A HMM Model Based on Perceptual Codes for On-Line Handwriting Generation

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***Abstract*—This paper handles the problem of synthesis of online handwriting that can be reconstructed by several methods such as those of movement or shape simulation techniques and computational methods. Indeed, this work presents a probabilistic model using the Hidden Markov Models for the classification of perceptual sequences, starting from global perceptual codes as input and ending with a class of number probabilities as output. In fact, the algorithm analyzes and learns the handwriting visual codes features. In order to recover the original handwriting shape, and to generate new ones via the generated perceptual sequences, we investigate the polynomial approximation methods such us the Bezier curves and Bspline interpolation. The performance of the proposed model is assessed using samples of scripts extracted from Mayastroun Database. In experiments, good quantitative agreement and approximation is found between human handwriting data and the generated trajectories and more reduced representation of the scripts models are designed.**

Keywords-Human reading; Cursive handwriting synthesis; Hidden Markov Models; Global perceptual codes; Beta-elliptic model.

I. INTRODUCTION

Despite the invasion of the computer and various technologies, such as keyboard and mouse, reducing the importance of handwriting, this latter attaches a great value on communication for a large majority of individuals. As a result, the emergence of so different and significant progress, such as tablet PC, interactive whiteboard or pen-based devices allow the implementation of recognition, production and handwriting synthesis systems.

Various cognitive and psychological studies have been performed on a number of participants to observe their behavior during the reading process. It has been demonstrated that vision is considered as the most advanced sensor for human and provides the widest range of information to our body in general and to writing in particular. As stated in many previous studies, perception and handwriting synthesis are highly correlated and formed a cognitive loop: the visual perception process extracts relevant features from the external environment to enable the motor system to act [14]. So the handwriting process can be considered as a sensory-motor task in which the component of perception corresponds to the shape of the letter and the neuromuscular system assumes the formation of the script trajectory [7][8]. On the other hand, the problem of handwriting generation has been addressed for a long time and there are many studies in this field such as [9][11][12][16][18]. These ones try to generate handwriting scripts focusing on movement dynamic characteristics or handwriting shape features. Other research studies suppose that generation and visual perception of human movements are strongly correlated and that complex handwriting trajectories should be formed from the superposition of elementary building blocks [5]. In such a way, the visuo-motor correspondence could explain that Humans tend to provide motor commands which have similar kinematic properties to their neuro-muscular system. In this paper we have to analyze the interactions between the handwriting synthesis task and the cognitive functions which are integrated during learning of handwriting movements.

The most of previous studies focused respectively on one of the perception or the production processes involved in handwriting generation task without handling with the interaction between both of them as two interrelated parts of the complete feedback loop [10]. As well, because the handwriting depends on both the perceptual skills and production, the formation of an online acquired script was more effective in the improvement of these two capabilities. In this context, the purpose of our work is to develop a handwriting generation model by using the visual codes extracted from on-line handwritten scripts. We propose to use specific statistical models: the hidden Markov models for their dominant impact in the treatment of writing and their handling of any type of sequences (image, speech, writing traces, etc...). Because of their nature, these models are deemed the most adequate to our task involving the handwriting classification and the generation of sequential data.

This paper is organized as follows: in Section 2, the proposed approach is detailed and we present the principle characteristics of the detection of the global perceptual codes. In the second part of Section 2, we briefly introduce the Hidden Markov model, we summarize the observation

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extraction step. Then we detail the training and classification process. In the third part of Section 2, we describe our approach for fitting the partial contours i.e. GPC through conventional geometrical approach. Then we deal with some experimental results made on MAYASTROUN database [15]. Finally, we present some conclusions and further works.

II. THE PROPOSED MODEL

In this section, we describe on-line handwriting classifier and model for synthesis of scripts. An appropriate HMM model is used for each sample of the lexicon. In our concept, only one model is constructed for the different class of digits (each digit in lexicon), then for the classification task of an input digit, the score for matching the digit to each model is computed, and the class related to the model that has the maximum score gives the result of the classification. This approach is reasonable for small lexicon size. The samples of digits are extracted from the MAYASTROUN database [15].

