlation matrix. Furthermore, the $\mathcal{I}_i(k)$ in (3) is the amplified MAI at the FRS forwarded to the BS:

$$\mathcal{I}_i(k) = \mathrm{g}(k)\sum_{\substack{j=1 \\ j\neq i}}^{U} p_j(k)\mathrm{h}_j(k)\,|\mathbf{d}_i^H\mathbf{A}\mathbf{s}_j(k)|^2 \quad (5)$$

and $\mathcal{N}_{\mathrm{T}}(k)$ is the normalized noise at the BS and treated through the linear multiuser receiver:

$$\mathcal{N}_{\mathrm{T}}(k) = \left[\sum_{i=1}^{U} p_i(k)\mathrm{h}_j(k) + \mathrm{F}_i(k)\sigma^2\right]\sigma^2||\mathbf{d}_i||^2 \quad (6)$$

where $p_i(k)$ is the allocated power, associated to the respective users (i) and sub-carrier (k), and σ^2 is the power noise.

III. Energy and Spectral Efficiencies and Problem Formulation

The SE of each user through the N sub-channels can be computed as the number of bits per second that may be transmitted for a single Hertz. Considering a practical approach for the theoretical bound obtained through the Shannon channel capacity equation, the SE of the ith user can be defined as:

$$\mathcal{S}_i = \sum_{k=1}^{N}\log_2\left(1+\delta_i(k)\right), \; i = 1,\ldots,U \quad \left[\frac{\text{bits}}{\text{s}\cdot\text{Hz}}\right] \quad (7)$$

Finally, the user rate at each sub-channel is given by:

$$r_i(k) = \mathrm{w}\cdot\mathcal{S}_i = \mathrm{w}\log_2\left(1+\delta_i(k)\right) \quad \left[\frac{\text{bits}}{\text{s}}\right] \quad (8)$$

In MC-DS/CDMA systems with a single FRS, the EE function for user i can be formulated as [8]:

$$\xi_i = \sum_{k=1}^{N}\frac{r_i(k)\ell_i(k)\,\cdot f(\delta_i(k))}{\varrho\, p_i(k) + \varrho_{\mathrm{R}}\, p_{\mathrm{R}} + p_{\mathrm{C}} + p_{\mathrm{C_R}}} \quad \left[\frac{\text{bit}}{\text{Joule}}\right], \quad (9)$$

$\forall i = 1,\ldots,U$, where i and k are the user and sub-channel indexers respectively; $\ell \leq 1$ is the code rate. $p_i(k)$ is the MT transmission power, p_{R} is the re-transmission power from the FRS to the BS, assumed a fixed and equal power quantity per user overall the subcarriers; p_{C} and $p_{\mathrm{C_R}}$ are the circuitry power consumption at MT and FRS, respectively; $\varrho > 1$ and $\varrho_{\mathrm{R}} > 1$ are the power amplifier inefficiency factors at MT and FRS, respectively. The efficiency function $f(\delta_i(k))$ expresses the probability of error-free packet reception. Considering M-QAM square constellation modulations of order $M = M_i(k)$ and Gray coding, the bit error rate is approximately [9], [10]:

$$\mathrm{BER}_i(k) = \frac{2(\sqrt{M}-1)}{\sqrt{M}\log_2 M}\left(1-\sqrt{\frac{3\delta_i(k)\log_2 M}{2(M-1)+3\delta_i(k)\log_2 M}}\right) \quad (10)$$

A. Problem Formulation

To maximize the EE of each user in the MC-DS/CDMA system with a single FRS, the overall EE maximization problem with MT's power constraint is posted as:

$$\begin{aligned}
\underset{\mathbf{P}\in\wp}{\text{maximize}} \quad \sum_{i=1}^{U}\xi_i &= \sum_{i=1}^{U}\sum_{k=1}^{N}\frac{r_i(k)\ell_i(k)f(\delta_i(k))}{\varrho\, p_i(k) + \varrho_{\mathrm{R}}\, p_{\mathrm{R}} + p_{\mathrm{C}} + p_{\mathrm{C_R}}} \\
&\equiv \sum_{i=1}^{U}\frac{\sum_{k=1}^{N}\ell_i(k)\,\mathrm{w}\log_2\left[1+\delta_i(k)\right]\cdot(1-\mathrm{BER}_i(k))}{p_{\mathrm{C}} + p_{\mathrm{C_R}} + \varrho_{\mathrm{R}}\, p_{\mathrm{R}} + \varrho\sum_{k=1}^{N}p_i(k)}, \\
\text{s.t.(C.1)} \quad & \delta_i(k) \geq \delta_{i,\min}(k), \quad \forall\, k, i
\end{aligned} \quad (11)$$

