A New Coarse Timing Estimation Method for OFDM Signals

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***Abstract*—In this paper, a data aided timing synchronization scheme for Orthogonal frequency division multiplexing (OFDM) signals is proposed. The algorithm uses multiple preambles for initial (coarse) timing estimation, and it works in the time domain. We also propose a timing estimation method using one preamble which is independent of the structure of the preamble, and that works better than the other existing methods in the presence of carrier frequency offset (CFO). The performance is compared in terms of probability of erasure, probability of correct estimation and mean square error (MSE) with the existing timing synchronization methods for OFDM systems.**

Keywords—Timing synchronization; OFDM; Preamble; MSE; Probability of erasure; Probability of correct estimation.

I. INTRODUCTION

The main impairments in a wireless communication system are multipath fading and noise. Multipath fading introduces inter symbol interference (ISI). The major requirements of a digital communication system is to maximize the bit rate, minimize bit error rate, minimize transmit power and minimize transmission bandwidth [1]. Orthogonal frequency division multiplexing (OFDM) has emerged as a powerful technique which meets the above requirements in multipath fading channels [2]. However, OFDM is known to be very sensitive to timing and carrier frequency synchronization errors [3].

Timing estimation in OFDM systems can be achieved by either data aided (DA) or non data aided (NDA) method. In this paper, we focus on the data aided (DA) timing estimation methods. Data aided timing estimation methods proposed in the literature can be broadly classified into two categories.

1) Approaches that depend on the special structure of the preamble [3]–[7]
2) Approaches that work independent of the structure of the preamble [8]–[13].

The technique proposed in the literature by Schmidl and Cox [3] uses a preamble with two identical halves to estimate the timing synchronization. The variance of the timing estimation proposed in [3] is large due to the timing metric plateau. The performance of timing estimation is further improved by the authors in [4]–[7]. All these methods are dependent on the special structure of the preamble; hence they cannot work with other preambles and moreover the variance of the timing estimation of these methods is high in multipath fading scenario. Kang [8] proposed a technique to estimate timing offset which is independent of the preamble structure. In [8], a delayed correlation of the preamble is used for timing synchronization. The performance is further improved in [9] [10] by utilizing all correlation points without repetition. In [12] [13], a new timing estimation method using a matched filter is proposed which gives better performance than [8] [9] with less computational complexity. All these methods proposed in the literature [8]–[10] [12] [13], which are independent of the structure of the preamble utilize only one preamble for timing synchronization. Hamed [11] first proposed a timing estimation method by utilizing more than one preamble. The main drawback of these methods [8]–[12] is that the coarse timing estimation is severely degraded in the presence of carrier frequency offset (CFO). Here, we propose a new timing estimation method using multiple preambles and we also propose a modified timing estimation method which is robust to CFO.

This paper is organized as follows. The system model is presented in Section II. The proposed method is presented in Section III. The simulation results are given in Section IV and finally, the conclusions in Section V.

II. SYSTEM MODEL

Fig. 1 shows the typical structure of a OFDM frame in the time domain. An OFDM frame contains preamble, cyclic prefix (CP) and data. The preamble is used for synchronization purpose. The mth preamble in the frequency domain can be

$\mathbf{x}_0$	$\mathbf{x}_1$	. . . .	$\mathbf{x}_{M-1}$	***CP***	***DATA***

Figure 1: OFDM frame structure in the time domain

represented in vector form as follows.

$$\mathbf{X}_m = [X_m(0)\ X_m(1)\ ...\ X_m(N-1)] \tag{1}$$

where $0 \leq m \leq M-1$. The IFFT of the mth preamble is given by

$$x_m(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_m(k)\, \mathrm{e}^{j2\pi nk/N}. \tag{2}$$

The mth preamble in the time domain can be represented in vector form as follows

$$\mathbf{x}_m = [x_m(0)\ x_m(1)\ ...\ x_m(N-1)] \tag{3}$$

where $0 \leq k, n \leq N-1$. Let $\mathbf{x}_0$ to $\mathbf{x}_{M-1}$ be the preambles of the frame in the time domain. Let the transmitted frame be given by (Fig. 1)

$$\mathbf{x} = [\mathbf{x}_0\ ..\ \mathbf{x}_{M-1}\ \mathbf{CP\ DATA}]\,. \tag{4}$$