The system that we have developed for modelling and classification of online handwritten scripts is shown in Fig.1.

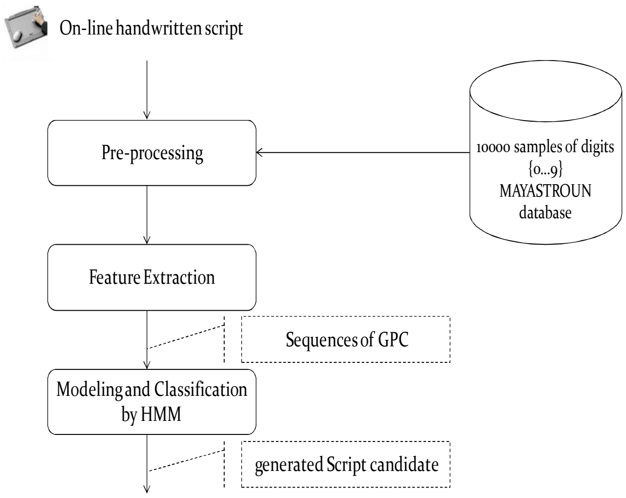

Figure 1. The architecture of the proposed model.

Firstly, the scripter writes on a digitizing tablet using a special stylus. So that the user's written scripts are captured as they are being formed by sampling the (x,y) coordinates corresponding to the pen position. Then, the acquired handwriting data is smoothed by a low pass-filtering type Chebychev. In this connection, each handwriting trace "isolated digit" is composed of at least one segment and each segment comprises multiple strokes. Moreover a segment is defined as a trace which is drawn continuously and during such movement the pen is touching the digitizing tablet.

Regarding to the feature extraction stage, each script, i.e trace, is represented as a feature vector, which becomes its identity. In other words, it resides in the selection and the extraction of important features. Our purpose is to achieve a transformation of the original data space into a fixed dimension vector space which contains all of the relevant information necessary to the modeling step with HMM. Since the proposed model is classified as well as generative model, we can choose the most probable digit from the HMM. The most probable digit can be interpreted as the most representative digit pattern that each model has. If our proposed model successfully learns the concept of digits from training data, then it can generate natural shapes.

A. Feature Extraction

Because of the human visual system is selectively activated in response to global form, we have investigated the properties of the GPC extractor composed of ten GPCs which have been already accomplished in previous studies [13]. As shown in [13], a GPC is a combination of a set of elementary perceptual codes (EPCs) according to well defined criteria. For this task, the authors have used the beta-elliptic model for on-line handwriting segmentation scripts [2].

1) The Beta-Elliptic Model for Handwritten Segmentation

In the context of the beta-elliptic theory, a rapid handwriting movement is resulted from the activation of a sufficient number of neuromuscular subsystems, to get a smoothest trajectory characterized by a curvilinear velocity profile fitted by a Beta function and an elliptic stroke in the static domain.

As shown in (1), the Beta equation depends on the set of parameters *(t, t_0, t_1, t_c, p, q)* where t_0 is the starting time, t_1 is the ending time, t_c is the instant where the curvilinear velocity reaches its maximum. p and q are intermediate parameters which describe the beta profile shape [1][3][4].

$$\beta(t, t_0, t_1, p, q) = \left[\frac{t_1 - t}{t_1 - t_c}\right]^p \left[\frac{t - t_c}{t_c - t_0}\right]^q \quad (1)$$

and

$$t_c = \frac{p * t_1 + q * t_0}{p + q} \quad (2)$$

According to these kinematic features, we check the different static characteristics. Based on the assumption that each stroke is represented in the static domain by an elliptic arc characterized by a and b which are respectively the dimensions of the large and the small axes of the elliptic shape. x_0 and y_0 are the Cartesian coordinates of the elliptic center relative to the orthogonal reference *(o, x, y)*. The angle θ defines the deviation of the elliptic portion as presented in (3) [2].

$$\theta = tan^{-1}\left(\frac{(y_1 - y_0)}{(x_1 - x_0)}\right) \quad (3)$$

For complex handwriting movements, each handwriting trace is composed of at least one segment and each segment comprises multiple strokes. Each stroke is characterized by ten features in both the kinematic and static domain. The reader is referred to [2] for more details.

