A Novel Location Estimation Method based on an Apollonian Circle with Robust Filtering

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***Abstract*— Noise, deviations, and outliers with varying distribution characteristics exist in measured data for outdoor location estimation, propagation characteristics that make source location estimation difficult. The estimation error of conventional methods (typically a least-squares method) is increased by such outliers. To solve this problem, this study proposes a novel location estimation method, specifically a modified trilateration technique based on Apollonian circles that does not require knowing the exact transmission power of the source or carrying out a calibration procedure. The proposed method results in improved location estimates compared to existing methods, which is confirmed with robust filtering in verification experiments.**

Keywords- Robust location estimation; received signal strength indication; random sample consensus; Apollonian circle

I. INTRODUCTION

Wireless geolocation refers to the problem of finding the location of mobile subscribers in different radio systems, such as cellular networks, wireless local area networks, and wireless sensor networks [1]. Researchers have proposed several methodologies for estimating the location of unknown radio frequency (RF) sources based on different physical characteristics, including received signal strength indication (RSSI), time-of-arrival (TOA), time-difference-of-arrival (TDOA), and angle-of-arrival (AOA). Of these, RSSI-based location estimation methods can be implemented easily with no additional hardware; thus, this method is frequently used. The main drawback to algorithms that use RSSI as a range measurement is that they are highly dependent on the propagation conditions that exist between the transmission point (TP) and each measurement point (MP).

The most widely used positioning mechanism is the Global Positioning System (GPS). GPS requires four or more satellite signals to function properly. These signals can be impossible to obtain indoors, in downtown city centers with tall buildings, under poor atmospheric conditions, or in geographically obstructed outdoor areas, such as deep valleys. Satellite-based localization services may also be disabled at any time by intentional jammers [2]. Therefore, new positioning techniques are needed in environments that cannot use GPS but require highly accurate, reliable results.

Moreover, a reliable location estimation algorithm must adapt to unknown channel conditions. For this reason, the least-squares (LS) method [3] is generally used to determine source location. However, this method generates significant error when using attenuated data based on the characteristics of the radio channel. Data exhibiting distinct distribution characteristics are referred to as outliers. Outlier data are a major factor that interferes with accurate location estimation. The random sample consensus (RANSAC) algorithm was proposed by Fischler and Bolles [4]; it performs local parameter estimation to search the inliers group after removal from a target estimated by identifying outliers. The method is effectively used for the estimation of various models [5][6]. Even when the ratio of outliers is very high compared to the inliers, this algorithm can be used for robust location estimation.

Exiting localization studies can be classified as either range-based or range-free. In range-based methods, the TP at an unknown location determines the distances to MPs based on signal strength; they then use trilateration [7]. These methods achieve high localization accuracy, but require the availability of line-of-sight (LOS) propagation conditions between any three MPs and the TP. Range-free methods rely only on the locations of MPs; they do not use the distances to these nodes. To determine the TP location, the centroid algorithm [8] uses information from neighboring *MPs* instead of distance information.

This paper presents a novel probability-based approach to estimating location based on Apollonian circles [9] that does not use any map information or calibration stage. The proposed algorithm modifies existing trilateration techniques for a field environment dealing with extensively physical phenomena. Outlier data are removed to improve performance by applying the RANSAC algorithm. We provide simulation results that compare the estimation performance of the original and improved algorithms.

The paper is organized into four sections. Section 2 provides the methodology for and Section 3 describes the results of a performance analysis. The main conclusions are listed in Section 4.

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II. PROPOSED LOCATION ESTIMATION ALGORITHM

A. Training Phase

In this stage, the MP locations and RSS values are recorded from each environment to serve as the training data set. Assume that the number of MPs is N, there is one unknown TP, the locations of the MPs are denoted by $\{m_1, \ldots, m_N\}$, and the location of the unknown TP is denoted by x. For simplicity, it is assumed that each MP is equipped with a non-directional antenna. We consider a log-distance path-loss model [10], which is widely used for the analysis of outdoor wireless channels. The measured RSS value at each MP, P_i, may be formulated as the following expression:

$$P_i = P_0 - 10\gamma \log_{10}\left(\frac{d_i}{d_0}\right) + n_i \tag{1}$$

where P_0 is the power measured at a reference distance d_0 from the TP, γ is a path loss index. n_i is a zero-mean Gaussian and unit variance. The values of γ can be set depending on the propagation environment. Consequently, this phase is generally accomplished with the direct inversion of (1), i.e.:

$$\hat{d}_i \overset{def}{=} d_0 10^{\frac{P_0 - P_i}{10\gamma}} \tag{2}$$

which is a maximum likelihood estimator of $\widehat{d_i}$ [11], asymptotically unbiased (and normal) but biased for a finite sample.