Now, $\mathbf{x}$ is transmitted through the frequency selective channel. The channel is assumed to be quasi static and it is fixed for

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one frame and varies independently from frame to frame. Its impulse response for a given frame can be expressed as:

$$\mathbf{h} = [h(0)\, h(1)\, h(2) \,.....\, h(L-1)] \tag{5}$$

where L is the number of channel taps. The received signal $r(n)$ in the time domain is given by:

$$r(n) = y(n)\, \mathrm{e}^{j2\pi n\epsilon/N} + w(n) \tag{6}$$

where

$$\begin{aligned} y(n) &= h(n) \star x(n) \\ &= \sum_{l=0}^{L-1} h(l)\, x(n-l)\,. \end{aligned} \tag{7}$$

and $w(n)$ is zero mean Gaussian noise sample and ϵ is the normalized frequency offset.

III. PROPOSED MODEL

The received signal $r(n)$ is used to estimate the start of the frame $\hat{\theta}_t$. It is assumed that the preambles $\mathbf{x}_0, \mathbf{x}_1, ..., \mathbf{x}_{M-1}$ are known to the receiver. We define the correlation function given by

$$T(d) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} r^{\star}(d+n+mN)\, x_m(n) \tag{8}$$

$$R = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} |\, x_m(n)\,|^2 \tag{9}$$

$$G(d) = \frac{|\, T(d)\,|^2}{M.R} \tag{10}$$

The estimated start of the frame is given by

$$\hat{\theta}_t = \max_d [G(d)] \tag{11}$$

Note that in the special case of $M = 1$ in (8), $T(d)$ reduces to

$$T(d) = \sum_{n=0}^{N-1} r^{\star}(d+n)\, x_0(n) \tag{12}$$

It is equivalent to the method proposed in [12] which is a matched filtering approach using one preamble. The performance of the proposed timing metric (10) is severely degraded in the presence of CFO. Hence, we propose a modified timing metric which performs better than (10) in the presence of CFO. Let the frequency offset ϵ lie within $[-I, I]$. We divide the interval $[-I, I]$ into B sub intervals. The length of the each sub interval is 0.1. The modified correlation function $T_{CFO}(d)$ is given by

$$T_{CFO}(d) = \sum_{p=1}^{p=P} \left\{ \left| \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} r^{\star}(d+n+mN)\, x_m(n)\, \mathrm{e}^{(j2\pi(i(p))(n+mN)/N)} \right|^2 \right\} \tag{13}$$

where $i(p)$ takes equally spaced points within the interval $[-I, I]$. The spacing between two successive points is 0.1. $i(p)$ is defined as

$$i(p) = -I + (p-1)\, 0.1 \tag{14}$$

$$i(P) = I \tag{15}$$

where $1 \leq p \leq P$ and $P = B + 1$. The estimated start of the frame is given by

$$\hat{\theta}_t = \max_d [T_{CFO}(d)] \tag{16}$$

Note that in the special case of $M = 1$ in (13) $T_{CFO}(d)$ becomes

$$T_{CFO}(d) = \sum_{p=1}^{p=P} \left| \sum_{n=0}^{N-1} r^{\star}(d+n)\, x_0(n)\, \mathrm{e}^{(j2\pi(i(p))n/N)} \right|^2 \tag{17}$$

Now (17) is the proposed timing estimation method using one preamble, which is independent of the structure of the preamble. Note that (17) gives better performance than (12) in the presence of CFO.

In the proposed method the computational complexity is high as the range of CFO increases because as the value of I increases, the value of P is also increases. If the estimated start of the frame $\hat{\theta}_t$ satisfies the condition $1 \leq \hat{\theta}_t \leq L$ then the frame is processed further, otherwise frame is discarded and considered as an erasure. Let $F1$ be the total number of frames that is considered as erasure and $F2$ be the total number of frames that is transmitted. Probability of erasure (PE) is given by

$$PE = \frac{F1}{F2}. \tag{18}$$

Let the number of detected frames is given by

$$F = F2 - F1. \tag{19}$$

The mean squared error (MSE) of the detected frames is given by

$$MSE = \frac{\sum_{f=0}^{F-1} \left(\theta_{tf} - \hat{\theta}_{tf}\right)^2}{F} \tag{20}$$

where θ_{tf} is the maximum absolute value of the channel impulse response for the fth detected frame given by

$$\theta_{tf} = \max(\mathrm{abs}(\mathbf{h}_f)) \tag{21}$$

where $\mathbf{h}_f$ is the channel impulse response for the fth detected frame and $\hat{\theta}_{tf}$ is the estimated start of the fth detected frame.

