# A New Preamble Aided Fractional Frequency Offset Estimation in OFDM Systems

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***Abstract*—Carrier frequency offset (CFO) in OFDM systems can be divided in two parts, the fractional part (FFO) and the integer part (IFO). In this paper, a data aided fractional frequency offset (FFO) synchronization scheme for OFDM system is proposed. Four different algorithms to estimate the FFO are proposed. The proposed algorithms work in the time domain. An independent Rayleigh fading multipath channel in the presence of AWGN is considered. The performance is compared in terms of mean square error (MSE) of the frequency offset estimation and the computational complexity with the existing FFO estimation methods.**

***Keywords*—*Carrier frequency offset (CFO); Frequency synchronization; OFDM; Preamble.***

## I. Introduction

It is well known that orthogonal frequency division multiplexing (OFDM) [1] converts a frequency selective (multipath) channel into a frequency flat channel, thereby eliminating intersymbol interference (ISI). However, the presence of a carrier frequency offset (CFO) introduces inter carrier interference (ICI), which severely degrades the performance of OFDM.

There have been several methods proposed in the literature for solving the problem of CFO estimation in OFDM systems. In [2]–[5] the frequency offset is assumed to be a uniformly distributed random variable over a certain range, and is detected using maximum likelihood techniques. In the other papers CFO is usually divided into two parts: the fractional part (FFO) and the integer part (IFO). In this paper, we focus on the preamble based FFO estimation schemes. To estimate the FFO, methods proposed in the literature can be broadly classified into two categories.

1) Methods that utilize the phase shift between the repetitive parts of a preamble in the time domain [6]–[13].
2) Methods that utilize the symmetrical correlation of the preamble [14]–[18]

Schmidl and Cox [6] and Lim [8] use a preamble with two identical halves to estimate the FFO. FFO is estimated by measuring the phase shift between two identical halves of the preamble. Minn [9], Wang [11], Shi [10] use a preamble with four identical halves to estimate the FFO. FFO is estimated by measuring the phase shift between the adjacent blocks of the preamble. Tufvesson [13] proposed a different method to estimate the FFO. In [13], the received signal is multiplied by the known preamble and the FFO is estimated by measuring phase shift of the resulting signal. Morelli and Mengali [7] estimate the FFO by using a best linear unbiased estimator (BLUE), which gives better performance than [6] [8]–[11] [13]. The main drawback of the BLUE estimator [7] is its computational complexity. Zhang [14], Zhang [15], Park [16], Kim [17], Shao [18] estimate the FFO by utilizing symmetrical correlation of the preamble. Here, we propose a new method to estimate FFO using a time domain repeated preamble. The proposed method is compared with the existing methods in terms of performance in the multipath Rayleigh fading channel and the computational complexity.

This paper is organized as follows. The system model is presented in Section II. Existing FFO estimation methods are presented in Section III. The proposed method is presented in Section IV. The simulation results are given in Section V and finally, the conclusions in Section VI.

## II. System Model

Fig. 1 shows the typical structure of a OFDM frame in the time domain. An OFDM frame contains preamble, cyclic prefix (CP) and data. Preamble is used for synchronization purpose. Let $\mathbf{x}_p$ denotes the time domain preamble of the


Figure 1: OFDM frame structure in the time domain

OFDM frame given by

$$\mathbf{x}_p = \left[\mathbf{A}_{N/4}\, \mathbf{A}_{N/4}\, \mathbf{A}_{N/4}\, \mathbf{A}_{N/4}\right] \tag{1}$$

where $\mathbf{A}_{N/4}$ is the sample of length $N/4$ in the time domain, which is obtained by $N/4$ point IFFT of the $N/4$ length frequency domain data. A cyclic prefix (CP) of length $N_g$ which is denoted by $\mathbf{CP}_{\mathrm{PRE}}$ is introduced in front of the preamble in the time domain. $\mathbf{CP}_{\mathrm{PRE}}$ of the preamble $\mathbf{x}_p$ is given by

$$\mathbf{CP}_{\mathrm{PRE}} = \left[x_p\left(N - N_g\right)\, ...\, x_p\left(N-1\right)\right] \tag{2}$$

