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Flow Time Prediction for a Single-Server Order Picking Workstation using Aggregate Process Times*

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***Abstract*—In this paper we propose a simulation modeling approach based on aggregate process times for the performance analysis of order picking workstations in automated warehouses with first-in-first-out processing of orders. The aggregate process time distribution is calculated from tote arrival and departure times. We refer to the aggregate process time as the effective process time. An aggregate model uses the effective process time distributions as input to predict tote and order flow times. Results from experimental settings show that the aggregate model accurately predicts the mean and variability of tote and order flow times. As a case study, we develop an aggregate model to predict flow times for a real, operating warehouse. The resulting flow time predictions give satisfactory accuracy for both tote and order flow times. Meaningful insights are obtained for improving the performance of the warehouse.**

***Keywords*—Order picking; Polling system; Simulation; Aggregation; Performance analysis**

I. Introduction

Order picking has been identified as the most expensive process in a warehouse. It is estimated that 55% of the total warehouse operating expenses is caused by order picking only [2]. Even in automated warehouses, order picking remains a very capital intensive operation [3]. This fact alone highlights the importance of performance analysis and improvement of order picking systems.

In this paper we consider a product-to-picker, end-of-aisle, unit-load order picking system [4], with *totes* as unit-loads. An AS/RS (Automated Storage/Retrieval System) is used to retrieve product totes from a storage area. The totes are then transported using conveyors to an order picking workstation. At the workstation, a picker takes the required amount of products from the totes. Afterwards, totes with remaining items are stored back by the AS/RS.

For automated warehouses, the existing literature mostly focuses on the AS/RS [5]. Koh, Kwon and Kim [6] developed an analytical model for a miniload AS/RS with a horse-shoe style buffer. Park, Foley, and Frazelle [7] analyzed the performance of a miniload AS/RS with two-class storage. Bozer and Cho [8] derived closed-form analytical results to evaluate the performance of AS/RS under stochastic demand. Hur et al. [9] developed an $M/G/1$ queueing model to estimate the performance of a unit-load AS/RS. Other references on performance analysis of similar AS/RS are available in the recent review by Roodbergen and Vis [4].

An order picking workstation can be regarded as a special type of *polling system*, where a number of queues is attended by a single server in a certain order. Several analytical queueing models of such systems exist, see e.g., references [10] [11] [12] [13]. Typically these methods consider gated or exhaustive service policies, or a combination of the two. Another variation is the $k-$limited polling system where the server continues to work at a queue until either a predefined number of customers k is served or until the queue becomes empty (see e.g., [14]). These polling variants, however, do not fully correspond to the order picking workstation we consider. In our case, a picker always completes an order before starting to pick items for the next order. Hence, a picker may be idle at one queue (i.e., waiting for the remaining totes to arrive) while other queues are filled with totes.

We present a simulation model for quantifying the mean and variability of tote and order flow times for this type of order picking workstation. A key aspect of our model is that we do not model in detail the various outages that contribute to the flow time performance. That is, the human pickers, picking faults, setup times, picking equipment failures, etc. are not modeled in every detail. In practice, these are typically difficult to quantify [15]. Instead, we model them by means of an aggregate process time distribution. The idea is that we want to obtain the aggregate process time distribution from tote arrival and departure events of the order picking workstation in operation. Here we start from the concept of EPT (Effective Process Time) by Hopp and Spearman [16] and the concept of measuring EPT distribution from arrival and departure events [17], using a sample path equation [18].

Gu et al. [19] concluded in their recent literature review that studies describing validated or applied design models, and practical case studies will give important contributions to warehouse research in the future. This paper includes an extensive warehouse case study based on data obtained from a real, operating warehouse.

The remainder of this paper is organized as follows. Section II describes the order picking workstation. Section III

*This article is an extended version of [1].

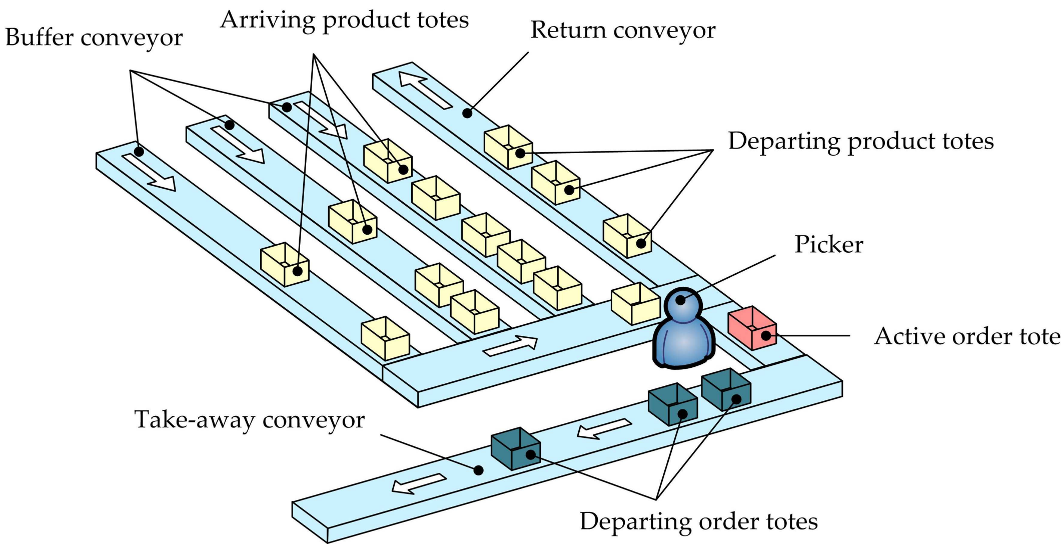


Fig. 1. Order picking workstation.

describes the simulation model. Section IV elaborates the aggregation method and the EPT measurement method. Section V discusses a number of validation experiments. Section VI provides a case study to see the performance of the proposed method in a realistic setting. Section VII concludes the paper.

II. System description

Figure 1 shows the layout of the order picking workstation under study. This system can be classified as a *product-to-picker* system [20]. Pickers work to fulfill *orders*. An order consists of a number of *order lines*. The number of order lines in an order is referred to as *order size*. Order sizes may vary significantly. Internet orders, for example, may have a small order size while orders from supermarkets may have a very large order size. An order line represents the required number of *items* from a certain SKU (Stock Keeping Unit). A *product tote* contains items of the same SKU type.

At the order picking workstation, arriving product totes form queues on buffer conveyors. Once the picker and the required product tote are available, the product tote will be removed from the buffer conveyor and transported to the pick position where the picker stands. The picker then picks a number of required items from the product tote and puts them in an *order tote*. The picker works on one order at a time until all lines of the order have been picked and the order is said to be finished. When an order is finished, the picker moves the finished order tote to a take-away conveyor that brings the order tote to a consolidation area. It is possible that more than one order tote is needed to fulfill an order due to the number of required items in that order or the size of the items being picked. In that case we assume the order has been split into suborders accordingly, which means that in the remainder of this paper we assume that every order corresponds to a single order tote. If a product tote is not yet empty after item-picking, the tote will be returned to the storage area using a *return conveyor*.

Order picking workstations have a typical characteristic that distinguishes them from ordinary manufacturing workstations. An order picking workstation receives a number of product totes for different orders. In the type of system that we consider here, the picker can only pick items from the product totes that belong to the order currently being processed, known as the *active order*. As such, only product totes required to fulfill the active order are sent in a FIFO (First-In-First-Out) sequence from the buffer conveyor to the pick position, while all other product totes wait in the queue. If there are no product totes in the queue that belong to the active order, then the picker will be idle although totes for other orders may be present. In this system, totes of three orders may arrive simultaneously at the buffer conveyor. They are sorted such that the picker always has access to the totes of the active order.

Once all order lines of the active order are finished, the next order is processed following a FIFO sequence. Subsequently, product totes for this new order are sent to the pick position. Note that only one active order is allowed in the system under consideration as shown in Figure 1. In other order picking systems it might be possible that more than one active order is processed, allowing the picker to pick items for multiple orders simultaneously. We do not consider these here.

Two performance measures are particularly of interest for this order picking workstation, namely the tote and order flow times. Tote flow time is defined as the total time spent by a tote at the order picking workstation, which starts when a tote arrives at the workstation and ends when it departs the workstation. Order flow time is defined as the time required to complete an order, which starts when the first product tote of an order arrives at the workstation and ends when the last product tote of the order has left the workstation. A complete order means that all items required for the order have been picked into the order tote.