B. Location Estimation Phase

Next, we apply the proposed algorithm with the calculated distance to estimate the location of the TP. In existing methods, the location of an unknown TP is estimated by means of the least squares criterion. The proposed location estimation method obtains the TP by a calculation based on an Apollonian circle. Figure 1 shows the Apollonian circle based on the ratio of the distance between MPs; Figure 2 shows the proposed location estimation method based upon these circles. The relationship between two MPs ((x_1, y_1), (x_2, y_2)), for which the distance ratio $a{:}b$ can be obtained based on the distance estimated using (2), and the estimated TP location (x, y) are represented in (5).

$$\sqrt{(x-m_{x,1})^2+(y-m_{y,1})^2} : \sqrt{(x-m_{x,2})^2+(y-m_{y,2})^2} = a:b \tag{5}$$

$$\sqrt{(x-m_{x,1})^2+(y-m_{y,1})^2} : \sqrt{(x-m_{x,3})^2+(y-m_{y,3})^2} = c:d \tag{6}$$

$$\sqrt{(x-m_{x,2})^2+(y-m_{y,2})^2} : \sqrt{(x-m_{x,3})^2+(y-m_{y,3})^2} = f:e \tag{7}$$

Rearranging the above equations, we have:

$$x^2+y^2-\frac{2(a^2m_{x,2}-b^2m_{x,1})}{a^2-b^2}x-\frac{2(a^2m_{y,2}-b^2m_{y,1})}{a^2-b^2}y+\frac{a^2m_{x,2}^2+a^2m_{y,2}^2-b^2m_{x,1}^2-b^2m_{y,1}^2}{a^2-b^2}=0 \tag{8}$$

where, $a{:}b$ is the ratio of distance MP_1 to MP_2. The internal division point $P(\cdot)$ and external division point $Q(\cdot)$ of the circle are as follows:

$$P_1\left(x=\frac{am_{x,2}+bm_{x,1}}{a+b}, y=\frac{am_{y,2}+bm_{y,1}}{a+b}\right) \tag{9}$$

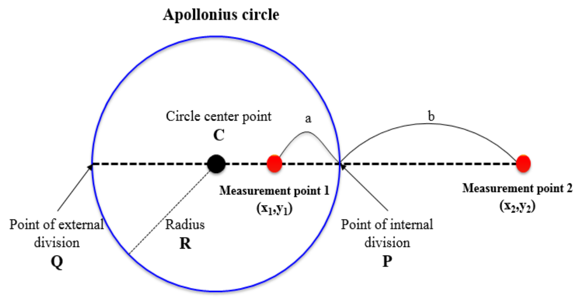

Figure 1. Apollonian Circle according to a certain ratio between the measurement points

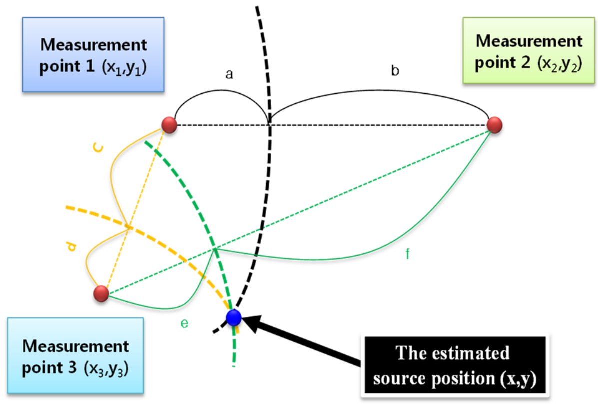

Figure 2. Proposed location estimation based on Apollonian circles

$$Q_1\left(x=\frac{am_{x,2}-bm_{x,1}}{a-b}, y=\frac{am_{y,2}-bm_{y,1}}{a-b}\right) \tag{10}$$

Through (9) and (10), the radius of the circle R and the center of the circle C can be determined as follows:

$$R_1=\frac{\sqrt{(P_{1,x}-Q_{1,x})^2+(P_{1,y}-Q_{1,y})^2}}{2}, \quad C_1\left(x=\frac{P_{1,x}+Q_{1,x}}{2}, y=\frac{P_{1,y}+Q_{1,y}}{2}\right) \tag{11}$$

Through (11), finally, the equation of a circle can be determined as follows:

$$\begin{aligned}(x-C_{1,x})^2+(y-C_{1,y})^2 &= R_1^2\\ (x-C_{2,x})^2+(y-C_{2,y})^2 &= R_2^2\\ (x-C_{3,x})^2+(y-C_{3,y})^2 &= R_3^2\end{aligned} \tag{12}$$