Let $F3$ be the total number of frames for which $\hat{\theta}_t = \theta_t$, then the probability of correct estimation $P(\hat{\theta}_t = \theta_t)$ is given by

$$P(\hat{\theta}_t = \theta_t) = \frac{F3}{F2} \tag{22}$$

where θ_t is the maximum absolute value of the channel impulse response for a given frame, given by

$$\theta_t = \max(\mathrm{abs}(\mathbf{h})) \tag{23}$$

where $\mathbf{h}$ is the channel impulse response for a given frame and $\hat{\theta}_t$ is the estimated start of that frame.

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IV. SIMULATION RESULTS AND DISCUSSION

In this section, the performance of the proposed method is compared with the major existing timing synchronization methods [8]–[13] for OFDM systems, which work independent of the structure of the preamble.

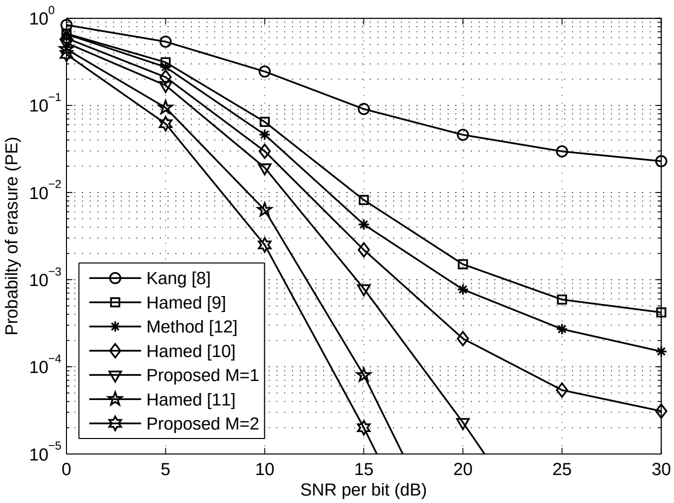

Figure 2: Probability of erasure of different estimators using randomly generated preamble in the presence of CFO [(M=1,2), I=0.5, P=11]

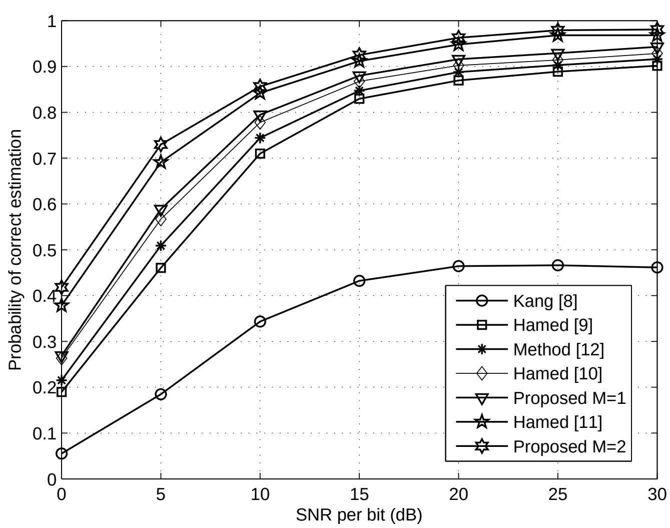

Figure 3: Probability of correct estimation of different estimators using randomly generated preamble in the presence of CFO [(M=1,2), I=0.5, P=11]

We have assumed N=64 and performed the simulations over $5 * 10^5$ frames. QPSK signaling is assumed. A frequency selective Rayleigh fading channel is assumed with $L = 5$ path taps and path delays $\mu_l = l$ for $l = 0, 1, ..., 4$. The channel has an exponential power delay profile (PDP) with an average power of $\exp(-\mu_l/L)$. The CFO takes random