where the total transmit power of the U mobile terminals across N subcarriers must be bounded (and be nonnegative) for any feasible power allocation policy, with the correspondent power allocation matrix described by:

$$\mathbf{P} \in \wp \stackrel{\text{def}}{=} \{[p_i(k)]_{U\times N} \mid 0 \leq \quad p_i(k) \leq P_{\max}\} \qquad (12)$$

where $N \cdot P_{\max}$ represents the maximum total transmit power available at each MT transmitter.

IV. Centralized RA Solutions

Analytical-iterative Dinkelbach method, as well as a heuristic PSO-based and Firefly-based algorithms are explored to solve the RA problem in a centralized way.

A. Dinkelbach Method

The Dinkelbach method is an iterative method to solve quasi-concave problems in a parameterized concave form [11]. The optimization problem in (11) is divided into N subproblems representing the EE problem in each subcarrier k:

$$\begin{aligned} &\text{maximize} \quad \frac{\mathcal{C}(\mathbf{p})}{\mathcal{U}(\mathbf{p})} = \frac{\sum_{i=1}^{U} \ell_i(k)\,\mathrm{w}\log_2\left[1+\delta_i(k)\right]\cdot(1-\mathrm{BER}_i(k))}{p_{\mathrm{C}} + p_{\mathrm{C_R}} + \varrho_{\mathrm{R}}\, p_{\mathrm{R}} + \varrho \sum_{k=1}^{N} p_i(k)} \\ &\text{s.t.}(\mathrm{C}.1) \quad \delta_i(k) \geq \delta_{i,\min}(k), \quad \forall\, i \qquad (13)\\ &\quad(\mathrm{C}.2) \quad 0 \geq p_i(k) \geq p_{\max}, \quad \forall\, i \end{aligned}$$

Each subproblem (13) is a quasi-concave problem [6]. The following parametric concave program is associated with the EE maximization problem in each subcarrier [12]:

$$\underset{\mathbf{p}\in\mathcal{X}}{\text{maximize}} \quad \mathcal{C}(\mathbf{p}) - \lambda\,\mathcal{U}(\mathbf{p}) \qquad (14)$$

where $\mathcal{X} = \{\mathbf{p} \in \mathcal{X} | 0 \leq p_i(k) \leq p_{\max}\ \forall i = 1, \ldots, U\}$. The objective function of the parameterized problem, denoted by $\mathcal{F}(\lambda)$ is a convex, continuous and strictly decreasing function. Without loss of generality, the maximum EE λ^* of the parameterized problem is [12]:

$$\lambda^* = \frac{\mathcal{C}(\mathbf{p}^*)}{\mathcal{U}(\mathbf{p}^*)} = \underset{\mathbf{p}\in\mathcal{X}}{\text{maximize}}\ \frac{\mathcal{C}(\mathbf{p})}{\mathcal{U}(\mathbf{p})} \qquad (15)$$

which is equivalent to find $\mathcal{F}(\lambda) = 0$. According to [13] the Dinkelbach method is the application of Newton's method to a nonlinear fractional program converging with a super-linear rate. The goal at each iteration of the method is to solve:

$$\mathcal{F}(\lambda_n) = \max_{\mathbf{p}\in\mathcal{X}} \{\mathcal{C}(\mathbf{p} - \lambda_n\,\mathcal{U}(\mathbf{p})\} \quad @n\text{th iteration.} \qquad (16)$$

Fig. 2 depicts a pseudo-code for Algorithm 1 based on iterative Dinkelbach's method. Moreover, in order to solve problem (16) we have deployed CvX tools [14].

