

# Performance Analysis of Multi-Antenna Hybrid Detectors and Optimization with Noise Variance Estimation

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**Abstract**—In this paper, a performance analysis of multi-antenna spectrum sensing techniques is carried out. Both well known algorithms, such as Energy Detector (ED) and eigenvalue based detectors, and an eigenvector based algorithm, are considered. With the idea of auxiliary noise variance estimation, the performance analysis is extended to the hybrid approaches of the considered detectors. Moreover, optimization for Hybrid ED under constant estimation plus detection time is performed. Performance results are evaluated in terms of Receiver Operating Characteristic (ROC) curves and performance curves, i.e., detection probability as a function of the Signal-to-Noise Ratio (SNR). It is concluded that the eigenvector based detector and its hybrid approach are able to approach the optimal Neyman-Pearson performance.

**Keywords**—hybrid detector; largest eigenvector; noise estimation; spectrum sensing; cognitive radio.

## I. INTRODUCTION

Spectrum sensing is the enabling unit of Secondary Users (SUs) in Cognitive Radio Networks (CRN) [1] for the accurate identification and exploitation of unused Primary User's (PU) spectrum in temporal and spatial domain. Precise identification of the spectrum holes is the major constraint for the establishment of Cognitive Radio, which ensures the dynamic exploitation of existing wireless spectrum. As an example, the Wireless Regional Area Network (WRAN) standard imposes stringent requirement on the probability of detection ( $P_d$ )  $\geq 0.9$  with probability of false alarm  $P_{Fa} \leq 0.1$  at Signal-to-Noise Ratio (SNR)  $-20dB$  (for Digital TV band) [3].

In order to satisfy the constraint of high performance and considering the dependence of noise uncertainty and the implementation complexity under wireless fading channels, several detection algorithms are put forward in context of Cognitive Radio applications including Uniformly Most Powerful (UMP) test derived according to the Neyman-Pearson Lemma known as Neyman-Pearson (NP) test [2], Energy Detection (ED) [4], Match Filtering [5], Feature Detection Algorithms [6] proposed for individual SU and their cooperative counterpart for multiple SU sensing. A multidimensional CR receiver has been studied considering multiple receive dimensions at the CR receiver in the form of multiple antennas, over-sampled branches and cooperative nodes [7]–[10]. These methods are mostly based on the statistics of the eigenvalues of the received signal covariance matrix and use recent results from Random Matrix Theory (RMT).

In the last few years, Eigenvalue Based Detection (EBD) techniques received considerable attention in spectrum sensing literature with improved performance and less dependent on noise uncertainty [7]–[16]. Some of the popular EBD based techniques in present literature include Maximum Eigenvalue

(ME) based [14], Eigenvalue Ratio Detector (ERD) [17], Signal Condition Number (SCN) based [10][12], Scaled Largest Eigenvalue (SLE) based [15][16], Akaike Information Criterion (AIC) [18], Minimum Description Length(MDL) [18]. Recently, more powerful techniques based on largest eigenvalue of the received covariance matrix, like Generalized Likelihood Root Test (GLRT) [20] and Roy's Largest Root Test (RLRT) [19] have been proposed and analyzed. Recently, a new algorithm known as EigenVEctor (EVE) test [21] has been introduced exploiting channel estimation parameter in the detection statistic whose performance is comparable with NP test.

Considering high performance detection algorithms like ED, RLRT and EVE test, the problem of unknown noise variance is crucial. In our previous work [21][22], the performance of hybrid approach of ED and Roy's Largest Root Test using estimated noise variance was carried out. It was suggested that, the optimum performance of ED and RLRT can be achieved even with the use of estimated noise variance by using a large number of slots for noise variance estimation.

In this work, we present a performance analysis of Roy's Largest Root Test (RLRT), Energy Detection (ED), EigenVEctor Test (EVE) and their hybrid approaches with noise variance estimation. Section II describes the system model and the NP test, which will be used as a benchmark. Section III illustrates the test statistics with known noise variance, while Section IV presents the hybrid approaches with estimated noise variance. Simulation results are presented in Section V, while in Section VI, some preliminary results of the optimization of Hybrid Energy Detection are presented. Finally, Section VII concludes the paper.

## II. SYSTEM MODEL

Let us denote by  $K$  the number of antennas or cooperative sensors and by  $N$  the number of samples per sensing slot. We focus on a single source scenario, which is of interest for many detection problems in cognitive radio networks. The  $K \times 1$  received vector at time  $n$  collects the baseband complex samples from the  $K$  antennas. The received samples are stored by the detector in the  $K \times N$  matrix  $\mathbf{Y}$ .