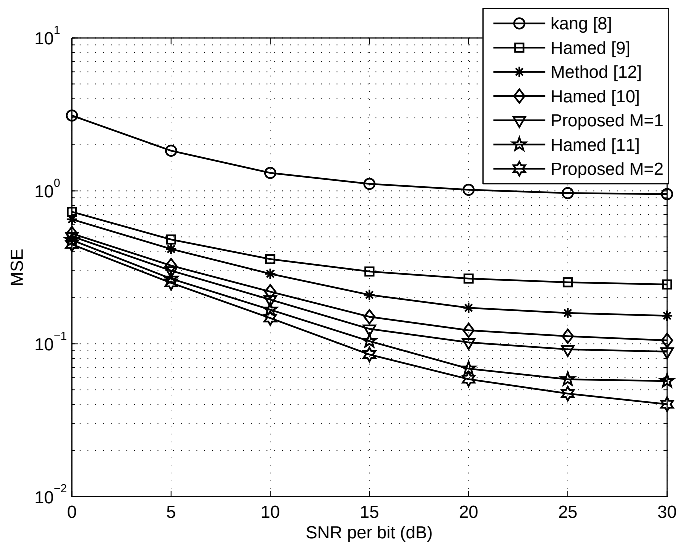

Figure 4: Timing MSE of different estimators using randomly generated preamble in the presence of CFO [(M=1,2), I=0.5, P=11]

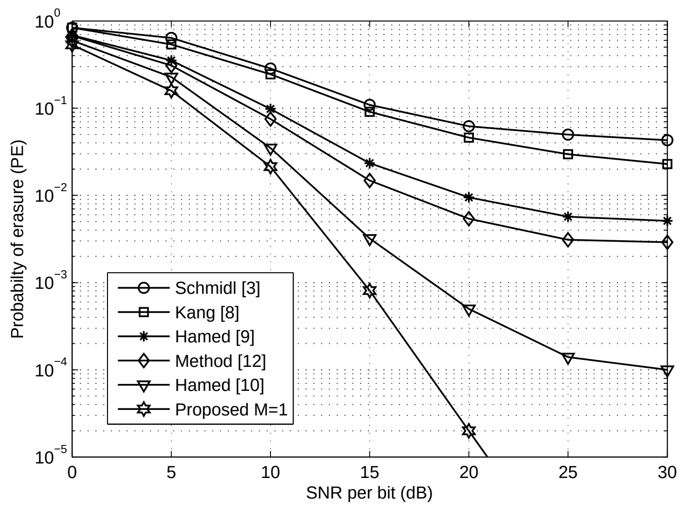

Figure 5: Probability of erasure of different estimators using Schmidl's preamble in the presence of CFO [M=1, I=0.5, P=11]

value within the range $[-I, I]$ and it varies independently from frame to frame. For the methods presented in [9] [10], we have considered all the available correlation points without repetition, i.e., $N(N-1)/2$=2016 and for the method presented in [11] we have considered all the available correlation points utilized by two preambles, i.e., N^2=4096. In order to compare with [8]–[10] [12], we consider M=1 and to compare with [11] we consider M=2. In Fig. 2, Fig. 3 and Fig. 4 probability of erasure, probability of correct estimation and timing MSE of the proposed method is compared with major existing timing synchronization methods in the presence of CFO using a randomly generated preamble. One randomly generated preamble is used for the methods presented in [8]–[10] [12] and the

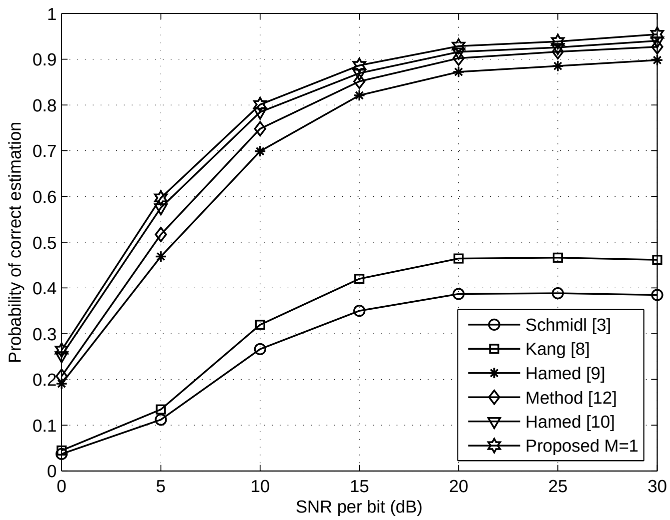

Figure 6: Probability of correct estimation of different estimators using Schmidl's preamble in the presence of CFO [M=1, I=0.5, P=11]