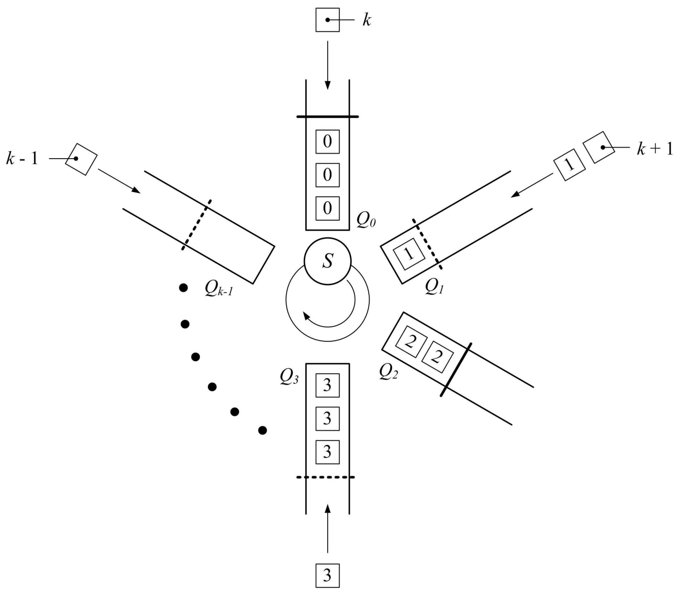


Fig. 2. Order picking workstation as a polling system.

III. Simulation model description

Figure 2 shows the simulation model representation of the order picking workstation. The workstation is modeled as a polling system with a single server S and k infinite queues. The queues are denoted by $Q_i,\ i = 0, 1, 2, ..., k-1$. The number of queues k indicates the maximal number of orders for which product totes *simultaneously* arrive in the workstation; that is, order IDs of arriving product totes may be shuffled. Totes arrive with a rate of λ. Each tote has an *id* that denotes the order ID to which the tote belongs. All arriving totes with the same *id* are put into the same queue.

When the first tote of a new order enters a queue, a *gate* is immediately set for that queue. The gate indicates the number of totes required for the order, which equals to the order length. The gate is kept open until all totes for the corresponding order have arrived at the queue. Once the last tote of the order has arrived, the gate is closed. In Figure 2 an open gate is represented by a dotted line and a closed gate is represented by a solid line in the queue.

A new order is created each time the gate for another order has been closed. The variable *id* is increased by one and the totes arriving for new order are put in queue Q_i where $i = id$ modulo k. In Figure 2, for example, the gates of orders 0 and 2 have been closed and thus two new orders can be started. If the number of queues $k = 5$ (as in Figure 2), then the new orders 5 and 6 are put into queues Q_0 and Q_1.

The server attends the queues in a cyclic direction, causing the orders to be served in FIFO sequence. The server will switch to the next queue only if the gate for the current queue has been closed and all totes in front of the gate have been served. If the server is done processing all totes in front of the gate but the gate is still open, then the server will become idle at the queue. In this case, the server waits until the remaining totes for the queue arrive.

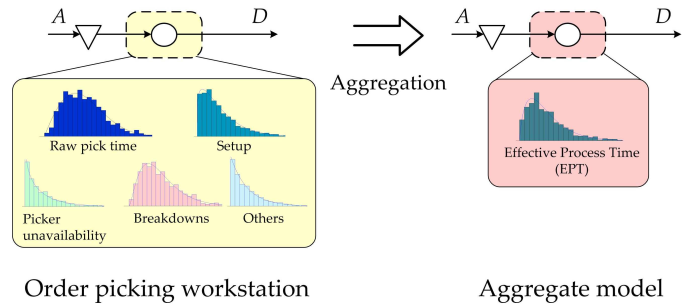


Fig. 3. Aggregation method.

IV. Aggregation method and EPT measurement

The process time used in this paper represents an aggregation of all components that contribute to the processing time at the order picking workstation. We refer to the aggregate process time as the effective process time or EPT for short (see [16]). Jacobs et al. [17] presented an algorithm to compute EPT realizations directly from arrival and departure events for infinitely buffered workstations with single-lot processing. Subsequent studies using this concept have been conducted for equipments in manufacturing lines with blocking [18], equipments in assembly lines [21], and batch equipments [22]. The former two studies employed sample path equations to calculate EPT realizations. We will do so here as well.

An order picking workstation is characterized by several process time components (see Figure 3). At the core of the process is the time required for picking items, which is referred to as the raw pick time. In addition to the raw pick time, pickers may require some setup time (change-over time) between processing of orders. Conveyor systems may break down, causing unavoidable delays. Picker availability is also an issue since it is likely that a picker is sometimes not present at the workstation. In our *aggregate model* (see Figure 3) these components are aggregated into a single EPT distribution. The idea is then to reconstruct the EPT distribution directly from tote arrival and departure times registered at the operating order picking workstation under consideration, with the obvious advantage that one does not need to quantify each component contributing to the process time.

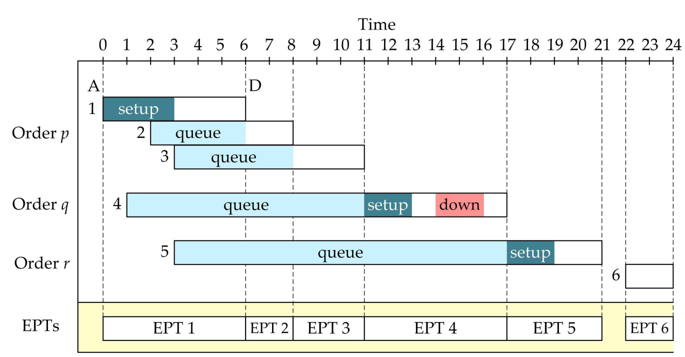


Fig. 4. Gantt chart example.

An EPT realization is calculated for each departing tote, which equals the total amount of time a tote *claims* capacity even if the tote is not yet in physical process. When EPT realizations for all departing totes have been obtained, an EPT distribution with mean t_e and squared coefficient of variation c_e^2 is created. We typically assume a gamma distribution, but other distributions may equally well be used. A gamma distribution is relatively easy to construct since the scale and shape parameters are readily obtainable from the mean and variance of the empirical EPT realizations.

Figure 4 shows an example of arrivals and departures of six totes at an order picking workstation. Totes 1, 2, and 3 belong to order p, Tote 4 belongs to order q, and Totes 5 and 6 belong to order r. An arrival A_i occurs at the moment a product tote i enters the buffer conveyor of the order picking workstation. A departure D_i occurs when item picking has been finished and the respective product tote i is moved to the return conveyor or to the take-away conveyor (see Figure 1).

EPT realizations are calculated using the following sample path equation:

$$\mathrm{EPT}_i = D_i - \max\{A_i, D_{i-1}\} \quad (1)$$

here D_i denotes the time epoch of i^{th} departing tote. A_i denotes the arrival epoch of the corresponding i^{th} departing tote. The bottom part of Figure 4 illustrates how EPT realizations are obtained using Equation (1).

The first tote of an order typically may have a different EPT distribution compared to the other totes in an order. The reason is that each time a picker starts working on a new order, a number of extra activities are performed. These activities include moving the active order tote to the take away conveyor, scanning the barcode of a new order tote to be used for the next order, and placing the order tote at the pick position. Furthermore, pickers may leave their workstations for a break after finishing an order. These are setup activities, which usually only take place in preparation of picking items from the first product tote of a new order. Consequently, EPTs of the first tote typically include a setup time whereas the remaining totes do not. Therefore, we sort EPTs into EPTs for the first totes and EPTs for the remaining totes. So in our aggregate model we will use two distribution functions, accordingly, which we refer to as the 1^{st} *tote difference* EPT approach.