Let us introduce the  $1 \times N$  signal matrix  $\mathbf{s} \triangleq [s_1 \cdots s_n \cdots s_N]$  and the  $K \times N$  noise matrix  $\mathbf{V} \triangleq [v_1 \cdots v_n \cdots v_N]$  where,

- $s_n$  is the transmitted complex signal sample at time  $n$ , modeled as Gaussian with zero mean and variance  $\sigma_s^2$ :  $s_n \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_s^2)$

- $\mathbf{v}_n$  is a noise vector at time  $n$ , modeled as Gaussian with mean zero and variance  $\sigma_v^2$  :  $\mathbf{v}_n \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{K \times 1}, \sigma_v^2 \mathbf{I}_{K \times K})$

As all the signal samples  $s_n$  of  $\mathbf{s}$  and the noise vectors  $\mathbf{v}_n$  of  $\mathbf{V}$  are assumed statistically independent, the detector must distinguish between Null and Alternate Hypothesis given by,

$$\mathbf{Y}|_{\mathcal{H}_0} = \mathbf{V} \quad \text{and} \quad \mathbf{Y}|_{\mathcal{H}_1} = \mathbf{h}\mathbf{s} + \mathbf{V}$$

where,  $\mathbf{h}$  is the complex channel vector  $\mathbf{h} = [h_1 \cdots h_K]^T$ ; assumed to be constant and memory-less during the sampling window.

Under  $\mathcal{H}_1$ , the average SNR at the receiver is defined as,  $\rho \triangleq \frac{\mathcal{E}\|\mathbf{x}_n\|^2}{\mathcal{E}\|\mathbf{v}_n\|^2} = \frac{\sigma_s^2 \|\mathbf{h}\|^2}{K \sigma_v^2}$  where,  $\|\cdot\|$  denotes the Euclidean norm and  $\mathcal{E}$  the mean operator. The sample covariance matrix is given by  $\mathbf{R} \triangleq \frac{1}{N} \mathbf{Y} \mathbf{Y}^H$  and  $\lambda_1 \geq \cdots \geq \lambda_K$  its eigenvalues sorted in decreasing order.

The usual criterion for comparing two tests is to fix the false alarm rate  $P_{fa}$  and look for the test achieving the higher  $P_d$ . The Neyman Pearson (NP) lemma is known to provide the Uniformly Most Powerful (UMP) test, achieving the maximum possible  $P_d$  for any given value of  $P_{fa}$ . The NP criterion is applicable only when both  $\mathcal{H}_0$  and  $\mathcal{H}_1$  are simple hypotheses. In our setting this is the case when both the noise level  $\sigma_v^2$  and the channel vector  $\mathbf{h}$  are a priori known. The NP test is given by the following likelihood ratio:

$$T_{NP} = \frac{p_1(\mathbf{Y}; \mathbf{h}, \sigma_s^2, \sigma_v^2)}{p_1(\mathbf{Y}; \sigma_v^2)} \quad (1)$$

and is known to be optimal, i.e., to achieve the maximum possible  $P_d$  for any given value of  $P_{fa}$ .

As an example, under the considered model, if the signal samples are independent Gaussian samples, the NP test is obtained by using:

$$p_0(\mathbf{Y}; \sigma_v^2) = \frac{1}{(\pi \sigma_v^2)^{NK}} \exp\left(-\frac{Ntr\mathbf{R}}{\sigma_v^2}\right) \quad (2)$$

and

$$p_1(\mathbf{Y}; \mathbf{h}, \sigma_s^2, \sigma_v^2) = \frac{1}{(\pi^K \det \boldsymbol{\Sigma})^N} \exp(-\mathbf{R} \boldsymbol{\Sigma}^{-1}) \quad (3)$$

where,  $\boldsymbol{\Sigma} = \sigma_v^2 \mathbf{I}_K + \sigma_s^2 \mathbf{h} \mathbf{h}^H$

The NP test provides the best possible performance, but requires exact knowledge of both  $\mathbf{h}$  and  $\sigma_v^2$ . For most practical applications, the knowledge of  $\mathbf{h}$  and  $\sigma_v^2$  is questionable.

### III. TEST STATISTICS WITH KNOWN NOISE VARIANCE

To make the decision between  $\mathcal{H}_0$  and  $\mathcal{H}_1$ , a test statistic compares a quantity  $T$  against a pre-defined threshold  $t$ : if  $T > t$ ,  $\mathcal{H}_1$  is selected, otherwise  $\mathcal{H}_0$  is chosen. The test performance is evaluated by the false alarm probability and the detection probability, defined as:

$$P_d = \mathbb{P}(T > t | \mathcal{H}_1) \quad (4)$$

$$P_{fa} = \mathbb{P}(T > t | \mathcal{H}_0) \quad (5)$$

In practice, the decision threshold  $t$  is typically computed as a function of the target  $P_{fa}$ , to guarantee the Constant false Alarm rate (CFAR) property.