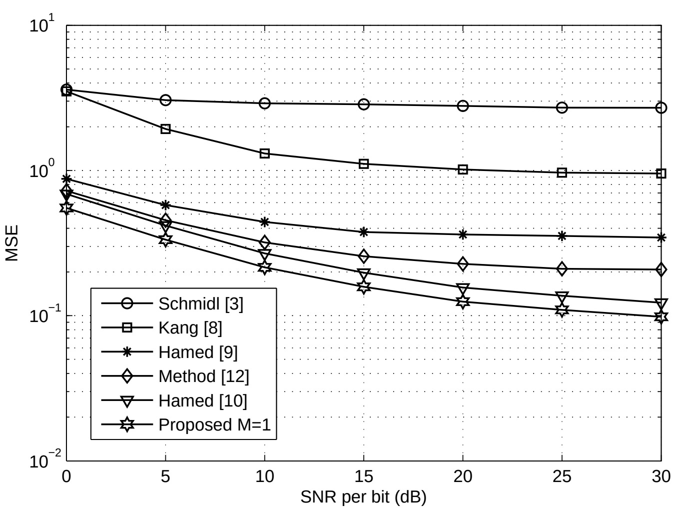

Figure 7: Timing MSE of different estimators using Schmidl's preamble in the presence of CFO [M=1, I=0.5, P=11]

proposed method with M=1 and two randomly generated preambles are used for the method presented in [11] and the proposed method with M=2. We assume $I = 0.5$ and in the proposed method $P = 11$. From Fig. 2, Fig. 3 and Fig. 4, it is observed that proposed method performs better than the existing methods in the presence of CFO. Note that in the presence of CFO ($I \neq 0$) with $M = 1$ in the proposed method, there is a significant improvement in the performance as compared to method presented in [12]. In Fig. 5 to Fig. 13, the probability of erasure, probability of correct estimation and timing MSE of the proposed method is compared with the major existing timing synchronization methods, for different preambles. In Fig. 5 to Fig. 7, the Schmidl's preamble is used considering $M = 1$. In Fig. 8 to Fig. 10, the Minn's preamble

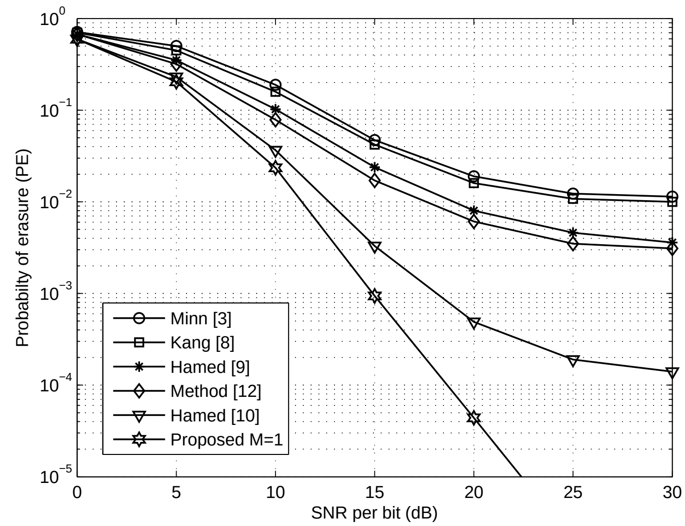

Figure 8: Probability of erasure of different estimators using Minn's preamble in the presence of CFO [M=1, I=0.5, P=11]