2) The Elementary Perceptual Codes Extractor

Following the approach of the beta-elliptic model, we carried on with the detection of the perceptual codes. Firstly, for each elliptic stroke, we assigned an elementary perceptual code (EPC) [13]. According to the to the deviation angle θ, we have identified four types of strokes: *Shaft, Valley, Left oblique shaft, Right oblique shaft*). Each one has the opportunity to belong to both separate intervals of the trigonometric circle and depending on the trigonometric sense, we have defined the positive part going from 0 to π and the negative range going from 0 to - π. i.e. the stroke number two (valley), belongs to both intervals containing each one a positive and a negative part. It takes into account both sides of the trigonometric circle (positive and negative) to indicate the direction of writing which is consistent with the trigonometric direction. The number of strokes of an appropriate handwriting trace is predefined by the Beta-elliptic model and this latter is equal to the number of EPC.

3) The Global Perceptual Codes Extractor

Assuming that human vision does not detect readily the elementary perceptual codes comprised in the original trace, but more general forms [13], we have used the global perceptual codes (GPCs) extracted previously in [13]. According to pre-defined criteria and by the means of genetic algorithms, these GPCs are obtained by collecting the EPCs [13] already detected. We have defined ten GPCs classified into both categories; simple and complex ones, as shown in Fig 2. In order to take into account the Arabic lexicon, we have considered the letter “Ain” as a complex perceptual code referenced by number ten. As a result, we obtained a set of CPGs forming the initial script that will be used for online handwriting classification modeling step [14][15].

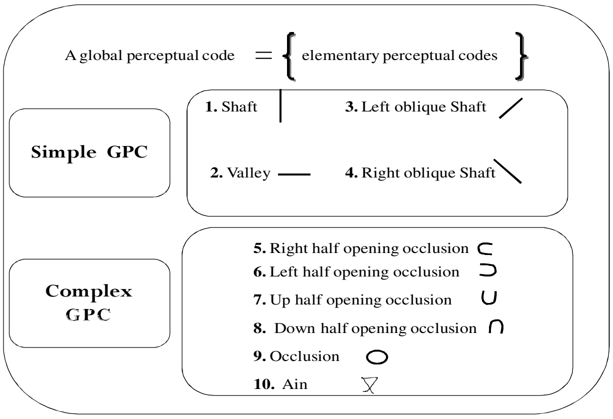

Figure 2. The different global perceptual codes.

In order to obtain more satisfying handwriting generation system and to simulate how Human understand and recognize handwriting, we adopt these global perceptual codes (GPCs) and the Hidden Markov Models to classify the different patterns of digits.

B. Modelling and Classification by HMM

The handwriting perception and learning can be viewed as a problem of probabilistic learning at high level. Indeed the learning of new characters affects not only the temporal characteristics but also influences the composition of the GPC sequences and the whole shape.

The extraction of knowledge from sequentially structured data is a complex problem appearing in various fields of applications [6]. In this study, we check to develop a single model of representation that comprehensively summarizes all structured data identifying correspondences or differences in data sets. Hidden Markov Models (HMM) appear as one of the best approaches adopted for sequences treatment, due to their ability to handle with sequences of variable lengths, and secondly their capability to model the dynamics of phenomena described by sequences of events. Thus, we propose to use a discrete HMM for the modeling and classification task, the observations sequences are identified to sequences of discrete values corresponding to sequences of GPCs. An important issue that must be resolved before putting the system into use is to decide how many states the digits model should composed. For our case, each digit has different number of states that reflects the number of GPCs that the digit has. As a result, the states number for a HMM model is varied with respect to the modeled handwritten digit proprieties.

1) Overview of Hidden Markov Models

A Hidden Markov Model (HMM) [17] is a kind of stochastic model that is similar to a finite states machine in which transitions and results are stochastic. Otherwise, a HMM is a sequence of observations as a piecewise stationary process. This model combines the advantages of a states machine and probability distributions between states. The HMM registers the observations as probability functions of an appropriate state and the input data is described as a hidden stochastic process.