#### A. Roy's Largest Root Test (RLRT)

Using the information of the received signal matrix  $\mathbf{Y}$  and assuming a perfect knowledge of the noise variance  $\sigma_v^2$  and the channel parameter  $\mathbf{h}$ , test statistic for RLRT is given by

$$T_{RLRT} = \frac{\lambda_1}{\sigma_v^2}. \quad (6)$$

If  $T_{RLRT} < t$  it decides in favor of Null Hypothesis  $\mathcal{H}_0$  otherwise in favor of Alternate Hypothesis  $\mathcal{H}_1$ . The detection probability  $P_d^{RLRT} > t | \mathcal{H}_1$  and false alarm  $P_{fa}^{RLRT} > t | \mathcal{H}_0$  probabilities for this detector are well-known in the literature (e.g., [24]).

The optimality of RLRT in the class of semi-blind algorithms was pointed out in [25]. For a single emitting source, if the SNR is above the identifiability threshold given by  $\rho > \rho_{Cric} = \frac{1}{\sqrt{KN}}$  [26], the signal is detectable by the largest eigenvalue  $\lambda_1$  value. Starting from the NP test and using the asymptotic expansion of the hypergeometric function, it was shown in that, under known noise variance, distinguishing between  $\mathcal{H}_0$  and  $\mathcal{H}_1$  in the asymptotic regime ( $N \rightarrow \infty$  with  $K$  fixed) depends to leading order only on  $\lambda_1$ .

#### B. Energy Detection (ED)

ED computes the average energy of the received signal matrix  $\mathbf{Y}$  normalized by the noise variance  $\sigma_v^2$  and compares it against a predefined threshold  $t_{ed}$ .

$$T_{ED} = \frac{1}{KN \sigma_v^2} \sum_{k=1}^K \sum_{n=1}^N |y_k(n)|^2. \quad (7)$$

If  $T_{ED} < t_{ed}$  it decides in favor of Null Hypothesis  $\mathcal{H}_0$  otherwise in favor of Alternate Hypothesis  $\mathcal{H}_1$ . The detection probability  $P_d = Prob\{T_{ED} > t | \mathcal{H}_1\}$  and false alarm  $P_{fa} = Prob\{T_{ED} > t | \mathcal{H}_0\}$  probabilities for this detector are well-known in the literature (e.g., [4]).

#### C. EigenVector Test (EVE)

The starting idea of the new test is that given a  $\mathcal{H}_1$  slot, the eigenvector  $\mathbf{e}_1$  associated to largest eigenvalue  $\lambda_1$  provides an estimation of the channel vector  $\mathbf{h}$ .

Given  $S_{aux}$  signal slots available before the current sensing slot, we can construct a matrix of size  $K \times (S_{aux} \cdot N)$  from all the samples and evaluate the eigenvector  $\mathbf{e}_{aux}$  corresponding to largest eigenvalue. The proposed statistical test known as EVE test [10], which exploits the channel estimation parameter  $\mathbf{e}_{aux}$  in its test statistic is defined as,

$$T_{EVE} = \frac{S_{aux} [\mathbf{e}_{aux}^H \mathbf{R} \mathbf{e}_{aux}] + [\mathbf{e}^H \mathbf{R} \mathbf{e}]}{\sigma_v^2 (S_{aux} + 1)} \quad (8)$$

Note that if  $S_{aux} = 0$ , the test reduces to

$$T_{EVE} = \frac{\mathbf{e}^H \mathbf{R} \mathbf{e}}{\sigma_v^2} = \frac{\|\mathbf{e}\|^2 \lambda_1}{\sigma_v^2} = \frac{\lambda_1}{\sigma_v^2} \quad (9)$$

and has the same statistical power of the RLRT.

#### IV. HYBRID TEST STATISTICS

It is evident that the knowledge of the noise variance is imperative for the optimum performance of RLRT, ED and EVE tests. Unfortunately, the variation and the unpredictability of noise variance is unavoidable. Thus, the knowledge of the noise variance is one of the critical limitations of those tests for their ideal operation in low SNR. Under the considered scenario, noise variance can be estimated from  $S_{aux}$  auxiliary noise-only slots in which we are sure that the primary signal is absent.