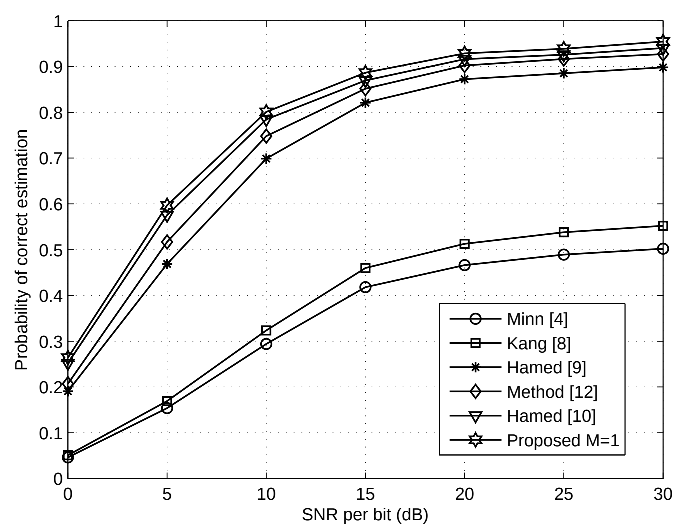

Figure 9: Probability of correct estimation of different estimators using Minn's preamble in the presence of CFO [M=1, I=0.5, P=11]

is used considering $M = 1$ and in Fig. 11 to Fig. 13, both Schmidl's and Minn's preamble are used considering $M = 2$. We assume $I = 0.5$ and in the proposed method $P = 11$. From Fig. 5 to Fig. 13, we find that the proposed method gives the best performance. In fig. 14, probability of erasure of different estimators along with the proposed method are compared with the method proposed in [13] assuming $M = 1$. Randomly generated preamble is used. A 10 tap channel with uniform power delay profile is considered. Preamble length (N) is assumed as 128 and the number of OFDM frames simulated is $5 * 10^5$. We assume I is 3.2. For the methods presented in [9] [10], we have considered all the available correlation points without repetition, i.e., $N(N-1)/2$=8128.

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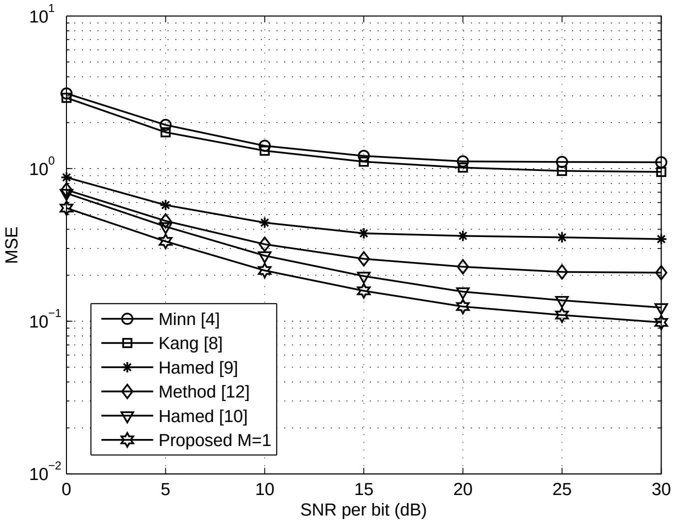

Figure 10: Timing MSE of different estimators using Minn's preamble in the presence of CFO [M=1, I=0.5, P=11]

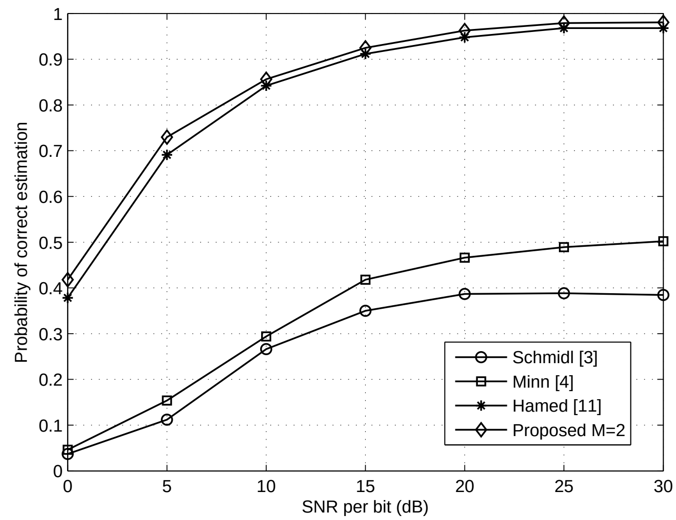

Figure 12: Probability of correct estimation of proposed estimator using Schmidl's and Minn's preamble in the presence of CFO [M=2, I=0.5, P=11]