Let

$$\mathbf{x} = \left[\mathbf{CP}_{\mathrm{PRE}}\, \mathbf{x}_p\right]. \tag{3}$$

Now, $\mathbf{x}$ is transmitted through the frequency selective channel. The channel is assumed to be quasi static and it is fixed for one frame and varies independently from frame to frame. Its impulse response for a given frame can be expressed as:

$$\mathbf{h} = \left[h(0)\; h(1)\; h(2)\; .....\; h(L-1)\right] \tag{4}$$

where $L$ is the number of channel taps. The received signal $\mathbf{r}$ in the time domain is given by:

$$r(n) = y(n)\,\mathrm{e}^{j2\pi n\epsilon/N} + w(n) \tag{5}$$

where

$$\begin{aligned} y(n) &= h(n) \star x(n) \\ &= \sum_{l=0}^{L-1} h(l)\,x(n-l)\,. \end{aligned} \tag{6}$$

where $w(n)$ is zero mean Gaussian noise sample and $\epsilon$ is the normalized frequency offset. $\epsilon$ can be divided into two parts the integer part denoted by $\epsilon_I$ (IFO) and the fractional part denoted by $\epsilon_F$ (FFO), where $-1/2 \leqslant \epsilon_F < 1/2$ [19] and $-N/2 \leqslant \epsilon_I < N/2$ or $0 \leq \epsilon_I < N$ [20]. Here, we consider the presence of FFO only. Let $z(n)$ denote the signal after discarding the CP of the received preamble $r(n)$. After discarding CP $z(n)$ is re indexed from 0 to $N-1$. We define the sub vectors $\mathbf{Z}_i$ given by

$$\mathbf{Z}_i = \left[z\left((i-1)\,N/4\right) \;.....\; z\left(iN/4-1\right)\right]^T \tag{7}$$

where $1 \leq i \leq 4$. We define the correlation functions given by

$$\mathbf{Z}_1^H\mathbf{Z}_2 = \sum_{n=0}^{N/4-1} z^\star(n).z(n+N/4) \tag{8}$$

$$\mathbf{Z}_2^H\mathbf{Z}_3 = \sum_{n=N/4}^{N/2-1} z^\star(n).z(n+N/4) \tag{9}$$

$$\mathbf{Z}_3^H\mathbf{Z}_4 = \sum_{n=N/2}^{3N/4-1} z^\star(n).z(n+N/4) \tag{10}$$

$$\mathbf{Z}_1^H\mathbf{Z}_3 = \sum_{n=0}^{N/4-1} z^\star(n).z(n+N/2) \tag{11}$$

$$\mathbf{Z}_2^H\mathbf{Z}_4 = \sum_{n=N/4}^{N/2-1} z^\star(n).z(n+N/2) \tag{12}$$

$$\mathbf{Z}_1^H\mathbf{Z}_4 = \sum_{n=0}^{N/4-1} z^\star(n).z(n+3N/4). \tag{13}$$

## III. EXISTING FFO ESTIMATION METHODS

In this section, we give a brief overview of fractional frequency offset estimation methods.

*1) Schmidl and Cox method:* Schmidl and Cox [6] use a preamble with two identical halves to estimate the FFO, which is given by

$$\mathbf{x}_{p(sch)} = \left[\mathbf{A}_{N/2}\;\mathbf{A}_{N/2}\right] \tag{14}$$

where $\mathbf{A}_{N/2}$ is the sample of length $N/2$. In this case the $\mathbf{Z}_i$ becomes

$$\mathbf{Z}_i = \left[z\left((i-1)\,N/2\right) \;.....\; z\left(iN/2-1\right)\right]^T \tag{15}$$

where $1 \leq i \leq 2$. FFO estimation proposed by Schmidl and Cox [6] is given by

$$\hat{\epsilon}_F = \frac{2}{\pi}\mathrm{arg}\left(\mathbf{Z}_1^H\mathbf{Z}_2\right) = \frac{2}{\pi}\mathrm{arg}\left(\sum_{n=0}^{N/2-1} z^\star(n)\,.z\left(n+N/2\right)\right). \tag{16}$$