Consider a sampling window of length  $M$  prior and adjacent to the detection window which contains noise-only samples for sure. Then the estimated noise variance from the noise-only samples using a Maximum Likelihood Estimation (MLE) can be written as,

$$\hat{\sigma}_v^2 = \frac{1}{KM} \sum_{k=1}^K \sum_{m=1}^M |v_{km}|^2 \quad (10)$$

If the noise variance is constant, the estimation can be averaged over  $S_{aux}$  successive noise-only slots and (10) can be modified by averaging over  $S_{aux}$  successive noise-only slots as,

$$\hat{\sigma}_v^2(S_{aux}) = \frac{1}{KS_{aux}M} \sum_{s=1}^{S_{aux}} \sum_{k=1}^K \sum_{m=1}^M |v_{km}|^2 \quad (11)$$

##### A. Hybrid RLRT (HRLRT)

Knowledge of the noise power is one of the critical limitation of RLRT for its operation in low SNR. Hybrid RLRT (HRLRT) [22] deals with the study of detection performance of the RLRT algorithm using noise variance estimated from  $S_{aux}$  auxiliary noise only slots where we are sure that the primary signal is absent. The test statistic of HRLRT can now be presented as,

$$T_{HRLRT} = \frac{\lambda_1}{\hat{\sigma}_{HRLRT}^2(S_{aux})} \quad (12)$$

where,  $\hat{\sigma}_{HRLRT}^2(S_{aux})$  is the Maximum Likelihood Estimate of the true noise variance  $\sigma_v^2$  given by (11).

Performance of HRLRT in terms of ROC parameters are derived and well justified in [22][23].

##### B. Hybrid ED (HED)

Hybrid ED (HED) [22] deals with the study of detection performance of the ED algorithm using noise variance estimated from  $S_{aux}$  auxiliary noise only slots where we are sure that the primary signal is absent. The test statistic of HED can be presented as,

$$T_{HED} = \frac{1}{KN\hat{\sigma}_{HED}^2(S_{aux})} \sum_{k=1}^K \sum_{n=1}^N |y_k(n)|^2 \quad (13)$$

where  $\hat{\sigma}_{HED}^2(S_{aux})$  is computed as in (11) for HRLRT. The detection probability  $P_d = Prob\{T_{HED} > t_{H_1}\}$  and false alarm  $P_{fa} = Prob\{T_{HED} > t_{H_0}\}$  probabilities for this Hybrid ED can be referred in literature [22][23].

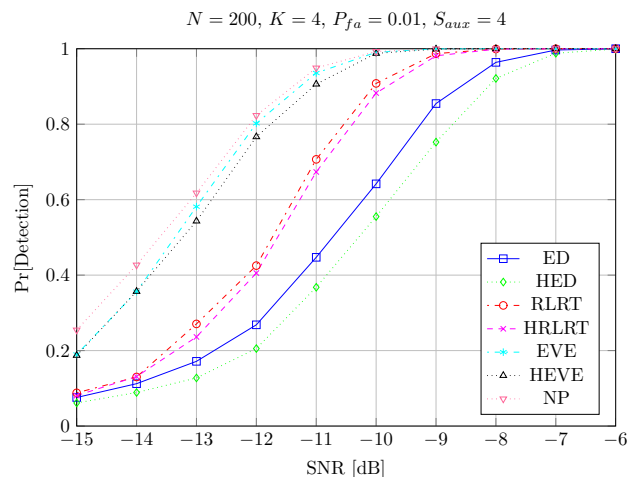


Figure 1. Performance curve,  $P_d$  vs. SNR,  $K = 4$ ,  $N = 200$

##### C. Hybrid EVE (HEVE)

If we apply the same hybrid approach for RLRT and ED of [22][23] to the new EVE test, we define a new Hybrid EigenVector (HEVE) test:

$$T_{HEVE} = \frac{S_{aux} [e_{aux}^H \mathbf{R} e_{aux}] + [e^H \mathbf{R} e]}{\hat{\sigma}_{HEVE}^2(S_{aux}) \cdot (S_{aux} + 1)} \quad (14)$$

where  $\hat{\sigma}_{HEVE}^2(S_{aux})$  is computed as in (11) for HRLRT and HED. In fact, we use in HEVE the same number of slots  $S_{aux}$  both to compute the eigenvector  $e_{aux}$  for channel estimation and to estimate the noise variance  $\hat{\sigma}_{HEVE}^2(S_{aux})$ . Similarly to (9) if  $S_{aux} = 0$ , the test reduces to  $\lambda_1 / \hat{\sigma}_{HEVE}^2(S_{aux})$ , which has the same statistical power of HRLRT.

#### V. SIMULATION RESULTS

Results are shown in terms of Receiver Operating Characteristic (ROC) curves ( $P_d$  vs.  $P_{fa}$ ) and performance curves, in which  $P_d$  is plotted against SNR, by fixing  $P_{fa}$ . All the tests described in Section III-IV have been simulated by using a Montecarlo approach with 10000 iterations for each SNR value. The primary signal has a Gaussian distribution and the typical Rayleigh flat fading channel scenario has been considered. In performance curves,  $P_{fa}$  is fixed to  $10^{-2}$  while in ROC curves,  $SNR = -12$  dB.