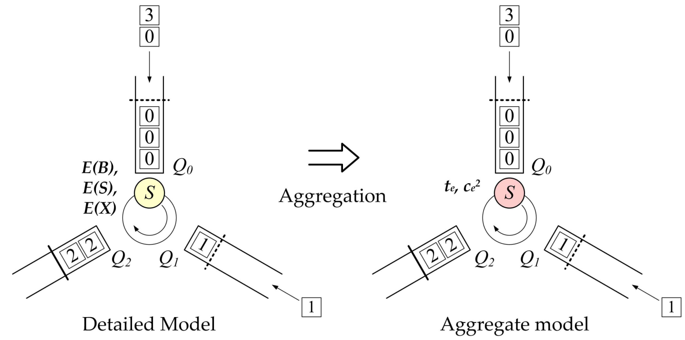


Fig. 5. EPT-based aggregation.

V. Validation experiments

This section presents simulation experiments to validate the proposed aggregate modeling method. First, we create a *detailed model* to be used as a test case representing the "real-life" operating order picking workstation. We simulate this detailed model at a certain utilization level (referred to as the *training point*) to generate tote arrival and departure events. Subsequently, these events are used as input for the sample path equation to calculate EPT realizations. Two gamma EPT distributions are constructed namely for the first totes and the remaining totes. Next, we simulate the detailed model at various utilization levels to measure the mean and variability of tote and order flow times.

In the aggregate model, we use the EPT distributions to sample the aggregate process time for totes that are being processed. The aggregate model is then simulated at the same utilization levels as the detailed model. We compare the mean and variability of tote and order flow times from the aggregate model with those of the detailed model. In this way, we assess the accuracy of flow time predictions by the aggregate modeling method.

A. Detailed model

The detailed model represents the real system under study, namely the order picking workstation with a number of process time components including raw picking time, setups and disturbances. This system is modeled as a polling system with three queues shown in Figure 5. As such, we assume

TABLE I
Data of order sizes and their frequencies.

Size	Freq.	Size	Freq.	Size	Freq.	Size	Freq.	Size	Freq.
1	331	11	80	21	20	31	11	41	11
2	243	12	67	22	20	32	6	42	10
3	257	13	41	23	12	33	5	43	11
4	181	14	24	24	13	34	12	44	12
5	195	15	34	25	7	35	11	45	7
6	208	16	42	26	10	36	12	46	2
7	147	17	19	27	6	37	7	47	3
8	91	18	14	28	12	38	9	48	5
9	134	19	17	29	20	39	8	49	3
10	90	20	27	30	5	40	6	50	1

that totes for three orders are generated simultaneously to the workstation, each with an exponential rate of $\frac{1}{3}\lambda$. Orders have different sizes as given in Table I.

The server S in Figure 5 represents a picker, which is characterized by the mean values of raw pick time $E(B)$, order tote setup $E(S)$, and other disturbances $E(X)$. The raw pick time is assumed to be gamma distributed with a mean of 17.5 seconds and an SCV (Squared Coefficient of Variation) of 0.8. The SCV is defined as the variance divided by the square of mean raw pick time. An order tote setup is performed each time a picker starts working on a new order. We assume that the order tote setup is uniformly distributed between 10.0 and 15.0 seconds. Other disturbances such as incorrect product tote administration, unreadable barcode on the product tote, distraction from other pickers, etc. occur during item picking. These disturbances are assumed to take place on average every 30 minutes, with a duration of on average 2 minutes. Both times are assumed to be exponentially distributed.

This model has been implemented using the process algebra based simulation language χ (Chi) 1.0 [23]. χ uses a pseudo-random number generator based on Mersenne Twister [24] to generate samples from distributions. But other simulation packages may of course be used as well.

The experimental setup used for the detailed model is as follows. The arrival and departure data are generated in a single simulation run of 1,000,000 totes. To measure the mean and variability of tote and order flow times we perform 30 simulation runs of 300,000 totes and a warm up period of 30,000 totes at utilization levels ranging from 0.30 to 0.95.

B. Measured EPT

To measure EPT realizations we first generate arrival and departure events from the detailed model at a training point of $0.8\delta_{\text{max}}$, where δ_{max} is the maximum throughput of the detailed model. Through simulation we obtain $\delta_{\text{max}} = 0.05$ totes per second. Arrival and departure events of 1,000,000 product totes are then generated. Subsequently, EPT realizations are calculated using Equation (1).

We apply the 1st tote difference as explained in Section IV. Two EPT distributions with parameters $t_{e,1} = 31.15$ seconds, $c^2_{e,1} = 0.59$ and $t_{e,2+} = 18.69$ seconds, $c^2_{e,2+} = 1.61$ are obtained for the first and remaining totes of orders, respectively. Suppose now we do not apply the 1st tote difference. That is, we do not distinguish between EPT realizations of the first totes and the remaining totes of orders. Without the 1st tote difference we obtain one EPT distribution with parameters $t_e = 20.08$ seconds and $c^2_{\text{e}} = 1.44$.

Figure 6 shows the CDF (Cumulative Distribution Function) of EPT realizations with and without 1st tote difference. With the 1st tote difference we obtain two significantly different EPT distributions for the first and remaining totes of orders. Without the 1st tote difference, the EPT distribution of all totes is very similar to the EPT distribution of the remaining totes using the 1st tote difference. This is because the number of EPT realizations of remaining totes is significantly larger than the first totes.

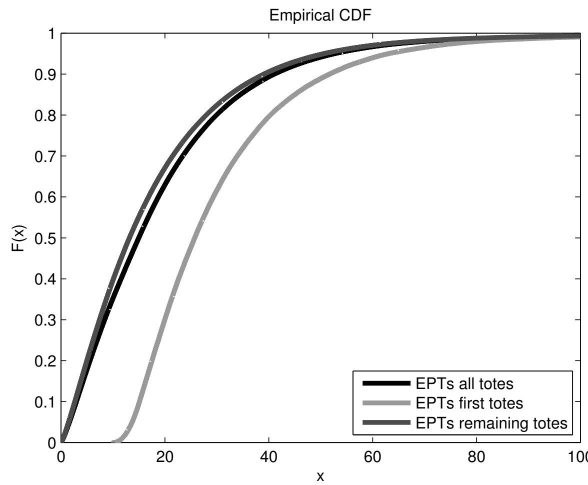


Fig. 6. CDF of EPT realizations.

C. Analytical EPT

The mean EPT for this system can be obtained analytically by applying the EPT formula for nonpreemptive and preemptive outages consecutively [16]. Order tote setup is a *nonpreemptive outage* because the setup only occurs *between* picking. Other disturbances, on the contrary, can be seen as a *preemptive outage* since they occur *during* picking. Since the formula in [16] assumes no distinction between job types, the resulting analytical mean EPT is as if the 1st tote difference is not applied.

Let t_0, σ_0^2, and c_0^2 be the mean raw pick time, its variance, and its SCV, respectively. The order tote setup is characterized by the mean setup time t_{s}, its variance σ_{s}^2 and the number of jobs between setup N_{s} (or the mean order size from Table I). The mean EPT t_{e}, effective variance σ_{e}^2, and effective SCV c_{e}^2 after including the order tote setup (nonpreemptive outage) are [16]:

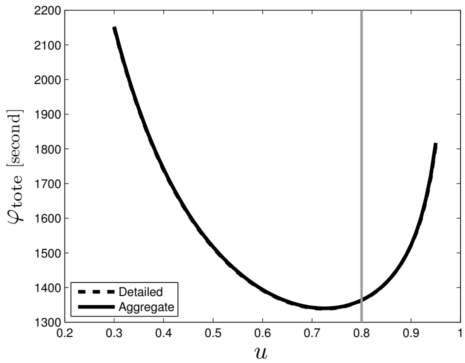
$$t_{\text{e}} = t_0 + \frac{t_{\text{s}}}{N_{\text{s}}},\ \sigma_{\text{e}}^2 = \sigma_0^2 + \frac{\sigma_{\text{s}}^2}{N_{\text{s}}} + \frac{N_{\text{s}}-1}{N_{\text{s}}^2} t_{\text{s}}^2,\ c_{\text{e}}^2 = \frac{\sigma_{\text{e}}^2}{t_{\text{e}}^2} \quad (2)$$

Next we include other disturbances (preemptive outage) in the EPT calculation. t_{e}, σ_{e}^2, and c_{e}^2 obtained previously become t_0, σ_0^2, and c_0^2. Mean EPT t_{e} and effective SCV c_{e}^2 after including the preemptive outage are [16]:

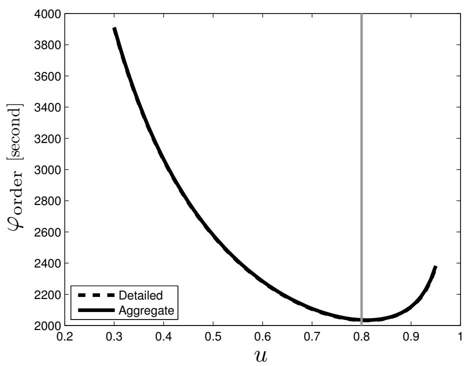
$$A = \frac{m_{\text{f}}}{m_{\text{f}} + m_{\text{r}}},\ t_{\text{e}} = \frac{t_0}{A},\ c_{\text{e}}^2 = c_0^2 + (1 + c_{\text{r}}^2)A(1-A)\frac{m_{\text{r}}}{t_0} \quad (3)$$

where m_{f} is the mean time between two consecutive disturbances, m_{r} is the mean repair time, and c_{r}^2 is the SCV of the repair times.