*2) Minn and Bhargava method:* Minn and Bhargava [9] use a preamble with four identical halves (as given in the system model) to estimate the FFO. FFO estimation proposed by Minn and Bhargava [9] is given by

$$\hat{\epsilon}_F = \frac{2}{\pi}\mathrm{arg}\left(\psi_1\right) \tag{17}$$

where $\psi_1$ is given by

$$\psi_1 = \mathbf{Z}_1^H\mathbf{Z}_2 + \mathbf{Z}_3^H\mathbf{Z}_4 \tag{18}$$

*3) Wang and Faulkner method:* Wang and Faulkner [11] use a preamble with four identical halves (as given in the system model) to estimate the FFO. FFO estimation proposed by Wang and Faulkner [11] is given by

$$\hat{\epsilon}_F = \frac{2}{\pi}\mathrm{arg}\left(\psi_2\right) \tag{19}$$

where $\psi_2$ is given by

$$\psi_2 = \mathbf{Z}_2^H\mathbf{Z}_3 + \mathbf{Z}_3^H\mathbf{Z}_4 \tag{20}$$

*4) Shi and Serpedin method:* Shi and Serpedin [10] use a preamble with four identical halves (as given in the system model) to estimate the FFO. FFO estimation proposed by Shi and Serpedin [10] is given by

$$\hat{\epsilon}_F = \frac{2}{\pi}\mathrm{arg}\left(\psi_3\right) \tag{21}$$

where $\psi_3$ is given by

$$\psi_3 = \mathbf{Z}_1^H\mathbf{Z}_2 + \mathbf{Z}_2^H\mathbf{Z}_3 + \mathbf{Z}_3^H\mathbf{Z}_4 \tag{22}$$

*5) Morelli and Mengali method:* Morelli and Mengali [7] use a preamble with $T$ identical halves to estimate the FFO, given by

$$\mathbf{x}_{p(morelli)} = \left[\mathbf{A}_{N/T}\;\mathbf{A}_{N/T}\;.....\;\mathbf{A}_{N/T}\right] \tag{23}$$

where $T$ is given by

$$T = 2^j \tag{24}$$

where $j$ is a positive integer and $\mathbf{A}_{N/T}$ is the sample of length $N/T$. FFO estimation proposed by Morelli and Mengali [7] is given by

$$\hat{\epsilon}_F = \frac{T}{2\pi}\sum_{m=1}^{H} w(m)\,\phi(m) \tag{25}$$

where $w(m)$ is given by

$$w(m) = 3\frac{(T-m)(T-m+1) - H(T-H)}{H\left(4H^2 - 6TH + 3T^2 - 1\right)} \tag{26}$$

$\phi(m)$ is given by

$$\phi(m) = \left[\mathrm{arg}\left\{R(m)\right\} - \mathrm{arg}\left\{R(m-1)\right\}\right] \tag{27}$$

and $1 \leq m \leq H$. where

$$R(k) = \sum_{n=kM}^{N-1} z^\star(n-kM)\,z(n) \tag{28}$$

and $0 \leq k \leq H$ and $H = T/2$ and $M = N/T$.

*6) Park and Cheon method:* Park and Cheon [16] use symmetrical correlation to estimate the FFO. Preamble used in [16] is given by

$$\mathbf{x}_{p(park)} = \left[\mathbf{A}_{N/4}\,\mathbf{B}_{N/4}\,\mathbf{A}^{\star}_{N/4}\,\mathbf{B}^{\star}_{N/4}\right] \tag{29}$$

where $\mathbf{A}_{N/4}$ is the sample of length $N/4$. $\mathbf{A}^{\star}_{N/4}$ is the conjugate of $\mathbf{A}_{N/4}$. $\mathbf{B}_{N/4}$ is designed to be the time reversed version (symmetric) of $\mathbf{A}_{N/4}$. $\mathbf{B}^{\star}_{N/4}$ is the conjugate of $\mathbf{B}_{N/4}$. The proposed FFO estimation in [16] is given by

$$\hat{\epsilon}_F = \frac{1}{\pi}\arg\left(\sum_{n=0}^{N/4-1} z^{\star}(n)\,.z(n+N/2)\right). \tag{30}$$