Figures 1 and 2 show respectively the performance and ROC curves of all the test statistics with 4 antennas, 200 samples per slot and 4 auxiliary slots. It can be noticed that EVE and HEVE are clearly capable to significantly reduce the gap with NP wrt RLRT. The gap between EVE and RLRT is 1 dB at  $P_d = 0.9$ . In general, the hybrid approaches HEVE, HRLRT and HED are very close in performance with their respective known-noise tests.

Figures 3 and 4 show how the number of slots affects the performance of these tests. The number of antennas is equal to 4, while we used 200 samples per slot. It is evident that there is an important gap between 2 and 4 auxiliary slots (especially for HEVE), while the curves with 4 and 6 auxiliary slots are almost overlapped.

Finally, we show 2 other performance curves. In Figure 5, the detection probability is plotted against the number of

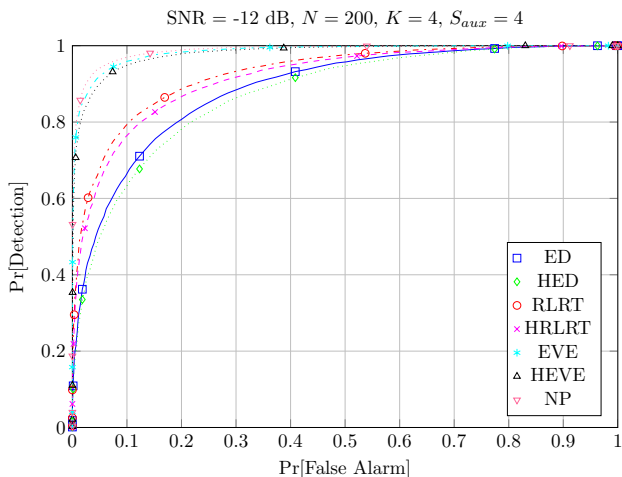


Figure 2. ROC curve,  $K = 4$ ,  $N = 200$

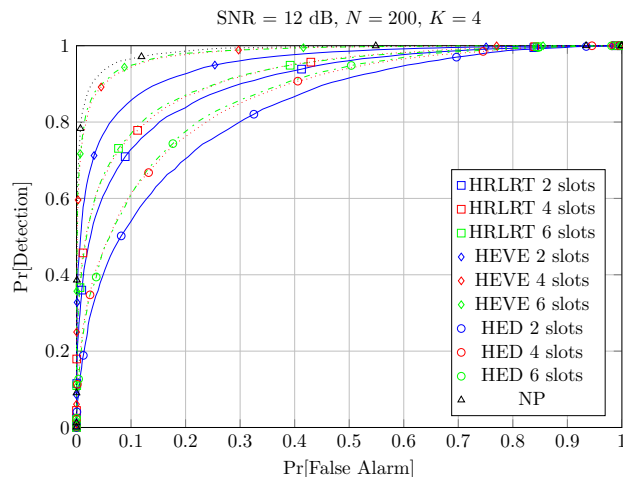


Figure 4. ROC curve, with 2, 4, 6 auxiliary slots

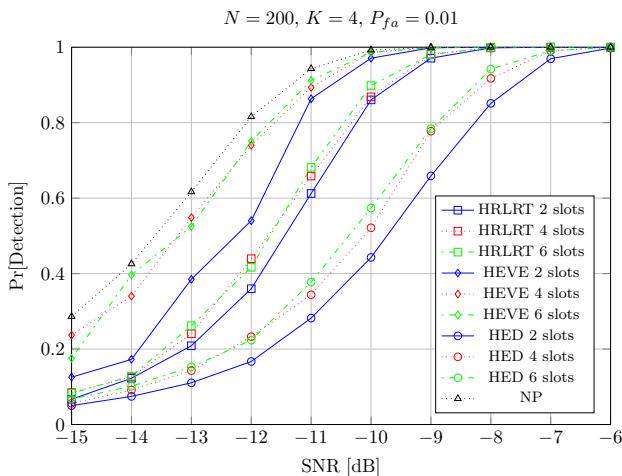


Figure 3. Performance curve,  $P_d$  vs. SNR, with 2, 4, 6 auxiliary slots

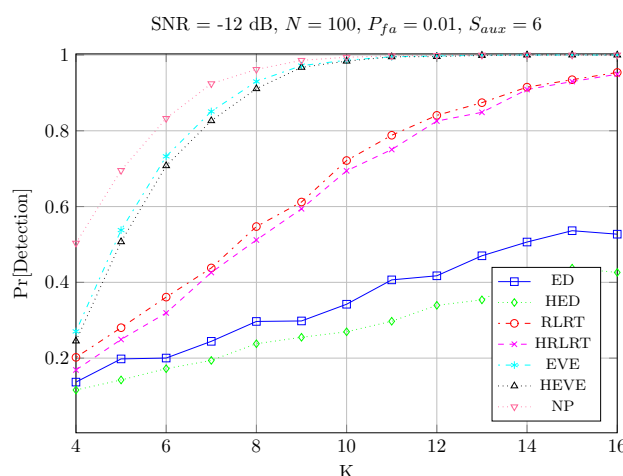


Figure 5. Performance curve,  $P_d$  vs.  $K$ ,  $N = 100$

antennas, with 100 samples per slot and 6 auxiliary slots, while in Figure 6,  $P_d$  is plotted against the number of samples, with 4 antennas and 6 auxiliary slots. NP and the EVE group tests require a much smaller number of antennas or sensors to reach  $P_d \simeq 1$  wrt to all other tests.

### VI. OPTIMIZATION OF HYBRID ENERGY DETECTION

In this section, we show some preliminary results on the optimization of Hybrid Energy Detection. Let us assume that the secondary user has a fixed time window for both noise estimation and detection, i.e., the number of samples that the SU can use for both noise estimation and signal detection is constant. For the sake of simplicity, the Maximum Likelihood expression of (10) will be considered for the