Algorithm 1 Dinkelbach's Method

Input: λ_0 satisfying $\mathcal{F}(\lambda_0) \geq 0$; tolerance ϵ_d
Initialize: $n \leftarrow 0$,
repeat
  Solve problem (16) with $\lambda = \lambda_n$ to obtain $\mathbf{p}_n^*$
  $\lambda_{n+1} \leftarrow \frac{\mathcal{C}(\mathbf{p}_n^*)}{\mathcal{U}(\mathbf{p}_n^*)}$;
  $n \leftarrow n+1$
until $|\mathcal{F}(\lambda_n)| \leq \epsilon_d$;
Output: λ_n; $\mathbf{p}_n^*$

Figure 2. Pseudo-Code for the Dinkelbach's Method

B. Particle Swarm Optimization (PSO)

Created by Kennedy and Eberhart in [15] the PSO algorithm is based on flocks behaviour when searching for food. In this algorithm each possible solution is analogously put as a particle in the swarm and each member of the population has two main attributes: position and velocity. Position itself is the candidate solution to the optimization problem and velocity is the parameter used to move each individual according to the best solution found by him and by the whole group.

In order to deal with unfeasible solutions the implemented algorithm discards any individual which leaves the problem domain or does not satisfy at least one of the problem constraints as in (11). Hence, at each iteration the M individuals move through the search space, being eliminated from the population if they leave the problem domain. At the t-th iteration the PSO updates particle m position through:

$$\mathbf{X}_m[t+1] = \mathbf{X}_m[t] + \mathbf{V}_m[t+1] \qquad (17)$$

where $\mathbf{X}_m[t]$ is the particle m position at iteration t and it is a $U \times N$ non-negative valued real matrix. $\mathbf{V}_i[t+1]$ is the velocity $U \times N$ matrix which can be computed by:

$$\begin{aligned} \mathbf{V}_m[t+1] \;=\; & \omega\mathbf{V}_m[t] + c_1\,\mathbf{U}_m^1[t] \circ \left(\mathbf{V}_m^{\text{best}}[t] - \mathbf{V}_m[t]\right) \\ & + c_2\,\mathbf{U}_m^2[t] \circ \left(\mathbf{V}_g^{\text{best}}[t] - \mathbf{V}_m[t]\right) \qquad (18) \end{aligned}$$

where $\circ$ is the Hadamard product, $\mathbf{V}_m^{\text{best}}[t]$ is the best solution found by individual m up to iteration t, $\mathbf{V}_g^{\text{best}}[t]$ is the best solution found by all individuals up to iteration t, c_1 and c_2 are weights for local and global solution candidates, ω is the inertia coefficient, $\mathbf{U}_m^1[t]$ and $\mathbf{U}_m^2[t]$ are random matrices uniformly distributed on the interval $[0, 1]$.

It is important to note that the best solution is stored based on which candidate solution has the best outcome in terms of objective function. The objective function herein for both heuristics is the system effective capacity in (11).

The PSO algorithm is the recurrent computation of equations (17) and (18) and updating process of $\mathbf{V}_i^{\text{best}}$ and $\mathbf{V}_g^{\text{best}}$ up until convergence or when a maximum number of iterations has been reached.

C. Firefly Algorithm (FA)

Another bio-inspired heuristic, the FA was created by Yang in 2008 [16]. It is based on the collective intelligence of fireflies and their light emission patterns. Firefly algorithm dynamics include two main properties of fireflies in some arbitrary environment: the distance between the firefly population individuals and the light intensity of their bioluminescence.

There are different species of fireflies that use their bioluminescence for different purposes. Fireflies attract each other for reproduction purposes and therefore they are more attracted by the most intense flash pattern. Therefore, the link between the firefly behaviour and the optimization problem is that their flash pattern increases its intensity the better the solution represented through the individual of the population which leads the population to improved solutions that yield better objective function outcomes.

To become a useful tool, three basic rules about the individual fireflies are assumed: a) fireflies have no gender definition, hence any firefly can attract any other firefly; b) attractiveness is proportional to the bioluminescence intensity such that less intense flash patterns fireflies are moved on the direction of more intense glints; c) if a particular individual has the most intense flash pattern then it moves randomly in the search universe. Instead of being an intrinsic characteristic to a firefly, the attractiveness is, in fact, a correspondence between firefly i and j which is mathematically defined as [16]:

$$\beta_{ij} = \beta_0 \, e^{-\gamma \, d_{ij}^2} \tag{19}$$

where j attracts i with intensity β_0 and an exponential decay with the distance between them. The parameter β_0 is the base attractiveness at null distance, common for all individuals of the population, and γ is the medium permissiveness or light absorption coefficient; d_{ij} is the Euclidean distance between the fireflies. Hence, at each iteration t, each pair i, j – where firefly j has a larger light intensity than i – performs a move such that i moves in the direction of j according to [16]:

$$\mathbf{X}_i[t+1] = \mathbf{X}_i[t] + \beta_0 \, e^{-\gamma \, d_{ij}^2} \, (\mathbf{X}_j[t] - \mathbf{X}_i[t]) + \alpha_f \, \left(\mathbf{U}_f[t] - 0.5\right) \tag{20}$$

where $\mathbf{X}_i[t]$ is the firefly i position at the t-th iteration and, in this paper, is a $U \times N$ matrix of power allocation, α_f is a random step coefficient parameter and $\mathbf{U}_f[t]$ is a random matrix at iteration t uniformly distributed on the interval $[0, 1]$.

V. GAME THEORETIC APPROACH

We have adopted two approaches to solve the optimization problem in (11) in a distributed fashion: a) solving N non-cooperative games aiming to find the best SINR response to a given interference level at each sub-channel and using the Verhulst based distributed power control algorithm (V-DPCA) [17] afterwards; b) considering a non-cooperative game and an average channel gain across N sub-carriers. The problem is solved also in two steps: finding the best SINR response and allocating the same power to all sub-channels using the V-DPCA. Note that in all power control algorithms analyzed herein system users adjust their own transmission power level selfishly.

A. Solving N Non-Coalitional Games

A different game for each sub-channel such that N non-coalitional games must be solved in order to find the best SINR response. The EE maximization game at the k-th subcarrier is:

$$\mathcal{G}(k) \quad = \quad \left[\mathcal{U}, \, \{\mathcal{A}_i(k)\}, \, \{u_i^k\}\right], \quad k = 1, 2, \ldots, N \tag{21}$$

where $\mathcal{U} = \{1, 2, \ldots, U\}$ is the player set, $\{\mathcal{A}_i(k)\} = [0, \, p_{\max,i}]$ is the strategy set for user i in the kth sub-channel where $p_{\max,i}$ is the maximal resource (transmission power) available at the ith MT; and $\{u_i^k\}$ is the utility function for the ith user at kth sub-carrier; in this case $\{u_i^k\}$ is given by:

$$u_i^k = r_i(k) \ell_i(k) \frac{\left(1 - \mathrm{BER}_i(k)\right)^{V_i(k)}}{\varrho \, p_i(k) + \varrho_{\mathrm{R}} \, p_{\mathrm{R}} + p_{\mathrm{C}} + p_{\mathrm{C_R}}}, \tag{22}$$

with $\mathrm{BER}_i(k)$ defined as Eq. (10). Consider the power allocated to the ith user at the kth sub-channel. The vector:

$$\mathbf{p}_{-i}(k) = [p_1(k), \cdots, p_{i-1}(k), p_{i+1}(k), \cdots, p_U(k)] \tag{23}$$

is the allocated power vector considering all users but user i. Thus, given the power allocated to all except ith user at the kth sub-carrier the best response for user i in a non-coalitional fashion may be expressed as [18]:

$$p_i^*(k) = \arg \max_{p_i(k)} \; u_i^k \left[p_i(k), \mathbf{p}_{-i}(k)\right] \tag{24}$$

Non-coalitional games can be easily solved finding the Nash Equilibrium (NE) of the problem [19]. An NE is a set of strategies such that any unilateral change will not increase the user utility function without decreasing the other players payoffs. Let $\mathbf{p}_{-i}^*(k)$ be the set of optimal strategies for all users but user i. The vector $\mathbf{p}^*(k)$ is a NE if and only if:

$$\forall p_i(k) \neq p_i^*(k) \Rightarrow u_i^k \left[p_i(k), \mathbf{p}_{-i}^*(k)\right] \leq u_i^k \left[p_i^*(k), \mathbf{p}_{-i}^*(k)\right] \tag{25}$$