*7) Shao method:* Shao [18] uses symmetrical correlation to estimate the FFO. Preamble used in [18] is given by

$$\mathbf{x}_{p(shao)} = \left[\mathbf{A}_{N/4}\,\mathbf{B}^{\star}_{N/4}\,\mathbf{A}^{\star}_{N/4}\,\mathbf{B}_{N/4}\right] \tag{31}$$

where $\mathbf{A}_{N/4}$ is the sample of length $N/4$. $\mathbf{A}^{\star}_{N/4}$ is the conjugate of $\mathbf{A}_{N/4}$. $\mathbf{B}_{N/4}$ is designed to be the time reversed version (symmetric) of $\mathbf{A}_{N/4}$. $\mathbf{B}^{\star}_{N/4}$ is the conjugate of $\mathbf{B}_{N/4}$. The proposed FFO estimation in [18] is given by

$$\hat{\epsilon}_F = \frac{1}{\pi}\arg\left(\sum_{n=0}^{N/2-1} z^{\star}(n)\,z(N-n)\right). \tag{32}$$

## IV. PROPOSED MODEL

The proposed method uses a preamble with four identical halves as mentioned in the system model. Correlation functions between the adjacent blocks are given by $\mathbf{Z}_1^H\mathbf{Z}_2$, $\mathbf{Z}_2^H\mathbf{Z}_3$, $\mathbf{Z}_3^H\mathbf{Z}_4$ and the correlation functions between the nonadjacent blocks are given by $\mathbf{Z}_1^H\mathbf{Z}_3$, $\mathbf{Z}_2^H\mathbf{Z}_4$, $\mathbf{Z}_1^H\mathbf{Z}_4$. In [9]–[11], the FFO is estimated by utilizing the correlation between the adjacent blocks of the preamble. In proposed method, we utilize only the non adjacent blocks of the preamble.

### A. Proposed algorithm 1

The FFO estimate is

$$\hat{\epsilon}_F = \frac{1}{\pi}\arg(\psi_4) \tag{33}$$

where $\psi_4$ is either given by

$$\psi_4 = \mathbf{Z}_1^H\mathbf{Z}_3 \tag{34}$$

or

$$\psi_4 = \mathbf{Z}_2^H\mathbf{Z}_4 \tag{35}$$

### B. Proposed algorithm 2

The FFO estimate is

$$\hat{\epsilon}_F = \frac{1}{\pi}\arg(\psi_5) \tag{36}$$

where $\psi_5$ is given by

$$\psi_5 = \mathbf{Z}_1^H\mathbf{Z}_3 + \mathbf{Z}_2^H\mathbf{Z}_4 \tag{37}$$

### C. Proposed algorithm 3

The FFO estimate is

$$\hat{\epsilon}_F = \frac{2}{3\pi}\arg(\psi_6) \tag{38}$$

where $\psi_6$ is given by

$$\psi_6 = \mathbf{Z}_1^H\mathbf{Z}_4 \tag{39}$$

### D. Proposed algorithm 4

The FFO estimate is

$$\hat{\epsilon}_F = \frac{1}{2\pi}\arg(\psi_5) + \frac{1}{3\pi}\arg(\psi_6) \tag{40}$$

## V. SIMULATION RESULTS AND DISCUSSION

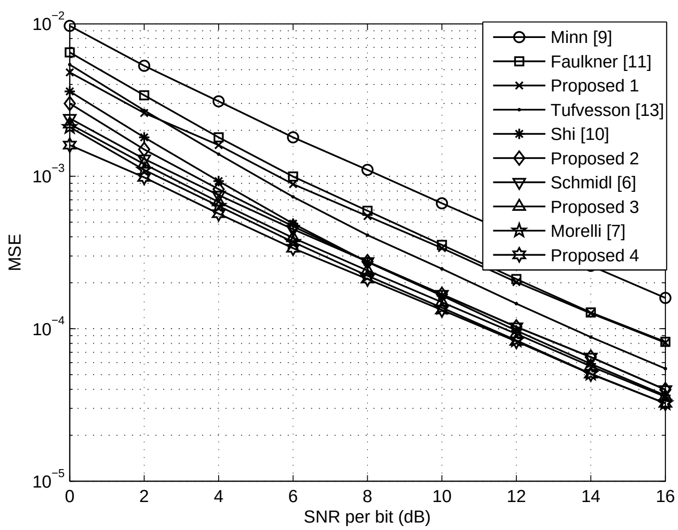


Figure 2: Mse performance of the proposed method in comparison with the previous methods that use time domain repeated preamble to estimate FFO in AWGN channel

In this section, the performance of the proposed method is compared with the major existing fractional frequency offset synchronization methods We have assumed $N$=128 and performed the simulations over $10^4$ frames. Length $N_g$ of the cyclic prefix (CP) is 16. QPSK signaling is assumed. Frequency selective Rayleigh fading channel is assumed with path taps $L = 5$ and path delays $\mu_l = l$ for $l = 0, 1, ..., 4$ . The channel has an exponential power delay profile (PDP) with an average power of $\exp(-\mu_l/L)$. The CFO takes random value within the range $[-0.5, 0.5]$ and it varies from frame to frame. In order to compare with the methods in [9]–[11] along with the proposed method the value of $j$ and $T$ for the method proposed in [7] are set to 2 and 4 respectively.