Applying the above formula with assumptions used in the detailed model, we obtain $t_{\text{e}} = 20.06$ seconds and $c_{\text{e}}^2 = 1.43$. Comparing these values with the measured EPT without 1st tote difference (see Section V-B), we get errors of 0.11% and 0.81% for t_{e} and c_{e}^2, respectively. This result validates our method of measuring EPT realizations from tote arrival and departure events.

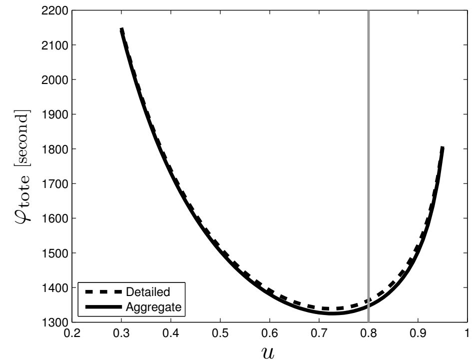


(a) Tote flow time.

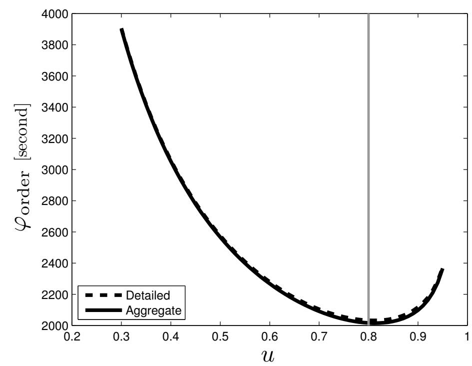


(b) Order flow time.

Fig. 7. Flow time prediction with 1st tote difference.



(a) Tote flow time.



(b) Order flow time.

Fig. 8. Flow time prediction without 1st tote difference.

D. Aggregate model

The aggregate model comprises a single-server with aggregate process times sampled from EPT distributions. With the 1st tote difference, two EPT distributions with means and SCVs $t_{e,1}$, $c^2_{e,1}$, and $t_{e,2+}$, $c^2_{e,2+}$ are used for the first and remaining totes, respectively. Without 1st tote difference, the EPT distribution has mean t_e and SCV c^2_e. The sampled aggregate process time represents the duration in which the capacity is claimed by a tote.

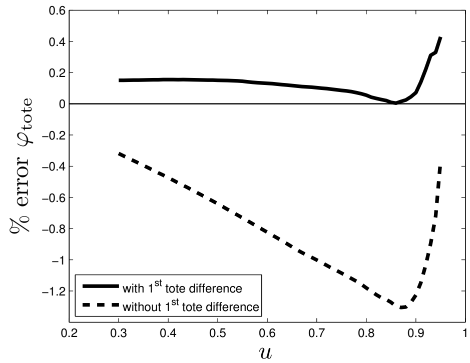
The aggregate model is simulated at the same utilization levels as the detailed model (see Section V-A). At each utilization level, 30 simulation runs of 300,000 totes and a warm up period of 30,000 totes are performed. We evaluate the flow time prediction accuracy of the aggregate model with and without 1st tote difference by comparing the flow times from the aggregate model with that of the detailed model.

E. Flow time prediction

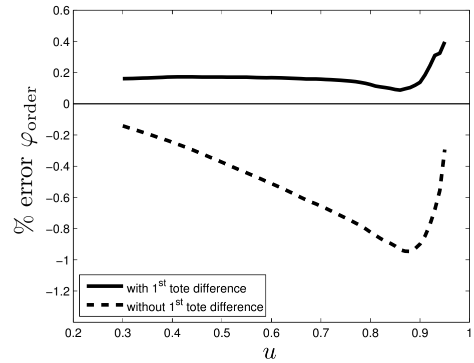
Figure 7 shows the tote and order flow time predictions *with* 1st tote difference. The first tote of an order is assigned with an aggregate process time that is significantly larger than the remaining totes (see the values of $t_{e,1}$ and $t_{e,2+}$ in Section V-B). This imposes a longer flow time for the first totes of orders. Therefore the remaining totes have to wait longer before they are processed. Consequently, the aggregate model correctly predicts both tote and order flow times. Errors for mean and variability of flow time prediction are less than 0.5% and 3.0%, respectively for both tote and order flow times.

Figure 8 shows the tote and order flow time predictions *without* 1st tote difference. Flow time predictions by the aggregate model are consistently lower than the flow times from the detailed model. This observation can be explained as follows. The processing time for all totes in the aggregate model are sampled from an EPT distribution with parameters $t_e = 20.08$ seconds and $c^2_\mathrm{e} = 1.44$ (see Section V-B). However, this t_e is significantly lower than the mean EPT of the first totes of orders $t_{e,1} = 31.15$ seconds when using 1st tote difference. This causes the aggregate model to underestimate the flow times of the first totes of orders because they are processed much faster in the aggregate model than in the detailed model. The flow times of the remaining totes are affected as well. These totes have shorter waiting time in the buffer and therefore their flow times become lower as well.

Figure 9 compares the percentage error in flow time predictions with and without 1st tote difference. It is clear that the 1st tote difference approach increases the flow time prediction accuracy.



(a) Totes.



(b) Orders.

Fig. 9. Percentage error of flow time prediction

F. Effect of order size distribution

We investigate the effect of using different order size distributions on the accuracy of flow time prediction by the aggregate model. Geometric and uniform distributions are used for this purpose. A geometric distribution allows us to model an order pattern with many small orders (e.g., internet orders, slow-moving products) or an order pattern with many large orders (e.g., supermarket orders and fast-moving products). A uniform distribution allows us to model an order pattern with a predefined maximum order length.

The geometric distribution for order size is given by its probability mass function:

$$P\{X = n\} = (1 - p)^{(n-1)}p$$

where n is the order size and two values of p are used namely 0.8 and 0.2. With $p = 0.8$ the order size distribution is short-tailed and most orders will have a size of 1 tote. On the contrary, with $p = 0.2$ the order size distribution is long-tailed and most orders will have a size larger than 1 tote.

For the uniform distribution, we set the maximum order size $n_{\max} = 20$. As such, the probability that an order size takes any value between 1 and 20 is fixed at 0.05.

Each order size distribution is used in a simulation experiment with 30 replications of 300,000 totes and a warm up period of 30,000 totes. Again, we evaluate flow time predictions from two aggregate models namely with and without 1st tote difference. In each simulation replication, the mean and variability of individual flow times are calculated as φ and c^2_φ, respectively. We take the average of all 30 replications to get the mean values of both performance measures $\bar{\varphi}$ and $\bar{c}^2_\varphi$. Subsequently, a two sample t-test at significance level $\alpha = 0.05$ is conducted to compare the mean flow time from the detailed model $\bar{\varphi}_{\mathrm{D}}$ with that of the aggregate model $\bar{\varphi}_{\mathrm{A}}$. The following two-sided hypothesis is tested.

$$H_0 : \bar{\varphi}_{\mathrm{D}} = \bar{\varphi}_{\mathrm{A}}$$
$$H_1 : \bar{\varphi}_{\mathrm{D}} \neq \bar{\varphi}_{\mathrm{A}}$$

The results are shown in Tables II and III. Prediction errors (in %) are indicated in the columns labeled with $\%\ e$.