Note that in the context of distributed energy-efficient power allocation problem the strategy set for each user is $p_i(k) \in [0, P_{\max}]$; thus, it is non-empty, compact and convex. Furthermore, if the utility function is quasi-concave it has been proved in [20] using Glicksberg generalization of the Kakutani fixed point theorem [21] that the non-cooperative game has at least one Nash Equilibrium. As a consequence, distributed energy-efficient power allocation problem under non-cooperative game perspective may be posed as:

$$\begin{aligned} \max_{p_i(k)} \; u_i^k \quad &= \quad r_i(k) \ell_i(k) \frac{\left(1 - \mathrm{BER}_i(k)\right)^{V_i(k)}}{\varrho \, p_i(k) + \varrho_{\mathrm{R}} \, p_{\mathrm{R}} + p_{\mathrm{C}} + p_{\mathrm{C_R}}} \\ \text{s.t.} \quad (\text{C.1}) \quad & \quad 0 \leq p_i(k) \leq P_{\max} \\ (\text{C.2}) \quad & \quad \delta_i(k) \geq \delta_{i,\min}(k), \quad \forall \, k, i \end{aligned} \tag{26}$$

where $\mathrm{BER}_i(k)$ function is defined in Eq. (10).

Since function u_i^k in Eq. (22) depends on both the user allocated power and its SINR from Eq. (3) follows the relation:

$$p_i(k) = \delta_i(k) \frac{\mathcal{I}_i(k) + \mathcal{N}_{\mathrm{T}}(k) + \sigma^2 \mathrm{g}(k) ||\mathbf{A}^H \mathbf{d}_i||^2}{F_i(k) \cdot \mathrm{h}_i(k) \mathrm{g}(k) |\mathbf{d}_i^H \mathbf{A} \mathbf{s}_i(k)|^2} = \delta_i(k) \Gamma_i(k) \tag{27}$$

The fact that the power domain is an interval, *i.e.*, $p_i(k) \in [0, p_{\max}]$, and the relation between power and SINR is linear over the optimization window as shown in (27), the SINR domain is also an interval such that $\delta_i(k) \in [0, \delta_{\max}]$, where $\delta_{\max}$ is related to the SINR level when transmitting with the highest power level allowed. So, utility function is rewritten:

$$u_i^k = r_i(k) \ell_i(k) \frac{\left(1 - \mathrm{BER}_i(k)\right)^{V_i(k)}}{\varrho \, \delta_i(k) \Gamma_i(k) + \varrho_{\mathrm{R}} \, p_{\mathrm{R}} + p_{\mathrm{C}} + p_{\mathrm{C_R}}}, \tag{28}$$

Finding the best response strategy for each user is equivalent to maximize the utility function Functions maxima have a null derivative; hence, applying the derivative in (28):

$$\frac{\partial u_i^k}{\partial \delta_i(k)} = 0 \tag{29}$$

Fig. 3 presents the proposed Algorithm 2 for finding the best SINR response, while simultaneously allocating the corresponding transmission power levels for each user i and subcarrier k.

Algorithm 2 Iterative EE-Maximization Algorithm

Input: $\mathbf{p}$, I, ϵ; **Output:** $\mathbf{p}^*$

begin
1. initialize first population and set $n = 0$;
2. while $n \leq I$ or $error > \epsilon$
3. find $\delta_i(k)$, $\forall i = 1, \ldots, U \quad k = 1, \ldots, N$ through (29)
4. allocate $p_i(k)$ for all i and k in order to achieve $\delta_i(k)$
5. calculate $error = ||\mathbf{p}_i[n] - \mathbf{p}_i[n-1]||_2$
6. $n = n + 1$
7. end while

——————————————-

$\mathbf{p}$ = initial power vectors;
$\mathbf{p}[n]$ = power vector at the nth iteration;
$\mathbf{p}^*$ = power vector solution;
I = maximum number of iterations;
ϵ = least expected precision;