More formally, it may be represented by a set of features namely N, M, A, B and π. The values associated to these characteristics can be used to produce the observations sequences. These characteristics are detailed below:

- N : the number of hidden states within the model.
- M: the number of observations symbols per state.
- $A = \{a_{ij}\}$: the state transition probability distributions;

$a_{ij} = P(q_{t+1} = S_j \mid q_t = S_i), \;\; 1 \le i, j \le N$

- $B = \{b_j(k)\}$: the emission probability distribution in the state j;

$b_j = P(v_k \text{ at } t \mid q_t = S_j), \;\; 1 \le j \le N, 1 \le k \le M$

- $\pi_i = \{\pi_i\}$: the prior probability of being in the state i at the beginning of the observations;

$\pi_i = P(q_1 = S_i), 1 \leq i \leq N.$

- $O = \{ O_1\ O_2\ O_3 \ldots O_T\}$ where O_t is an observation from the set of possible symbol observations and T is the number of observations in the sequence.

To initiate a HMM characterized by a compact notation $\lambda = (N, A, B, \pi)$, an initial state will be chosen based on the prior distribution π and t is set at 1. $\sum_J a_{ij} = 1$, $\sum_t b_i(O_t) = 1$, $\sum_i \pi_i = 1$, $a_{ij}, b_i(O_t), \pi_i \geq 0$, for all i, j, t.

2) Observations Extraction

We summarized the different steps of feature extraction as shown in Fig 3, starting with an original handwritten digit and ending with an observation which is equivalent to a sequence of global visual codes as output.

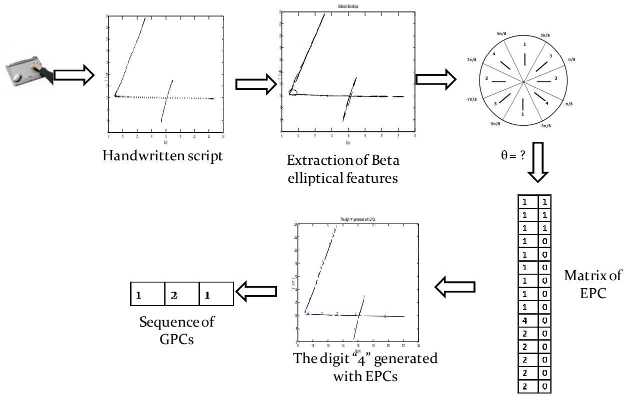

Figure 3. Example of the feature extraction step.

A GPC is a combination of a set of elementary perceptual codes (EPCs) according to well defined criteria. In other words, just like a script is defined as a sequence of GPC, each GPC may be defined as an alternating sequence of EPCs. In fact, we extend the conventional representation of observation sequence as following:

Handwritten digit = {GPC}+

GPC = EPC. EPC.{EPC}+

where "+" denotes repetition and "." indicates concatenation. Indeed, we consider the list of GPCs extracted from a handwritten digit as observations for our model. These observations are then classified into corresponding digit categories.

3) HMMs Structure and Training

To define the architecture of the model, we must take into account the topology as well as the number of states according to our case. The most adopted topology to our system is of ergodic type. In this type of structure, each state can be reached or visited by any other state and the transition to itself is permitted: that is to say that all states communicate among themselves. According to a graphical representation, all states are interconnected by arrows which indicate the direction of transition and the corresponding probability.

Only one model is built to represent every handwritten digit. Such one is formed with an ergodic digit models. Figure 4 shows the modeling of each digit class by a HMM.

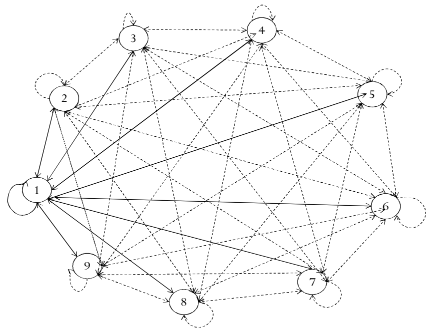

Figure 4. Graphical representation of the proposed model: an ergodic HMM with nine states.