The aggregate model with 1st tote difference predicts the mean flow times of both tote and order significantly better than the one without 1st tote difference. This can be seen from the resulting p-value of the t-test. Without 1st tote difference, the p-value is significant at some utilization levels ($p < 0.05$). At those values we reject H_0 and conclude that the mean flow times from the detailed and aggregate model are different. However, all p-values are not significant for the aggregate model with 1st tote difference. Hence, we cannot reject H_0 and accept that the mean flow time of the detailed model is similar to the mean flow time predicted by the aggregate model.

The errors for flow time SCV are larger for the order size distribution that has high probability of small orders (see columns $\%\ e\ \bar{c}^2_{\varphi_{\mathrm{A}}}$ and $\%\ e\ \bar{c}^2_{\varphi_{\mathrm{A1}}}$ in Tables II and III). For this type of order size distribution, setups between orders are performed more frequently. The error occurs because the gamma distributed EPTs do not correspond fully to the setup time, which is a convolution of a uniform and a gamma distribution. Consequently the errors of flow time variability increase as the EPT distribution is sampled more frequently. In this case, a more detailed fit for the EPT distribution of the first totes is required. We refer to [25] for alternative EPT distribution fits. However, if the probability of having small orders is low, then using a gamma distribution is sufficient.

For geometrically distributed order size with $p = 0.8$, the errors for mean flow time prediction without the 1st tote difference $\%\ e\ \bar{\varphi}_{\mathrm{A}}$ are all positive. That is, the predicted flow times from the aggregate model consistently overestimate the real flow times. This can be explained as follows. Recall that without 1st tote difference all EPT realizations are collected into a single bucket. In the case of geometric distribution with $p = 0.8$, most orders have a size of 1 tote. Therefore, most EPT realizations are high because EPTs include setup time for orders with size of 1. The resulting EPT distribution has a high mean EPT t_{e}. Since only one EPT distribution is used for sampling the aggregate process time, totes that do not require setup time (remaining totes of an order) also have high aggregate process times. This causes extra waiting for totes in the buffer and consequently higher flow times.

TABLE II
TOTE FLOW TIME (IN SECONDS) AND ITS VARIABILITY.

u	Detailed (D)		Aggregate without 1st tote difference (A)					Aggregate with 1st tote difference (A1)				
	$\bar{\varphi}_{\text{D}}$	$\bar{c}^2_{\varphi_{\text{D}}}$	$\bar{\varphi}_{\text{A}}$	$\bar{c}^2_{\varphi_{\text{A}}}$	$\%\ e\ \bar{\varphi}_{\text{A}}$	$\%\ e\ \bar{c}^2_{\varphi_{\text{A}}}$	p-value	$\bar{\varphi}_{\text{A1}}$	$\bar{c}^2_{\varphi_{\text{A1}}}$	$\%\ e\ \bar{\varphi}_{\text{A1}}$	$\%\ e\ \bar{c}^2_{\varphi_{\text{A1}}}$	p-value
Geometric $p = 0.8$												
0.3	200.47	2.35	203.47	2.30	1.50	−2.32	**0.00**	200.42	2.34	−0.02	−0.73	0.92
0.4	179.69	1.81	182.82	1.76	1.74	−3.06	**0.00**	179.62	1.79	−0.04	−1.39	0.86
0.5	174.53	1.38	177.71	1.33	1.82	−4.09	**0.00**	174.37	1.35	−0.09	−2.54	0.67
0.6	180.91	1.05	184.06	0.99	1.74	−5.69	**0.00**	180.68	1.00	−0.12	−4.49	0.58
0.7	200.80	0.80	203.87	0.73	1.53	−8.07	**0.00**	200.57	0.74	−0.11	−7.21	0.63
0.8	245.98	0.63	248.88	0.57	1.18	−10.33	**0.00**	246.08	0.57	0.04	−9.24	0.89
0.9	379.11	0.60	381.60	0.54	0.66	−10.03	0.27	380.24	0.55	0.30	−8.44	0.62
0.95	625.63	0.66	630.62	0.62	0.80	−5.61	0.57	637.12	0.63	1.84	−3.70	0.25
Geometric $p = 0.2$												
0.3	1071.00	1.22	1064.40	1.23	−0.61	0.86	**0.03**	1070.40	1.22	−0.06	0.08	0.83
0.4	875.53	1.07	868.49	1.08	−0.80	1.16	**0.01**	874.89	1.07	−0.07	0.07	0.80
0.5	771.43	0.91	763.87	0.93	−0.98	1.43	**0.00**	770.88	0.91	−0.07	−0.02	0.81
0.6	719.18	0.76	710.85	0.77	−1.16	1.71	**0.00**	718.62	0.76	−0.08	−0.17	0.80
0.7	707.24	0.61	697.83	0.62	−1.33	1.87	**0.00**	706.43	0.60	−0.11	−0.43	0.72
0.8	744.15	0.46	732.88	0.46	−1.52	1.49	**0.00**	742.79	0.45	−0.18	−1.32	0.61
0.9	899.89	0.33	886.05	0.33	−1.54	−0.41	**0.01**	896.99	0.31	−0.32	−4.03	0.56
0.95	1184.60	0.31	1173.30	0.32	−0.96	2.79	0.46	1187.20	0.31	0.22	−0.67	0.87
Uniform $n_{\textbf{max}} = 20$												
0.3	1161.10	0.90	1157.60	0.90	−0.30	0.22	**0.05**	1163.00	0.90	0.16	−0.21	0.28
0.4	938.48	0.80	934.43	0.80	−0.43	0.29	**0.01**	939.91	0.79	0.15	−0.26	0.31
0.5	815.37	0.69	810.86	0.69	−0.55	0.29	**0.00**	816.49	0.69	0.14	−0.38	0.36
0.6	746.40	0.58	741.42	0.58	−0.67	0.18	**0.00**	747.17	0.58	0.10	−0.59	0.51
0.7	716.53	0.48	710.72	0.48	−0.81	−0.09	**0.00**	716.55	0.47	0.00	−0.94	0.99
0.8	730.08	0.37	722.78	0.37	−1.00	−0.90	**0.00**	728.51	0.36	−0.22	−1.79	0.26
0.9	845.82	0.29	836.96	0.28	−1.05	−4.59	**0.01**	841.58	0.28	−0.50	−5.57	0.21
0.95	1089.00	0.31	1089.40	0.31	0.05	0.40	0.97	1087.90	0.30	−0.10	−3.29	0.94

TABLE III
ORDER FLOW TIME (IN SECONDS) AND ITS VARIABILITY.