The circumference of a circle determined using (12) can be estimated to be the TP located between the two MPs. As seen in Figure 2, the TP (x, y) can be estimated by intersecting the circle between three or more MPs and a non-iterative solution can be found by linearizing the system. The results from (12) can be written in matrix form:

$$\mathrm{A}\theta = \frac{1}{2}b \tag{13}$$

where

$$\mathrm{A} = \begin{bmatrix} C_{1,x}-C_{2,x} & C_{1,y}-C_{2,y} \\ C_{1,x}-C_{3,x} & C_{1,y}-C_{3,y} \\ C_{2,x}-C_{3,x} & C_{2,y}-C_{3,y}\end{bmatrix}, \quad \theta = \begin{bmatrix} x \\ y \end{bmatrix}$$

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$$b=\begin{bmatrix} C^2_{1,x}-C^2_{2,x}+C^2_{1,y}-C^2_{2,y}-R_1^{\ 2}+R_2^{\ 2} \\ C^2_{1,x}-C^2_{3,x}+C^2_{1,y}-C^2_{3,y}-R_1^{\ 2}+R_3^{\ 2} \\ C^2_{2,x}-C^2_{3,x}+C^2_{2,y}-C^2_{3,y}-R_2^{\ 2}+R_3^{\ 2} \end{bmatrix} \quad (14)$$

where the set of A and b can be expressed in C_{xy} and R_{sh}, respectively, the solution equation is given by [12] :

$$\mathbf{z}=\left(C_{xy}{}^{T}C_{xy}\right)^{-1}C_{xy}{}^{T}R_{sh} \quad (15)$$

Next, the outliers in $\mathbf{z}$ are filtered by applying the RANSAC algorithm. First, k samples are selected from the random measurement data. The point in the parameter space is defined by repeatedly selecting random subsets of the data and generating model hypotheses for each subset. The number of data points below a pre-determined threshold value is calculated. After S repetitions of this process, the best score model B is returned as the solution. The RANSAC algorithm must determine two main parameters, the sampling number of iterations S and threshold T, of inliers and outliers. The number of repetitions needed to guarantee a success probability η_0 is calculated as follows [7]:

$$S \geq \frac{\log(1-\eta_0)}{\log(1-\rho^m)} \quad (16)$$

where the probability η_0 that at least one sample is selected from within the S^{th} inlier is typically set to 0.95 or 0.99; ρ is the percentage of inliers in the data; and m is the number of samples used to generate a hypothesis. The threshold value T can be selected empirically. If the residual variance of the inliers is σ^2, T is set to 2σ or 3σ. First, experimental data composed of inliers are applied to the RANSAC algorithm and the best approximation model is obtained. After obtaining the residual between the best approximation model and inliers, T is determined in proportion to this variance (or standard deviation). If the residual of the inliers is assumed to follow a normal distribution, when $T = 2\sigma$, 97.7% of inliers are included, and when $T = 3\sigma$, 99.9% of inliers are included. Finally, it is possible to obtain a refined solution equation from the inliers obtained by filtering the outliers in $\mathbf{z}$.

$$\hat{\boldsymbol{z}}=\left(\hat{C}_{xy}{}^{T}\hat{C}_{xy}\right)^{-1}\hat{C}_{xy}{}^{T}\hat{R}_{sh} \quad (17)$$

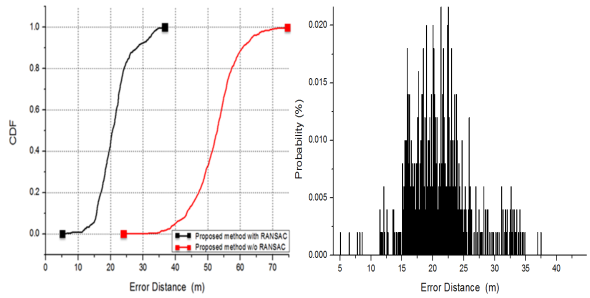
III. PERFORMANCE EVALUATION

In this section, estimation accuracy is tested in a field environment. The TPs were stationary and the RSSI dataset was acquired in practical experiments by car on the Korea Advanced Institute of Science and Technology (KAIST) campus in Daejeon, South Korea. The antenna was non-directional and fixed on the roof of the car, which moved at an average of 60 km/h. The resolution bandwidth was 12.5 KHz. Measurement data were stored every second. To reduce statistical variability, the saved data were averaged over 30 repetitions. The center frequencies of the RF signal used in our experiments was 421.5 MHz, the band used in amateur, industrial/business, public safety, and radio-location radio services. We used an arbitrary frequency from the amateur stations for the localization test. We set the TP, which transmitted a single tone signal, in the center of a building covering an area of 0.9 km × 0.95 km. Figure 3 shows the measurement environment, where the red dot is the actual TP and yellow line is the measurement path.