We handle the different types of GPCs as possible symbols that can formed the set of ten digits and we notice the absence of GPC "ain" in the composition of the different sequences and the presence of other ones. So we treat nine GPC instead of ten. In others words, GPCs are considered as states.

Taking into account the above remarks, the design of HMM characterized by a compact notation $\lambda = (N = 9, A, B, \pi)$ has led to the diagram shown in Fig. 4, with:

• Nine discrete states: 1, 2, 3 ... 9, which define the hidden states of the HMM.

• O_1, O_2, O_3 and O_{10} denote the observations of the HMM.

• a_{ij} denote the transition probabilities between the different states of the model .

• $P(O_j|i)$ represent the emission probabilities of observation.

In the training phase, the goal is the estimation of model parameters (A, B, π) for all possible digits that best approximate the digit. The learning of the HMM model parameters corresponding to the classes of sequences is performed by the Baum- Welch algorithm [17]. This latter allows us to adjust and recompute the parameters of each sequence by maximizing the likelihood value over several iterations. In another words, it adjusts the parameters of the HMM model in order to maximize the probability $P(O|\lambda)$ of generating a sequence O of observations which is already contained in the training data.

Thus, a set of reasonable re-estimation values for π, A and B is given. We save the new settings of each class model for the use in the next step as depicted in Fig.5.

Various digits prototypes were used for the experiments. These ones were collected from college and secondary school students without imposing any stylistic constraints or restriction while writing. The system is trained with 10000 digits written by 100 writers. The number of training samples per class was 700.

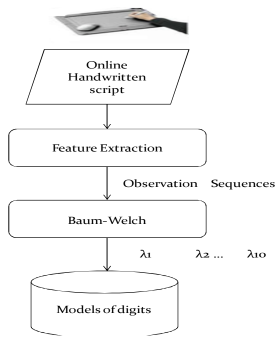

Figure 5. Scheme of the learning data.

4) HMMs Classification

Up to now, we have defined the structure of a HMM, its characteristics and how it can be created. Starting from many kinds of sequences, we would like to distinguish between them. To solve this task, we need a sequence classifier.

Given a sample of sequence or an observation digit obtained from an on-line handwriting digit or generated by HMM, our task can be expressed as the determination of the best observation sequence out of the HMM model and its classification.

During this phase, and in order to select the best class and make a decision, we use the classical classification likelihood values which are computed in the previous step by the HMM model such as described in Fig. 6.

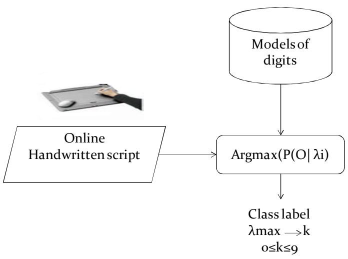

Figure 6. Digits classifier and the evaluation of HMMs Models.

The classifier takes the digit to be classified as a sequence of discrete observations O and the decision function is based on the maximum likelihood criterion. For each HMM model λi, the classifier calculates the probability P(O|λi) and we collect the correspondent probability value. So, in order to find the most near optimal sequence, we use the maximum likelihood criterion and the best one represents the observation of digit with maximum probability.

C. Handwriting Generation

Various approximating methods have been proposed including the interpolation and the B-spline [21] or Bezier curves [19][20], to generate the cursive handwriting. The main objective of the interpolation, for example, is to interpolate unknown data from points. In this case, the value of the approximated function between these points can be estimated. In other case, the interpolation involves passing a curve by a subset of points extracted from the set of GPCs composing a given script. Our task of generation can be expressed as the fitting of handwriting trajectory by a small number of points in the 2D plane defining the different GPCs. We describe our approach for fitting the partial contours i.e. GPCs through those conventional geometrical approach such that: Bezier curves and B-splines.

A simple GPC is a straight or nearly straight trace that has different directions and is composed of points. For the case of the simple GPCs, the linear interpolation method have been used, in which each pattern of data is fitted by a line segment connecting the extreme points of two neighboring GPC i.e. the tail and head of each of them. These ones correspond to local features representing maximums or minimums of the curvilinear velocity signal of the handwriting trace as shown in Table.I.