optimization of HED described in IV-B. Considering  $K$  antennas,  $M$  samples are used for estimation and  $N$  samples for detection. Our fixed time constraint implies  $M+N = c$  where  $c$  is a constant, hence our goal is to find the optimal  $M$  (and consequently optimal  $N$ ) that gives the maximum detection probability.

In [22][23] the mathematical analysis of HED was performed, the false alarm and detection probability expressions

are the starting point of our optimization task. The False Alarm Probability  $P_{fa}^{(HED)}$  for number of sensors  $K$ , number of samples  $N$ , number of noise estimation samples  $M$  and threshold  $t$  is given by,

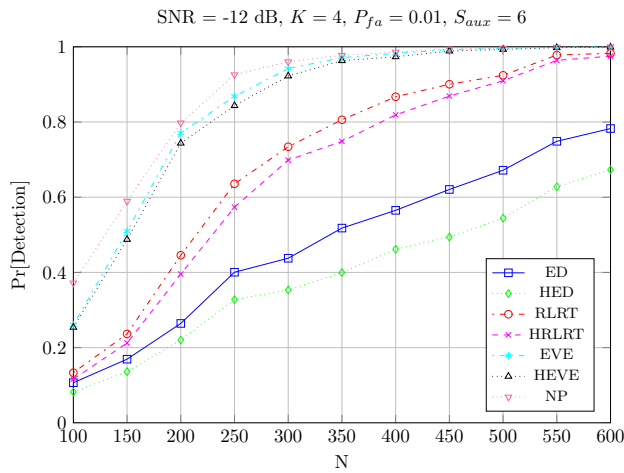
$$P_{fa}^{(HED)} = Q \left[ \frac{t - 1}{\sqrt{\frac{M+Nt^2}{KMN}}} \right] \quad (15)$$

Similarly, the Probability of Detection  $P_D^{(HED)}$  in similar scenario is given by,

$$P_d^{(HED)} = Q \left[ \frac{(t - 1 - \rho)}{\sqrt{\frac{t^2}{KM} + \frac{K\rho^2 + 2\rho + 1}{KN}}} \right] \quad (16)$$

where  $\rho$  is the signal-to-noise ratio.

First of all, from (15) we find the threshold  $t$  expression


 Figure 6. Performance curve,  $P_d$  vs.  $N$ ,  $K = 4$ 

as a function of the  $P_{fa}$ ,

$$t = \frac{M \left( K + \epsilon \sqrt{\frac{KM + KN - \epsilon^2}{MN}} \right)}{KM - \epsilon^2} \quad (17)$$

where  $\epsilon = Q^{-1}[P_{fa}]$ . This is the only acceptable solution ( $t > 1$ ) of a second degree equation. Unless  $KM$  is smaller than  $\epsilon^2$  (which is of no interest), this is always true.

By substituting (17) in (16) we obtain the following expression:

$$P_d = Q \left[ \frac{\frac{M \left( K + \epsilon \sqrt{\frac{KM + KN - \epsilon^2}{MN}} \right)}{KM - \epsilon^2} - 1 - \rho}{\sqrt{\frac{M \left( K + \epsilon \sqrt{\frac{KM + KN - \epsilon^2}{MN}} \right)^2}{K(KM - \epsilon^2)^2} + \frac{K\rho^2 + 2\rho + 1}{KN}}} \right] \quad (18)$$

Let us now use the following substitutions:

$$M = x \quad (19)$$

$$N = c - x \quad (20)$$

where  $x \in \mathbb{N}$  and  $c = M + N$ .