u	Detailed (D)		Aggregate without 1st tote difference (A)					Aggregate with 1st tote difference (A1)				
	$\bar{\varphi}_{\text{D}}$	$\bar{c}^2_{\varphi_{\text{D}}}$	$\bar{\varphi}_{\text{A}}$	$\bar{c}^2_{\varphi_{\text{A}}}$	$\%\ e\ \bar{\varphi}_{\text{A}}$	$\%\ e\ \bar{c}^2_{\varphi_{\text{A}}}$	p-value	$\bar{\varphi}_{\text{A1}}$	$\bar{c}^2_{\varphi_{\text{A1}}}$	$\%\ e\ \bar{\varphi}_{\text{A1}}$	$\%\ e\ \bar{c}^2_{\varphi_{\text{A1}}}$	p-value
Geometric $p = 0.8$												
0.3	272.23	1.65	275.21	1.64	1.09	−0.75	**0.00**	272.16	1.64	−0.02	−0.56	0.91
0.4	233.51	1.33	236.61	1.31	1.33	−1.17	**0.00**	233.42	1.31	−0.04	−1.14	0.84
0.5	217.59	1.05	220.76	1.03	1.46	−1.96	**0.00**	217.41	1.02	−0.08	−2.23	0.68
0.6	216.79	0.81	219.94	0.78	1.45	−3.44	**0.00**	216.55	0.78	−0.11	−4.20	0.59
0.7	231.56	0.63	234.62	0.59	1.32	−6.02	**0.00**	231.32	0.58	−0.10	−7.12	0.64
0.8	272.85	0.51	275.78	0.47	1.07	−8.99	**0.00**	272.98	0.46	0.05	−9.49	0.86
0.9	402.98	0.52	405.54	0.47	0.64	−9.52	0.26	404.19	0.47	0.30	−8.54	0.60
0.95	648.27	0.61	653.30	0.57	0.78	−5.37	0.57	659.89	0.58	1.79	−3.54	0.25
Geometric $p = 0.2$												
0.3	1917.30	0.42	1911.20	0.43	−0.32	1.29	0.12	1917.20	0.42	0.00	0.15	0.99
0.4	1510.30	0.38	1503.50	0.39	−0.45	1.83	**0.03**	1510.00	0.39	−0.02	0.17	0.92
0.5	1279.30	0.34	1271.80	0.35	−0.58	2.34	**0.01**	1278.90	0.34	−0.03	0.07	0.90
0.6	1142.40	0.30	1134.20	0.31	−0.72	2.91	**0.00**	1142.00	0.30	−0.04	−0.11	0.87
0.7	1070.10	0.25	1060.70	0.26	−0.88	3.46	**0.00**	1069.40	0.25	−0.07	−0.44	0.77
0.8	1061.80	0.20	1050.40	0.21	−1.07	3.36	**0.00**	1060.50	0.20	−0.13	−1.62	0.64
0.9	1182.30	0.16	1168.30	0.16	−1.18	0.91	**0.01**	1179.50	0.15	−0.24	−5.15	0.58
0.95	1452.10	0.19	1440.80	0.19	−0.78	4.00	0.46	1454.80	0.18	0.19	−0.86	0.87
Uniform $n_{\textbf{max}} = 20$												
0.3	2728.70	0.15	2729.70	0.15	0.04	0.39	0.75	2732.20	0.15	0.13	−0.33	0.25
0.4	2106.20	0.14	2106.40	0.14	0.01	0.56	0.94	2108.90	0.14	0.13	−0.46	0.25
0.5	1742.50	0.13	1742.10	0.13	−0.02	0.69	0.84	1744.60	0.13	0.12	−0.69	0.26
0.6	1512.30	0.11	1511.30	0.12	−0.06	0.75	0.58	1513.90	0.11	0.11	−1.03	0.34
0.7	1366.50	0.10	1364.60	0.10	−0.13	0.60	0.26	1367.10	0.10	0.05	−1.65	0.66
0.8	1292.30	0.09	1288.90	0.09	−0.27	−0.27	**0.04**	1291.10	0.08	−0.09	−3.07	0.45
0.9	1339.30	0.09	1334.00	0.08	−0.40	−5.35	0.13	1335.10	0.08	−0.32	−8.11	0.20
0.95	1553.20	0.13	1556.90	0.13	0.23	1.90	0.81	1551.70	0.12	−0.10	−3.76	0.91

VI. Case Study

A case study with data obtained from an operating automated warehouse is used to illustrate the applicability of our method in a real warehouse setting. The warehouse shown in Figure 10 distributes slow-moving products to a number of supermarkets in the Netherlands. Three processing units are present in the warehouse, namely miniloads, a conveyor loop, and order picking workstations. Miniloads provide temporary storage spaces for product totes. The conveyor loop transports product totes from the miniload to the order picking workstations, and the other way around. Three order picking workstations, with a similar structure as shown in Figure 1, are available to process customer orders. We predict the flow time of totes and orders at the order picking workstation using an aggregate simulation model. These flow times exclude the time spent while retrieving the totes from the miniload and the time spent by the totes while traveling on the conveyor loop. That is, the tote and order flow times start when a tote and the first tote of an order arrive at the order picking workstations, respectively.

A. Data processing

The data consist of event logs collected via Programmable Logic Controllers (PLC) from all processing units in the warehouse. From this PLC data we extract tote arrival and departure events at the order picking workstations. EPT realizations are then calculated using Equation 1. Other parameters are also extracted from the event data, including the interarrival times of totes, order lengths, and the order release strategy. These parameters are the input for the aggregate simulation model to predict tote and order flow times. Subsequently, we compare the predicted flow times with the flow times measured from the data. This demonstrates the prediction accuracy of the method when applied to the data from a real, operating warehouse. Figure 11 depicts the flow chart of activities performed in this case study.

Arrival and departure events from three working days are extracted from the PLC data for all three order picking workstations. An event consists of type (arrival or departure), time, order ID, order length, and tote ID. Some recorded events may be inconsistent or extreme outliers; e.g., a tote may have an arrival recorded without a departure, or the other way around. Extreme outliers are present when some exceptionally large delays occur between two events, for instance due to lunch breaks. These breaks occur also when there are some totes still waiting in the buffer. The outliers cause very large values for the EPT of the next required tote waiting to be served, the tote flow times of all totes in the buffer, and the order flow times pertaining to the totes in the buffer. Therefore, we filter the arrival and departure events to exclude inconsistent events and large delays between two events if they are longer than 60 seconds. This threshold has been chosen based on the observation that it is very unlikely that there is no arrival or departure event at all within 60 seconds from the previous event. Only 2.8% of all arrival and departure events are discarded due to data filtering. The remaining arrival and departure events (97.2%) are used to extract EPTs, interarrival times, order lengths, and order release strategy.

The calculated EPTs are sorted based on the 1st tote difference rule into EPTs of the first totes and EPTs of the remaining totes. We observed that many large EPTs occur when not all totes for the active order are present in the buffer as the picker starts picking. That is, the EPTs are smaller when the picker finds all totes for the active order present in the buffer. This observation holds for both the EPTs of the first totes and the EPTs of the remaining totes. As such, we further sort the EPTs based on the *completeness* of totes in the buffer when the picker starts picking a tote. This results

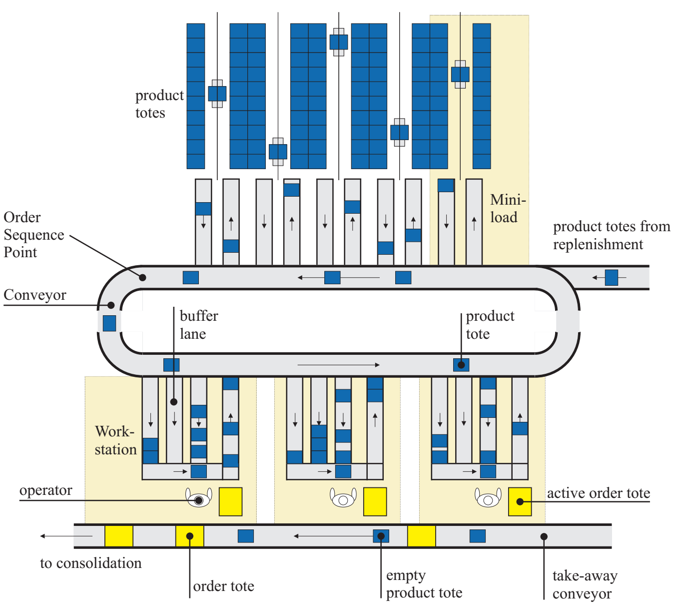


Fig. 10. Layout of an automated warehouse.

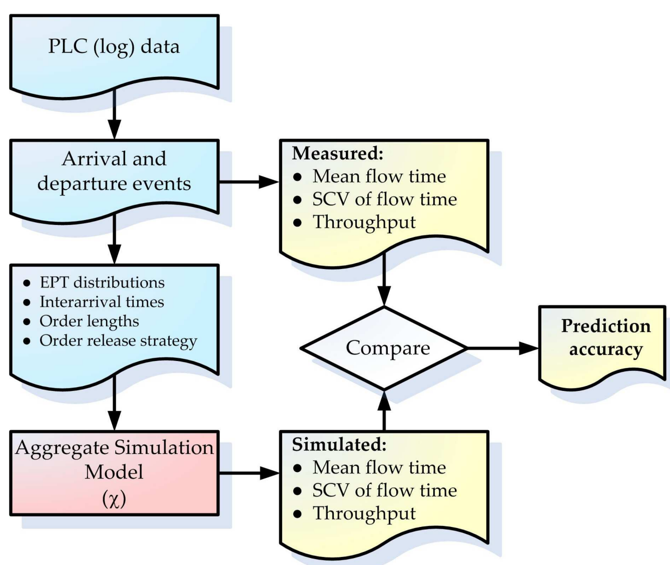


Fig. 11. Case study flow chart.