TABLE I. THE DIFFRENT GPC AND THE CORRESPONDING CONTROL POINTS

Code	GPC Shape	Control Points
1	|	$\{(x_i, y_i), (x_f, y_f)\}$
2	—	$\{(x_i, y_i), (x_f, y_f)\}$
3	/	$\{(x_i, y_i), (x_f, y_f)\}$
4	\	$\{(x_i, y_i), (x_f, y_f)\}$
5	⊂	$\{(x_i, y_i), (x_k, y_k), \dots (x_f, y_f)\}$
6	⊃	$\{(x_i, y_i), (x_k, y_k), \dots (x_f, y_f)\}$
7	∪	$\{(x_i, y_i), (x_k, y_k), \dots (x_f, y_f)\}$
8	∩	$\{(x_i, y_i), (x_k, y_k), \dots (x_f, y_f)\}$
9	O	$\{(x_i, y_i), (x_k, y_k), \dots (x_f, y_f)\}$

With $2<=k<=4$

According to Table.1, the local features representing the spatial information of the different GPCs namely two points for the simple GPC such as the starting point $P_i(x_i, y_i)$ and the ending point $P_f(x_f, y_f)$. While a complex GPC may be defined by five points. In addition to the starting and ending points, we have k points having the maximum or minimum curvature as well as points of zero crossing curvature.

In fact, each GPC may be expressed as follow:

Simple GPC $= \{(x_i, y_i), (x_f, y_f)\}$

Complex GPC $= \{(x_i, y_i), (x_k, y_k), \dots (x_f, y_f)\}$

Due to the variability of handwriting style, the number of main control points is different from a script to another and from one sample to another and we are obliged to add new ones especially when the original points are sparse i.e. assuming that many points are missed between both P_i and P_f, it is necessary to generate new points between them. We explore the idea of modeling the sequence of control points positions of handwritten digit to best fit via Bezier curve or B-spline. In this sense, we investigate the feature extraction step to compute these latter which are required for the implementation of the polygonal approximations.

We conducted experiments which show the ability of handwriting modeling of the proposed method. First of all, we applied our system to digit generation and classification. Moreover, and in order to test our system, we have carried out several experimentations on MAYASTROUN Database [15].

As depicted in Fig.7, an example of the handwriting digit "4" is generated. The associated controls points computed for such digit are represented in Fig 7.(a). , There are four control points for all the extracted GPCs namely Left oblique Shaft, Valley and Shaft. These ones are generated from the learned model. All the sample points are integrated together by respectively Bezier Curve in Fig. 7.(b). and B-spline in Fig.7(c) to yield a cursive handwritten digit.

(a) (b) (c)

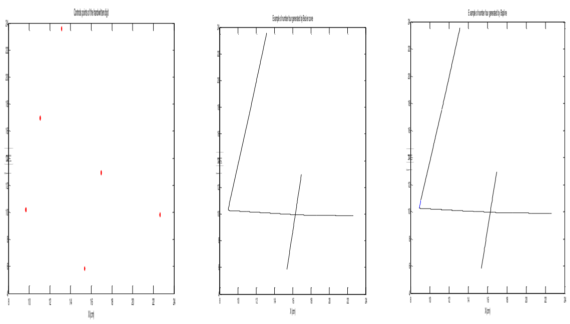

Figure 7. (a) The controls points. (b) The digit “4” generated by Bezier curves, (c) The digit “4” generated by Bsplines.

Another example of script is carried out, which is the digit "3", as shown in Fig.8. This one is made of two GPCs: two Left half opening occlusion. Figure 8.(b). presents the model fitted by Bezier curve while Fig8.(c). shows the approximation of the digit via B-spline.