We first rewrite the threshold expression in (17):

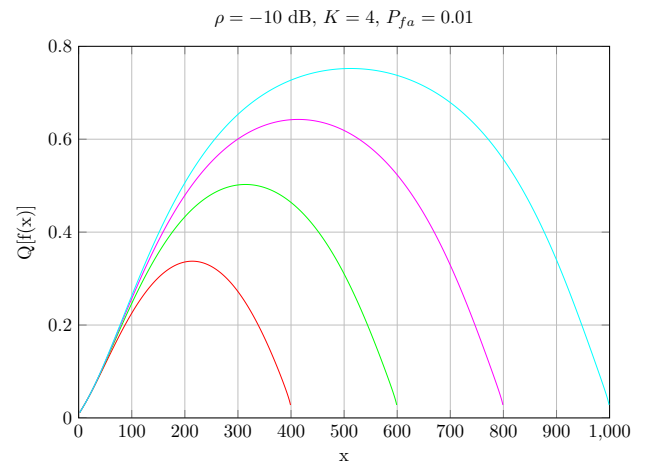
$$t = \frac{x \left( K + \epsilon \sqrt{\frac{Kc - \epsilon^2}{xc - x^2}} \right)}{Kx - \epsilon^2} \quad (21)$$

Then, we rewrite the argument of the Q-function of (18):

$$f(x) = \frac{\frac{x \left( K + \epsilon \sqrt{\frac{Kc - \epsilon^2}{xc - x^2}} \right)}{Kx - \epsilon^2} - 1 - \rho}{\sqrt{\frac{x \left[ K^2 + \frac{\epsilon^2(Kc - \epsilon^2)}{cx - x^2} + 2KQ \sqrt{\frac{Kc - \epsilon^2}{cx - x^2}} \right]}{K(K^2x^2 + \epsilon^4 - 2\epsilon^2Kx)} + \frac{K\rho^2 + 2\rho + 1}{Kc - Kx}}} \quad (22)$$

Figure 7 shows the probability of detection of HED as a function of  $M$  for different values of  $M + N$ , with 4 antennas, SNR equal to -10dB and  $P_{fa}$  equal to  $10^{-2}$ . It is clear to see that, when  $c$  samples are available for both estimation and detection, the highest probability of detection occurs for:

$$M \approx N \approx \frac{M + N}{2} \quad (23)$$


 Figure 7. Probability of detection as a function of  $M$  given  $M + N = \text{const.}$ 

Hence, given a time slot for spectrum sensing, the best performance occurs when estimation and detection slots are equally split.

## VII. CONCLUSIONS

In this paper, some important classes of multi-antenna spectrum sensing algorithms have been considered. The hybrid approach of method based on the eigenvector of the covariance matrix has been introduced. Performance of the new hybrid test has been compared with the well-known RLRT and ED together with their hybrid approaches HRLRT and HED. It is shown that the EVE test and its hybrid approach are able to outperform RLRT, ED and their respective hybrid approaches, furthermore it can significantly reduce the gap with the NP test. Finally, given a fixed time slot or number of samples for HED, it is concluded that estimation and detection slots should be equally divided in order to achieve the optimal performance.

## REFERENCES

- [1] J. Mitola, III, "Cognitive radio. An integrated agent architecture for software defined radio", Ph.D. dissertation, Royal Institute of Technology (KTH), May 2000.
- [2] J. Neyman and E. Pearson, "On the Problem of the Most Efficient Tests of Statistical Hypotheses," Philosophical Transactions of the Royal Society of London, 1933.
- [3] IEEE Standard for Information technology Local and metropolitan area networks Specific requirements Part 22: Cognitive Wireless RAN Medium Access Control (MAC) and Physical Layer (PHY) specifications: Policies and procedures for operation in the TV Bands, IEEE Std 802.22-2011, July 2011, pp. 1-680.
- [4] H. Urkowitz, "Energy detection of unknown deterministic signals," Proceedings of the IEEE, Apr. 1967, pp. 523-531.
- [5] D. Cabric, A. Tkachenko, and R.W. Brodersen, "Spectrum Sensing Measurements of Pilot, Energy, and Collaborative Detection," Military Communications Conference, IEEE, 23-25 Oct. 2006, pp. 1-7.
- [6] Jian Chen, A. Gibson, and J. Zafar, "Cyclostationary spectrum detection in cognitive radios," IET Seminar on Cognitive Radio and Software Defined Radios: Technologies and Techniques, 18 Sept. 2008, pp. 1-5.
- [7] Y. Zeng and Y.-C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio", IEEE Trans. Commun., vol. 57, no. 6, 2009, pp. 1784-1793.
- [8] W. Zhang, G. Abreu, M. Inamori, and Y. Sanada, "Spectrum sensing algorithms via finite random matrices", IEEE Trans. Commun., vol. 60, no. 1, Jan. 2012, pp. 164-175.