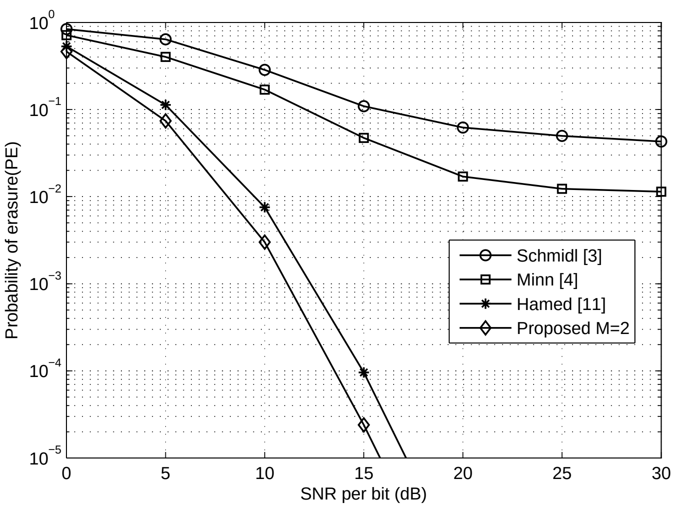

Figure 11: Probability of erasure of proposed estimator using Schmidl's and Minn's preamble in the presence of CFO [M=2, I=0.5, P=11]

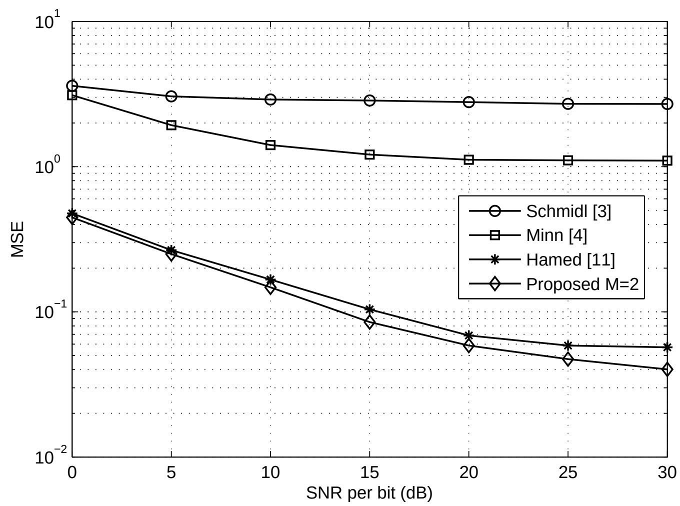

Figure 13: Timing MSE of proposed estimator using Schmidl's and Minn's preamble in the presence of CFO [M=2, I=0.5, P=11]

In the proposed method $P = 65$. It is observed that [13] gives the best performance since it is the optimum method.

V. CONCLUSION

In this paper, a new timing estimation method using multiple preambles has been proposed and the performance is investigated in the presence of CFO. It is observed that the proposed method performs better than most of the existing methods in the presence of CFO.

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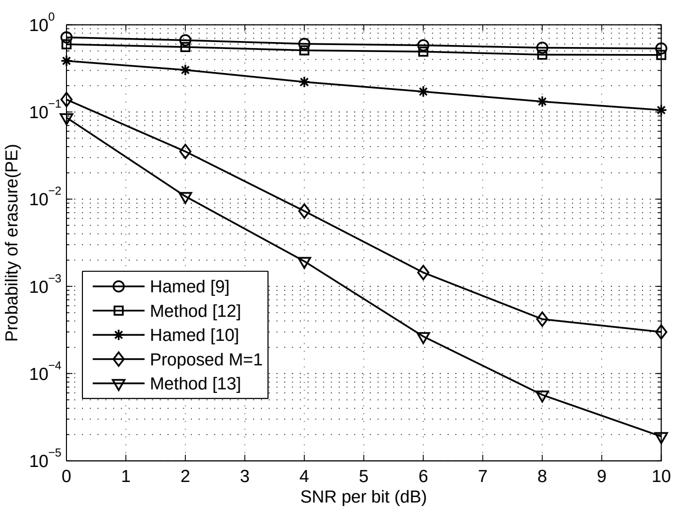

Figure 14: Probability of erasure of different estimators using randomly generated preamble in the presence of CFO [M=1, I=3.2, P=65]