Figure 3. Pseudo-Code for the Iterative EE-Maximization Algorithm

B. Solving One Non-Coalitional Game (M-DPCA)

As a second approach, the problem is simplified by taking the average channel gain over all N subcarriers. Thus, all variables which had a subcarrier indexer k are now identified by the operator $\bar{\cdot}$, indicating that the variable takes the average value over N subcarriers. Hence, the problem (11) can be re-written as:

$$\begin{aligned}
\underset{\bar{\mathbf{P}} \in \wp}{\text{maximize}} \quad & \sum_{i=1}^{U} \bar{\xi}_i = \sum_{i=1}^{U} \frac{\bar{r}_i \bar{\ell}_i \, f(\bar{\delta}_i)}{\varrho \, \bar{p}_i + \varrho_{\rm R} \, \bar{p}_{\rm R} + p_{\rm C} + p_{\rm C_R}} \\
& = \sum_{i=1}^{U} \frac{\bar{\ell}_i \, \mathrm{w} \log_2 \left[1 + \bar{\delta}_i\right] \cdot (1 - \bar{\mathrm{BER}}_i)^{\bar{V}_i}}{p_{\rm C} + p_{\rm C_R} + \varrho_{\rm R} \, \bar{p}_{\rm R} + \varrho \, \bar{p}_i}, \quad (30) \\
\text{s.t.} \quad (\text{C.1}) \quad & \bar{p}_i \leq p_{\max,i}, \quad i = 1, \ldots, U \\
(\text{C.2}) \quad & \bar{\delta}_i \geq \bar{\delta}_{i,\min}, \quad i = 1, \ldots, U
\end{aligned}$$

Problem (30) does not have the sub-carrier dimension, thus it is easier to solve when compared to the first approach. To find the best power allocation policy to (30) the same game theoretic approach presented in Section V-A is adopted. Therefore, one may solve this approach through our proposed Algorithm 2 depicted in Fig. 3 in combination with the V-DPCA. This approach will be referred hereafter as M-DPCA.

VI. Numerical Results

Simulations where conducted using MatLab 7.0 Mathworks to establish which are the best parameters for both heuristics. The scenario parameters values used in the simulations are presented in Table I. Before comparing the different methods' performance it is necessary to find the best parameters for both heuristics. Results from 1000 trials with at least 5 different values for each parameters shown that, for FA the best reference light intensity is $\beta_0 = 1$, the best light absorption coefficient is $\gamma = 1$ and the best random step coefficient is $\alpha_f = 10^{-3}$. While for the PSO, the local best and global best coefficients are $c_1 = c_2 = 2$.

The effect of population size for both PSO and FA was analyzed in Fig. 4. The population size and the maximum number of iterations impact was analysed in terms of the average EE obtained for both PSO and FA; also, such heuristic distributed solutions was compared to the centralized solution based on Dinkelbach method (Max EE). As shown, increasing the maximum number of iterations as well as population size leads to better EE results with the only counter effect being the increase on complexity. Furthermore, FA has a significant advantage in terms of achieve EE when compared to the PSO under small population sizes (5 to 10). This gap among each algorithms' solution is reduced when the population size increases and is almost equivalent when it reaches 50 individuals.

TABLE I. Multirate DS/CDMA system parameters

Parameters	Adopted Values
MC-DS/CDMA System	
Noise Power	$P_n = -90$ [dBm]
MT Circuitry Power	$p_c = 0.1$ [W]
Relay Circuitry Power	$p_{c_R} = 0.5$ [W]
Relay Transmission Power	$p_R = 25$ [W]
Power Amplifier Inefficiency	$\varrho = \varrho_R = 2.5$
Codification Rate	$\ell = \frac{3}{4}$
Processing Gain	$F = 128$
Sub-carriers	$N = 16$
Users	$U = 5$
Amplification Matrix	$A = \mathbf{I}_F * (\frac{p_R}{F})$ [1]
Bits per Packet	$V = 1$
Sub-channel Bandwidth	$w = 78$ KHz
Max. power per user	$P_{\max} = 125$ [mW]
# mobile terminals	$U = 5$
cell geometry	rectangular, with $x_{\rm cell} = 10$Km $y_{\rm cell} = 5$ Km
mobile term. distrib.	$\sim \mathcal{U}[0.5 * x_{\rm cell},\ y_{\rm cell}]$
Channel Gain	
path loss	$\propto d^{-2}$
shadowing	uncorrelated log-normal, $\sigma^2 = 6$ dB
fading	Rayleigh
User Types	
User Rates	$r_{i,\min} = [256;\ 512;\ 1024]$ [Kbps]
Modulation (For BER purposes)	$M = [4, 16, 64]$
Tolerable BER per class	$[10^{-3}; 10^{-5}; 10^{-8}]$