- [9] A. Kortun, T. Ratnarajah, M. Sellathurai, C. Zhong, and C. Papadias, "On the performance of eigenvalue-based cooperative spectrum sensing for cognitive radio", *IEEE J. Selected Topics Signal Process.*, vol. 5, no. 1, Feb. 2011, pp. 49-55.
- [10] S.K. Sharma, S. Chatzinotas, and B. Ottersten, "Eigenvalue based sensing and SNR estimation for cognitive radio in presence of noise correlation", *IEEE Trans. Veh. Technol.*, vol. 62, no. 8, Oct. 2013, pp. 3671-3684.
- [11] L. Cardoso, M. Debbah, P. Bianchi, and J. Najim, "Cooperative spectrum sensing using random matrix theory", in *proc. 3rd Int. Symp. Wireless Pervasive Comp.*, May 2008, pp. 334-338.
- [12] F. Penna, R. Garello, and M. Spirito, "Cooperative spectrum sensing based on the limiting eigenvalue ratio distribution in Wishart matrices", *IEEE Commun. Letters*, vol. 13, no. 7, July 2009, pp. 507-509.
- [13] S.K. Sharma, S. Chatzinotas, and B. Ottersten, "The effect of noise correlation on fractional sampling based spectrum sensing", in *Proc. IEEE ICC*, June 2013, pp. 1182-1187.
- [14] Y. Zeng, C. Koh, and Y.-C. Liang, "Maximum eigenvalue detection Theory and application", in *Proc. IEEE ICC*, May 2008, pp. 4160-4164.
- [15] P. Wang, J. Fang, N. Han, and H. Li, "Multiantenna-assisted spectrum sensing for cognitive radio", *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, May 2010, pp. 1791-1800.
- [16] P. Bianchi, M. Debbah, M. Maida, and J. Najim, "Performance of statistical tests for single-source detection using random matrix theory", *IEEE Trans. Info. Th.*, vol. 57, no. 4, 2011, pp. 2400-2419.
- [17] Y. Zeng and Y. C. Liang, "Maximum-Minimum Eigenvalue Detection for Cognitive Radio", *Personal, Indoor and Mobile Radio Communications, 2007. PIMRC 2007. IEEE 18th International Symposium on*, 3-7 Sep. 2007, pp. 1-5.
- [18] S. Sequeira, R.R. Mahajan, and P. Spasojevic, "On the noise power estimation in the presence of the signal for energy-based sensing," *Sarnoff Symposium (SARNOFF)*, 2012 35th IEEE, 21-22 May 2012, pp. 1-5.
- [19] S.N. Roy, "On a Heuristic Method of Test Construction and its use in Multivariate Analysis," *The Annals of Mathematical Statistics*, vol. 24, no. 2, Jan. 1953, pp. 220-238.
- [20] P. Bianchi, M. Debbah, M. Maida, and J. Najim, "Performance of Statistical Tests for Source Detection using Random Matrix Theory", *Information Theory, IEEE Transactions on*, vol. 57, no. 4, Apr. 2011, pp. 2400-2419.
- [21] R. Garello and Y. Jia, "Improving spectrum sensing performance by using eigenvectors", *The First Intl. Conf. on Advances in Cognitive Radio*, Budapest, Hungary, Apr. 2011, pp. 26-30.
- [22] P. Dhakal, D. Riviello, F. Penna, and R. Garello, "Impact of noise estimation on energy detection and eigenvalue based spectrum sensing algorithms." *IEEE International Conference Communications (ICC)*, Sydney, Australia, Jun. 2014, pp. 1367-1372.
- [23] P. Dhakal, D. Riviello, F. Penna, and R. Garello, "Hybrid Approach Analysis of Energy Detection and Eigenvalue Based Spectrum Sensing Algorithms with Noise Power Estimation," *The Fourth Int. Conf. on Advances in Cognitive Radio*, Nice, France, Feb. 2014, pp. 20-25.
- [24] B. Nadler, F. Penna, and R. Garello, "Performance of eigenvalue-based signal detectors with known and unknown noise level," in *Proc. of International Conference on Communications (ICC 2011)*, Jun. 2011, pp. 1-5.
- [25] Kritchman, Shira, and Nadler. "Non-parametric detection of the number of signals: hypothesis testing and random matrix theory." *IEEE Transactions on Signal Processing*, vol. 57, no. 10, 2009, pp. 3930-3941.
- [26] F. Penna, R. Garello, and M.A. Spirito, "Probability of Missed Detection in Eigenvalue Ratio Spectrum Sensing," *IEEE Int. Conf. on Wireless and Mobile Computing, Networking and Commun. (WIMOB)*, Marrakech, Morocco, Japan, 2009, pp. 12-14.