Fig. 2 and Fig. 3 show the comparison of the proposed method with the existing methods that use time domain repeated preamble to estimate FFO in AWGN channel and multipath channel respectively. As indicated in Fig. 2 and Fig. 3 the proposed algorithm 1 performs better than the methods in [9] and [11] with less computational complexity. The proposed algorithm 2 performs better than the methods in

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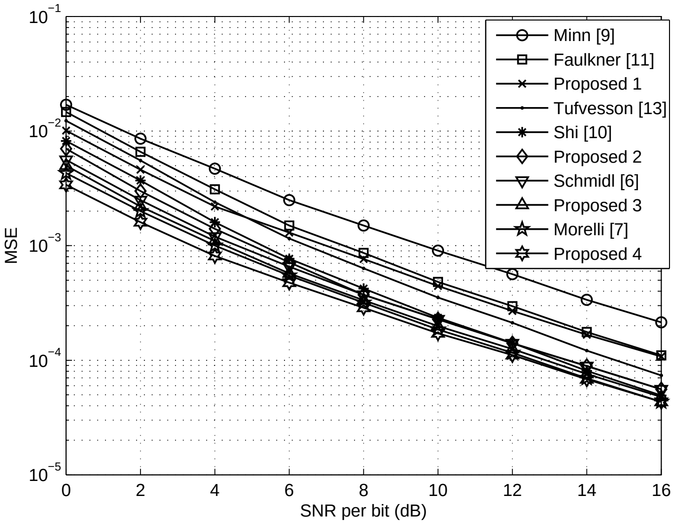


Figure 3: Mse performance of the proposed method in comparison with the previous methods that use time domain repeated preamble to estimate FFO in multipath channel

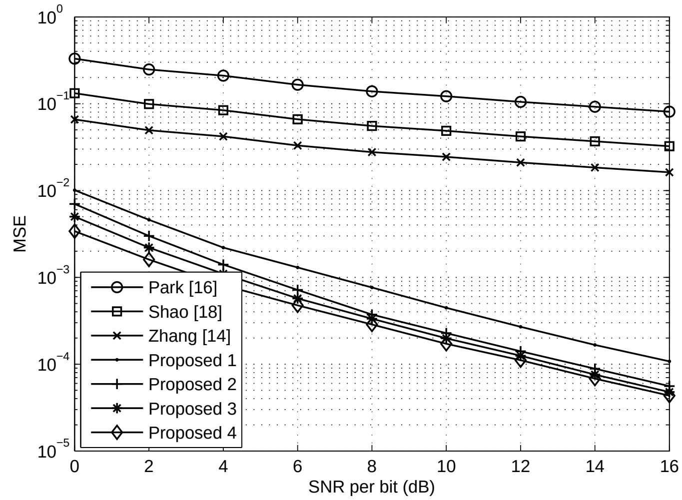


Figure 5: Mse performance of the proposed method in comparison with the previous methods that use symmetrical correlation of the preamble to estimate FFO in multipath channel