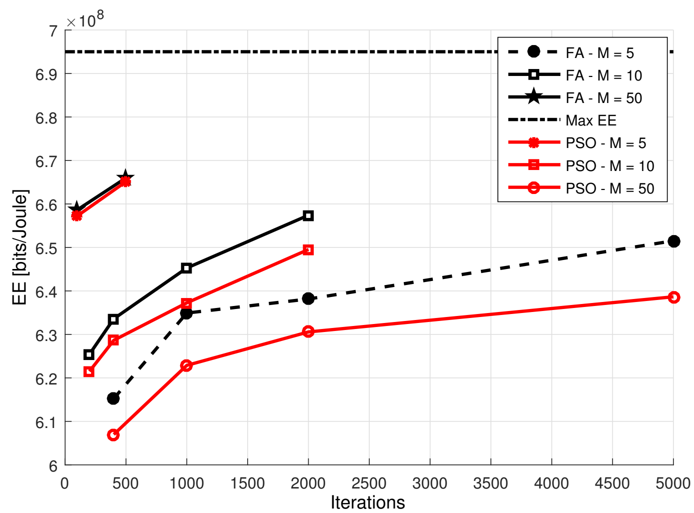

Figure 4. Effect of population size M and maximum number of iterations on the average Energy Efficiency.

For simulation purposes all users have been separated into three different QoS requirement classes. Basically, each class represents one type of multimedia service; thus, users may require low, average or high throughput associated with high, medium and very low maximal tolerable BER, respectively.

To effectively compare the performance of each method one must consider the computational complexity of each method. Some assumptions must be done: a) all algorithms are assumed to execute the same amount of iterations, hence the algorithm with the simplest iteration yield the simplest algorithm of all; b) computational complexity of Dinkelbach's method solution obtained via CvX tool is not considered in the analysis since its complexity cannot be securely obtained; c) number of users in the system is considered $U < N$ and therefore has no impact on the complexity asymptotically.

At each iteration of the V-DPCA algorithm a fixed number of operations are conducted which leads, in asymptotic terms, to a $\mathcal{O}(1)$ complexity. Hence, the M-DPCA approach has a static complexity while the V-DPCA is executed one time at each subcarrier which leads to linear complexity $\mathcal{O}(N)$. It is also easy to show that at each iteration of the PSO algorithm operations are computed for each of the individuals of the population which represents an asymptotic linear dependence of the population size, *i.e.*, $\mathcal{O}(M)$. Finally, the complexity of the FA is directly related to the fact that it must compare every possible pair of fireflies which gives an asymptotic complexity of $\mathcal{O}(M^2)$ where M is the population size. Furthermore, the performance comparison in terms of how close to the optimal solution (on average) each algorithm finds itself is sumarized in Table II. Such results represent the average over a thousand channel realizations and MTs geographical distribution.

TABLE II. AVERAGE EE AND ITERATION COMPLEXITY.

Algorithm	**% of the Optimal EE**	**Complexity**
Dinkelbach Method	100%	-
V-DPCA	57%	$\mathcal{O}(N)$
M-DPCA	96%	$\mathcal{O}(1)$
FA	96%	$\mathcal{O}(M^2)$
PSO	95%	$\mathcal{O}(M)$

N is the number of subcarriers in the system.
M is the population size of the heuristic method.

VII. CONCLUSION

Five different approaches solving the energy-efficient design in MC-DS/CDMA cooperative networks with FRS were analyzed. Also, the algorithms' complexity issues were briefly addressed. Our findings indicate that the best average EE-complexity tradeoff is achieved by the M-DPCA. The M-DPCA algorithm takes into account the average observed interference while performing the power control of the non-cooperative game in a distributed fashion, which is of paramount importance in uplink scenarios. Indeed, such algorithm presented the lowest iteration complexity and achieved similar or better performance results regarding the average EE than centralized solutions, such as the heuristic ones. Another advantage is that it simplifies the interference estimator procedure at the BS side while a low overhead information exchanging between MT and BS is held.

Future works include the problem of the joint resource block and power allocation in the uplink of multi-user multi-cell large scale multiple-input-multiple-output (or massive MIMO) under pilot contamination regime. Since the optimization problem is of mixed-integer nature, we are proposing different quasi-optimal resource allocation algorithms to achieve the better tradeoffs regarding average system spectral efficiency and energy efficiency, all of them supported by a game theoretic framework.

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