Figure 3. Measurement location in field environment

The simulation was performed 1000 times to represent the range of fluctuation in the distance error after the simulation. Figure 4a shows the probability density function (PDF) performance with or without the RANSAC algorithm. When the RANSAC is applied, the average estimated error distance is 21.16m, which is about 10.28m better than 31.44m without the RANSAC. Figure 4b is the cumulative distribution function (CDF) result of the proposed method with the RANSAC. The simulation resulted in a distance error range of 5–37.4 m, a mean distance error within 21.16 m, and a minimum distance error of 5.05 m. Figure 5 shows the location estimation methods used for performance comparison which are the trilateration [7] and triangle centroid location algorithms [8]. Tab. 1 summarizes the means and the 50$^{\text{th}}$, 75$^{\text{th}}$, and 90$^{\text{th}}$ percentile values of the error distance for each method. The proposed method performs better than both of the other methods, e.g., 50% of the distance error for the proposed method is within 20.62 m, compared with 129.36 m and 42.81 m for the trilateration and centroid methods, respectively. Similarly, for the proposed method, 90% of the distance error is within 28.18 m, and 157.25 m and 43.97 m for the trilateration and centroid methods.

If the measurement is performed in an outdoor environment, interference from a variety of factors is possible. Therefore, if position is estimated in an outdoor environment with only the RSSI value, the margin of error significantly increases. For the centroid method to result in precise localization, it must be widely distributed with a large number of MPs. However, its estimation performance is relatively poor in outdoor environments in which it is difficult to be widely distributed. Therefore, the centroid method is inefficient for use in high-precision localization. Trilateration is based on a simple



(a) (b)

Figure 4. PDF of the error distance of the proposed method, (a) Comparison of results with RANSAC, (b) PDF of the error distance of the proposed method

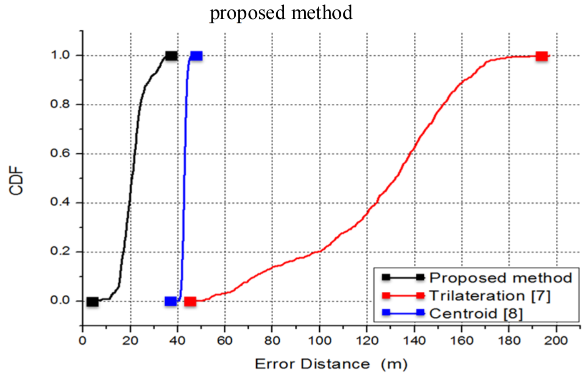


Figure 5. PDF of the error distance of the proposed method

TABLE I. ESTIMATION ERROR OF THE PROPOSED, TRILATERATION, AND CENTROID METHODS

	TRILATERATION [7]	CENTROID [8]	PROPOSED METHOD
MIN ERROR	42.75 m	39.7 m	5.05 m
MEAN ERROR	125.07 m	43.14 m	21.16 m
50th PERCENTRILE	129.36 m	42.81 m	20.62 m
75th PERCENTRILE	146.44 m	43.42 m	23.52 m
90th PERCENTRILE	157.25 m	43.97 m	28.18 m

mathematical calculation. Therefore, it is necessary to know the TP; if an error in distance estimation occurs due to obstacles between the TP and MP or in the surrounding environment, it is impossible to accurately estimate its position. In the proposed method, three or more TPs are calculated as a ratio of distances by applying the Apollonian circle. Therefore, it is possible to estimate position without the exact transmission power of the source, and precise position estimation compared with existing methods is made possible by removing outliers. In summary, in outdoor environments, it seems feasible to adopt our algorithm to estimate location based on the Apollonian circle scheme, which provides meaningful mapping of the topography in a large area.

IV. CONCLUSION

This paper presents a GPS-free scheme for outdoor localization. To overcome limitations caused by RSSI uncertainty, we describe a novel RSS-based outdoor location estimation method. The proposed scheme, based on Apollonian circles and RANSAC, improves upon both the accuracy and performance of conventional methods, particularly in complex environments. Additionally, it requires neither knowing the exact transmission power of the source nor any performing any calibration procedure. We verified our approach using computer simulation and practical experimentation, finding that the proposed algorithm has a considerable advantage in real-world precision and efficiency.

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