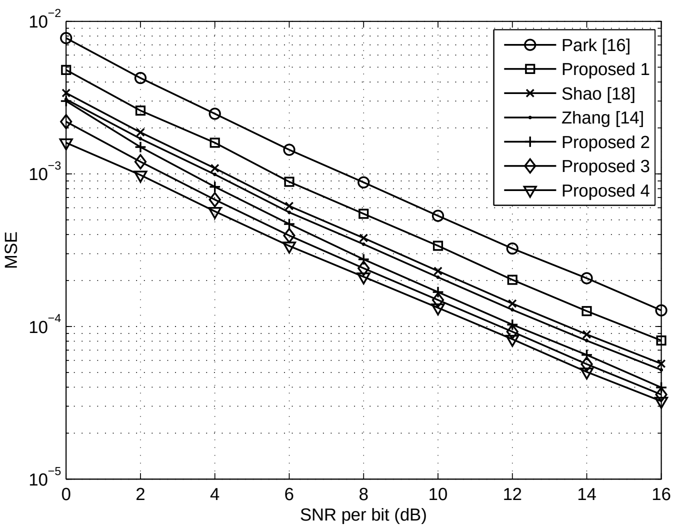


Figure 4: Mse performance of the proposed method in comparison with the previous methods that use symmetrical correlation of the preamble to estimate FFO in AWGN channel

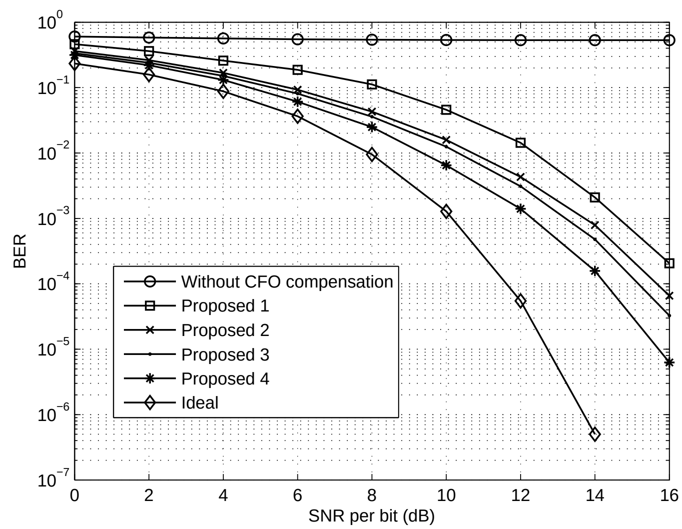


Figure 6: Ber performance of the proposed methods in AWGN channel

[9]–[11] and [13]. Computational complexity of the proposed algorithm 2 is the same as for the methods in [9] and [11] but less as compared to methods proposed in [13] and [10]. Proposed algorithm 3 performs better than the methods given in [6] [9]–[11] and [13] with less computational complexity. Proposed algorithm 4 gives the best performance because it gives the better result as compared to method in [7] with less computational complexity as indicated in Fig. 2 and Fig. 3.

Fig. 4 and Fig. 5 shows the comparison of the proposed method with the existing methods that use symmetrical correlation of the preamble to estimate FFO in AWGN channel and multipath channel respectively. It is observed that the performance of the symmetrical correlation methods [16] [14] and [18] are degraded in the presence of multipath as compared to AWGN channel. It is also observed that the proposed methods perform better than the existing symmetrical correlation methods in the presence of multipath. In table 1, the computational complexity of different estimators along with the proposed methods is given. Fig. 6 shows the ber performance of the proposed methods in AWGN channel. It is observed that proposed algorithm 4 performs better than the other methods.

## VI. CONCLUSION

In this paper, the performance of different existing data aided fractional frequency offset estimator schemes are com-

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TABLE I: COMPUTATIONAL COMPLEXITY

| Method | Multiplications | Addition | Division |
|---|---|---|---|
| Schmidl | $(N/2)+1$ | $(N/2-1)$ | $0$ |
| Minn | $(N/2)+1$ | $(N/2)-1$ | $0$ |
| Faulkner | $(N/2)+1$ | $(N/2)-1$ | $0$ |
| Shi | $(3N/4)+1$ | $(3N/4)-1$ | $0$ |
| Morelli | $9N/4+23, T=4, H=2$ | $9N/4-2, T=4, H=2$ | $2, T=4, H=2$ |
| Park | $N/4+1$ | $N/4-1$ | $0$ |
| Shao | $N/2+1$ | $N/2-1$ | $0$ |
| Proposed 1 | $(N/4)+1$ | $(N/4)-1$ | $0$ |
| Proposed 2 | $(N/2)+1$ | $(N/2-1)$ | $0$ |
| Proposed 3 | $(N/4)+1$ | $(N/4-1)$ | $0$ |
| Proposed 4 | $(3N/4)+2$ | $(3N/4-1)$ | $0$ |

pared with the proposed methods. The proposed methods give better result as compared to existing techniques.

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