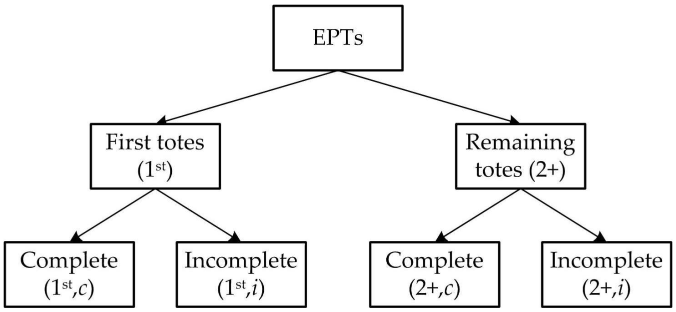


Fig. 12. Sorting of EPTs.

in four types of EPT as shown in Figure 12, namely EPTs first totes complete ($1^{\text{st}}, c$), EPTs first totes incomplete ($1^{\text{st}}, i$), EPTs remaining totes complete ($2+, c$), and EPTs remaining totes incomplete ($2+, i$). We use shifted gamma distributions to represent all four EPT distributions because the EPTs can never be smaller than a certain value. Hence, using a shifted gamma distribution with the minimum EPT value as offset should produce a better fit than using a gamma distribution as previously done in Section V. Figure 13 visualizes the EPT distributions gathered from workstation 1. The EPTs have been normalized due to data confidentiality. We can see that there are significant differences between all four EPT distributions. It is therefore important to distinguish the EPTs based on the 1^{st} tote difference approach and completeness.

The interarrival distribution can be easily extracted from the tote arrival times at the workstation. However, these interarrival times alone are not sufficient. We also need to reconstruct the order release strategy used in the operating warehouse. Such strategy determines to which order the next arriving tote belongs. Since tote and order flow times are affected by the sequence in which totes arrive, the flow time prediction accuracy also depends on how accurate the order release strategy is modeled in the aggregate simulation model as compared to the reality. The order release strategy is reconstructed also from the arrival and departure events, as follows. We create several so-called *buckets* for each order length. When the first tote a new order arrives at the workstation, the order length of that order is registered. Afterwards, we count the number of totes arriving subsequently for this order until the arrival of the first tote of the next order. The resulting number of totes is then collected into the corresponding bucket based on the order length. Next, an empirical distribution function of the number of totes is created for each bucket. These distributions will be used in the aggregate simulation model to sample the number of totes to be generated for the active order before generating the first tote of the next order.

To assess the quality of the reconstructed order release strategy, we compare the interarrival time of orders from the real data with the simulation. The order interarrival time is defined as the time between the arrival of the first totes of new orders. The result is depicted in Figure 14. The interarrival times have been normalized due to data confidentiality. The reconstructed order release strategy resembles the order release strategy used in the operating warehouse.

B. Flow time prediction

The aggregate simulation model shown in Figure 15 is used to predict the tote and order flow time of the operating warehouse. We model the system as a closed queueing network with two sequential servers namely the miniload and the workstation. The input parameters used in each server are shown in the figure.

The miniload generates new totes for the workstation if there is a space available in the finite buffer of the workstation. For each tote generated, the miniload determines the interarrival time of the tote, the tote ID, the order ID, and the order length. Since totes from multiple orders are generated simultaneously, the order release strategy reconstructed previously is used to determine the order ID indicating the order to which a tote belongs. The order release strategy works as follows. Suppose

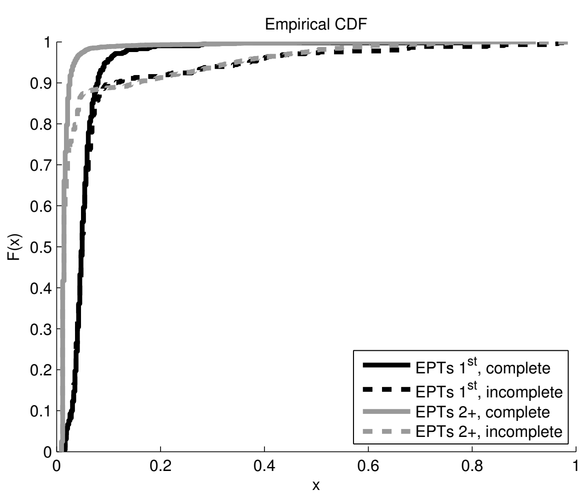


Fig. 13. CDF of four types of EPT from workstation 1.

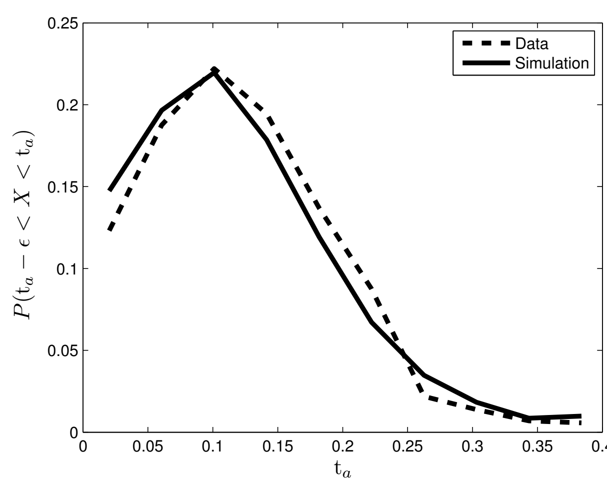


Fig. 14. Distribution of interarrival times of orders.

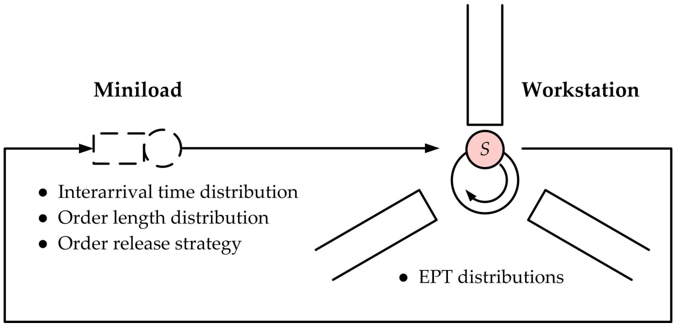


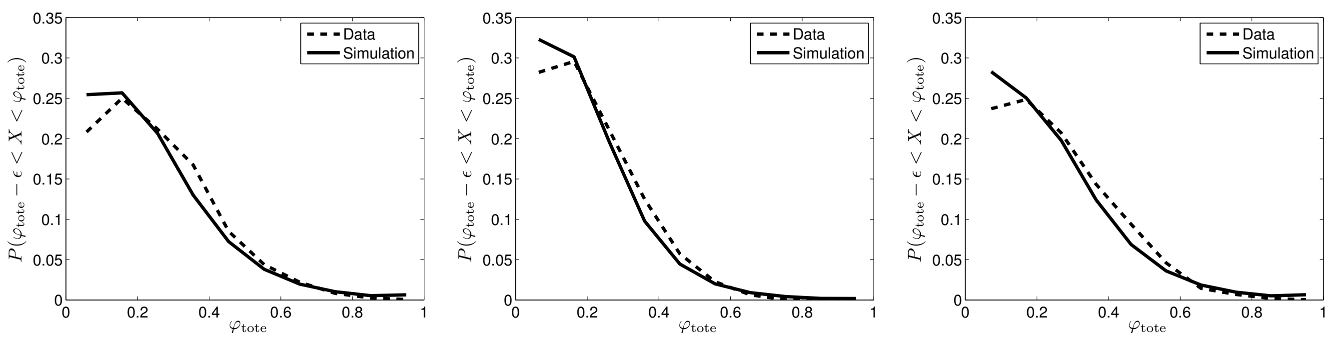
Fig. 15. Aggregate simulation model of the operating warehouse.

the system is empty and the miniload generates the first tote of order X with an order length of 5 totes. At that moment we say that a new order, i.e., order X, has entered the system. We sample from the corresponding bucket, that is the bucket for order length 5, the number of totes N to be generated for order X before generating the next order. Suppose we sample $N = 3$, this means the next three generated totes will belong to order X. The fourth generated tote belongs to a new order, e.g., order Y. Now two orders are in the system and the miniload will decide based on a certain probability whether the tote generated next belongs to order X, order Y, or another new order. If all totes of order X has been generated, then the value N will be sampled again from the correct bucket based on the order length of order Y. The interarrival time of totes are sampled from the interarrival time distribution.