(a) (b) (c)

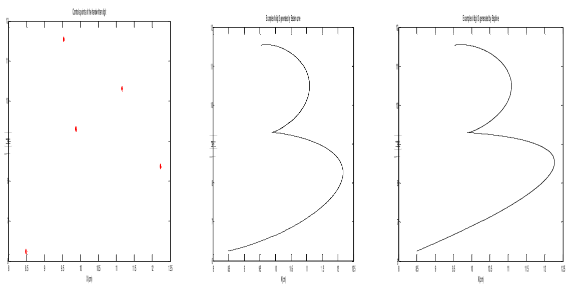

Figure 8. (a) The controls points. (b) The digit “3” generated by Bezier curves, (c) The digit “3” generated by Bsplines.

We give another example of digit "6" in Fig.9. which have some complex GPCs. Figure 9.(a) shows the distribution of the different control points that characterize these latter. An instantiation of the digit is then generated by Bezier curve as well as by B-spline.

(a) (b) (c)

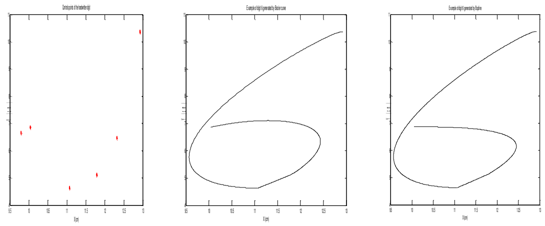

Figure 9. (a) The controls points. (b) The digit “6” generated by Bezier curves, (c) The digit “6” generated by Bsplines.

To evaluate our method, we use a measure of similarity between the original acquired data and regenerated one to analyze and provide information to help make a decision on the results provided. The degree of similarity [22] between the original script and the script generated is measured with the following equation (4):

$$S(S_o,S_g) = 1 - \frac{\sqrt{\sum_{i=1}^{n}((x_{oi}-x_{gi})^2+(y_{oi}-y_{gi})^2)}}{n} \qquad (4)$$

with:

$S(S_o,S_g)$ is the degree of similarity between the original script (S_o) and generated script (S_g).

(x_{oi},y_{oi}) are the i-th point coordinates in the original script.

(x_{gi},y_{gi}) are the i-th point coordinates in the generated script.

n is the number of points of the original writing.

We note that:

1. $S(S_o,S_g) \in [0, 1]$
2. $S(S_o,S_g) = 1 \Rightarrow S_o=S_g$
3. $S(S_o,S_g) = \alpha \Rightarrow S_g$ (generated script) is similar with α degree to the S_o (original script).

This degree helps us to compare different results between generated scripts via B-spline and Bezier curve.

TABLE II. THE SIMILARITY DEGREE OF SAMPLE OF DIGITS.

Digits	**The similarity degree with Bezier curves**	**The similarity degree with B-spline**
Digit ‘4’	0.121	0.215
Digit ‘3’	0.252	0.824
Digit ‘6’	0.432	0.864

As shown table II, we note the variance of the similarity degree from one digit to another and between the two adopted approximating method namely B-spline and Bezier curves. We remark that B-spline method gives better results for fitting than Bezier curves as the GPC curvature decreases. We conclude that the B-spline can generate handwritten digits with acceptable performance.

These results can be enhanced by increasing the number of control points.

III. CONCLUSION

In this paper, we presented a system for the synthesis of on line handwriting scripts using global perceptual codes. In such away, having a set of GPCs and in order to generate more variant and natural handwriting shapes, we have applied a conditional algorithm to obtain the set of variants scripts. For this task, the HMMs models have been successfully applied. Starting from many kinds of GPCs sequences we differentiate between them as possible symbols that can formed the set of ten digits. Then we have adopted the geometric fitting method to approximate the different GPCs sequences: we have tried to collect the perceptual codes by the means of geometrical methods i.e. a GPC is approximated by an n^{th} order polynomial.

The variability of the similarity degree is due to the style of the writer handwriting and to the complexity of extracted global perceptual codes. The representation of handwriting script by GPCs is a good way to minimize the amount of data, and promising results are obtained. In order to enhance the degree of similarity and the handwriting generation performance, we opt to use genetic algorithms to check the set of optimized control points belonging to every global perceptual code. Our proposed model can be applied for different applications purposes such us automated generating training data for recognition systems, and simplifying the understanding of human handwriting especially for young students. Therefore, it can be used for teaching of handwriting and integrated as a tool of handwriting learning.

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