The workstation is modeled as a polling system with a finite buffer. The EPT for each tote being processed is sampled from one of the four EPT distributions, depending on the type of the tote and the completeness of totes for the active order. For example, if the tote is the first tote of an order and not all totes for this order are present in the buffer at the start of picking, then the EPT for this tote will be sampled from the distribution of EPTs 1st incomplete.

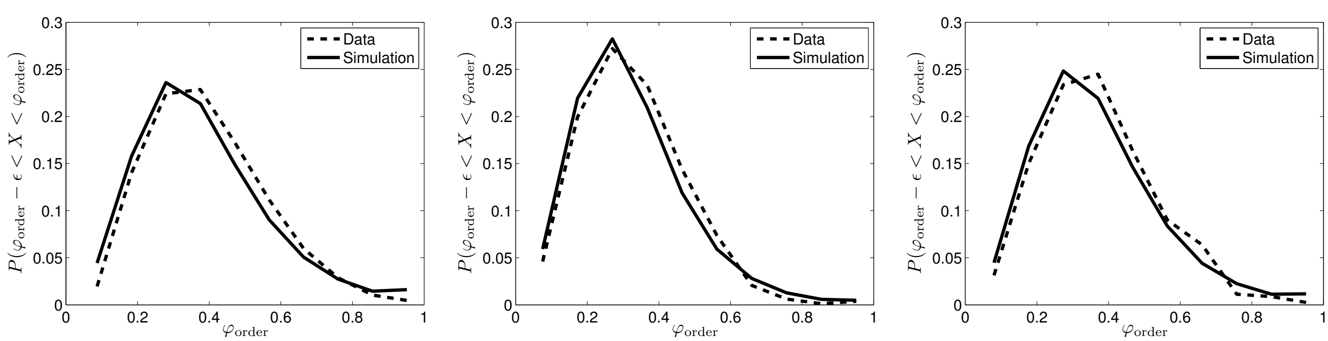
The aggregate simulation model has been run with 50 replications each with a run length of 1,000,000 totes excluding a warm-up period of 300,000 totes for all three workstations. The predicted flow times from the aggregate simulation model are then compared with the real flow times from the data.

The resulting flow time distributions shown in Figures 16 and 17 suggest that the aggregate simulation model accurately predicts the flow time of totes and orders. Indeed, the errors of mean tote and order flow time prediction are less than 5.5%. We also observe that the tote flow time variability is better predicted than orders flow time variability. That is, in Figure 16 the aggregate model consistently overestimates the occurrence of small values of tote flow times.



(a) Workstation 1. (b) Workstation 2. (c) Workstation 3.

Fig. 16. Tote flow time distributions.



(a) Workstation 1. (b) Workstation 2. (c) Workstation 3.

Fig. 17. Order flow time distributions.

C. Discussion

We found that tote and order overtaking exist in the real warehouse. Overtaking of totes occur when the picker picks a product tote that did not arrive the earliest for the active order. Any time loss due to overtaking is not accounted for in the EPT calculation using Equation 1. Overtaking of orders occur if upon completion of an order the picker processes the next order that is not the oldest in the buffer. We measured the average percentage of tote and order overtaking from the three workstations to be 18.3% and 4.6%, respectively. This may explain the underestimation of mean tote and order flow time prediction since overtaking is not considered in the aggregate simulation model.

Another insight from the case study is that pickers hesitate to wait for totes. If totes for the active order are not yet complete in the buffer by the time the picker starts picking, it is very likely that the picker will leave the workstation for a while. Suppose that a tote for the active order arrives shortly after the picker leaves, then the EPT for this tote will include the time when the picker was leaving the workstation. This will cause the EPT for this tote to become very large. To account for this phenomenon, we have sorted EPTs based on the completeness of totes in the buffer. In practice, one may want to improve the order release strategy such that the totes for the active order arrive more frequently than the totes for other orders. One may also consider allowing pickers to work on multiple orders simultaneously, hence reducing the likeliness that the picker becomes idle waiting for the required totes.

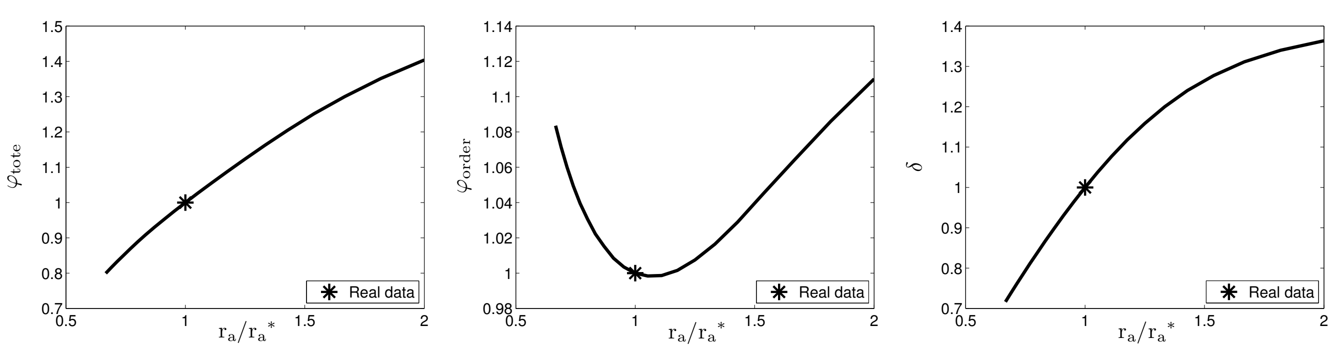
In this case study, the aggregate simulation model is able to predict the tote and order flow time with satisfactory accuracy based on real data of three working days. The aggregate model may further be used to analyze the effect of different interarrival rates, order length distributions, order release strategies, etc. on the system throughput and flow times. As an example, Figure 18 shows the predicted performance of workstation 1 under various interarrival rates. One may also be interested in analyzing the flow time distribution for a specific order length. Furthermore, EPTs can be calculated in real-time to monitor the performance of an operating order picking workstation such that any deviations from the expected performance (e.g., extremely large EPTs) can be detected and the necessary corrective actions can be performed timely.

VII. Conclusion and Future Work

In this paper, we propose a method to predict the mean and variability of tote and order flow times for a single-server order picking workstation by means of a simulation model that is based on an aggregate process time distribution. Arrival and departure data of totes are the only input required to calculate the aggregate process time distribution. Inspired by [16], we refer to the aggregate process time as Effective Process Time. We actually distinguish two types of EPT realizations namely for the first totes and for the remaining totes of orders, which we refer to as the 1st tote difference EPT method. We have demonstrated in the simulation validation study that this separation is important because the EPTs of the first totes are not identically distributed with the EPTs of the remaining totes. Therefore we sort the EPTs into two EPT distributions. We then fit a gamma distribution to these empirical EPT distribution data, but in principle any other suitable distributions may be used. The two gamma EPT distributions are used to sample the aggregate process time in the aggregate simulation model. We find that the proposed method accurately predicts the mean and variability of tote and order flow times.

We apply the method to data obtained from a real, operating warehouse. The flow time prediction by the aggregate simulation model has satisfactory accuracy, even with the relatively small amount of arrival and departure data. Practical insights for performance improvement are proposed based on the observation of EPTs from the real data. The resulting EPT distributions represent the actual pick rate of an order picking workstation, which for performance analysis purposes can be compared to the expected pick rate. The aggregate simulation model can further be used to evaluate the order picking workstation's performance under different settings.

The proposed method can also be used for manufacturing workstations processing a number of different product types, where switching from one product type to another requires



(a) Mean tote flow time. (b) Mean order flow time. (c) Throughput.

Fig. 18. Predicted performance of workstation 1 under various interarrival rates.

a setup time (see e.g., [26]). Here, the first job of a product type will have a different process time distribution than the remaining jobs. Hence, the 1st tote difference EPT approach used in this paper is also valuable in other systems.

Current advancements in order picking technology have allowed multiple orders to be processed simultaneously by a picker, which is often the case in warehouses with fast-moving products. Also, tote routings in large-scale automated warehouses may cause orders at the workstation to be processed not in a FIFO sequence. Performance analysis of these types of order picking workstation is subject to future work.

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