Study of Search Optimization Opportunities of Heuristic Algorithms for Solving Multi-Extremal Problems

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***Abstract*— The investigated objective of this paper is the search optimization task of multiextremal objects, which is considered to be more complicated than the optimization tasks of mono-extremal objects. This work postulates that in order to achieve this goal, the heuristics algorithms are the only ones able to provide suitable solutions. Therefore, 3 of the most popular and devised approaches have been considered: (1) the method of swarming particles, (2) evolutionary-genetic approach and (3) ant algorithm. The conducted research has established the common test environment for comparing the multi-extremal Rastrigin function, with the 3 investigated methods. It is clearly shown that all of these 3 methods are quite appropriate for solving the multiextremal tasks. However, in each of the addressed heuristic algorithms, we have applied their own specific characteristics to solve the problem of detection and identification of the global and local extrema. These approaches have been combined together due to the general need of data clustering. It is illustrated that, when solving an extremal task, each of these methods can provide the desired solution for a fairly wide range of imposed accuracies and available resource times.**

Keywords: searching optimization; multi-extremal; genetic algorithm; swarm algorithm; ant algorithm

I. INTRODUCTION

The most advanced state-of-the-art issues in science, technology, economics, military affairs and other applied modern trends are somehow connected with the solving tasks of achieving an optimum in designs, technologies, models and environments, through the possibility of controlling the dynamic and static states, as well as, other requirements put forward in the specifications of the design objects. In other words, the developers have to solve the problems of searching optimization (SO) [1][2][3]. It is very typical that most of the current known SO methods are developed and effectively used to find only one extremum, which is often the global one [3][4]. However, many design tasks in solving complex technological systems and transportation problems require optimization. Especially, the objects of discrete nature are characterized by multiextremal (ME) properties [4][5][6][7][8][9][10][11]. A significant distinctive property for solving such tasks requires specific methods to reach the solution. It is unlikely that these methods should be sought in the class of the SO deterministic methods, though such attempts are already well known. These methods are too sensitive to the sign variation of discontinuous functions within their continuum response factor spaces. However in the discrete factor spaces, they are described as the NP-complete algorithms. For solving real optimization problems, it has been common to apply methods marked “heuristics”. These methods are, according to the authors, the most perspective to obtain solutions for the multiextremal problems [5][6][7][8][9][10][11].

A. *Formulation of the problem*

As mentioned above, the motivation is to research the most common heuristic SO methods in an environment of a more typical, universal and complex ME problem. The performed research revealed the possibility of finding, some or all, extremes by applying each of the chosen methods. Along with this qualitative evaluation, it is necessary to numerically assess the accuracy of determining the extremes values, as well as their coordinates. Therefore, in the first stage of this research, we suggest to choose the ME test function that might provide a common environment for all the methods when solving ME tasks. In the second stage of this research, the exact heuristic approaches are chosen, which determine both, the well-known methods of solving ME tasks, and their implementation algorithms.

B. *Choosing multiextremal test function and a preliminary analysis of its properties*

The most common and effective test functions for developing and analyzing the SO methods are the Rosenbrock, Himmelblau and Rastrigin functions. The Rastrigin function (RF) is the most applied ME function between all of them. This universal function is not convex, and already proposed in 1974 by Rastrigin [12]. The equation of N function arguments is:

$$f(x) = A \cdot n + \sum_{i=1}^{n} \left[x_i^2 - A \cdot \cos(2 \cdot \pi \cdot x_i) \right], \quad (1)$$

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where: x=$(x_1,\ldots,x_n)^T$ – vector; A=10.

The global minimum of this function is at the point (0,0)=0. It is difficult to find a local minimum of this function, because it has many local minimums. The isolation and evaluation of extremais a complex task.

In Section 2, the 3 most popular approaches of finding the set of extremaproblem are discussed for the 2-dimensional Rastrigin function. Section 3 describes the related work. In Section 4, the conclusion of the conducted research is given.

II. SELECTING A GROUP OF HEURISTIC METHODS

In this article, the authors settled on 3 most relevant tasks that are common in practice, when solving various search optimization tasks.

A. RF using swarming particles method

The essence and reasons in using the method of swarming particles (MSP) in SO tasks are well known [13][14][15][16][17]. The classic MSP algorithm simulates the real behavior patterns of insects, birds, fishes, many protozoa, etc. However, ME objects require to know some specific properties of this algorithm.

The authors [18][19][20] and other students of R. Neudorf [7][8][9][10][11] have significantly reworked the canonical MSP version. In particular, a new modified version of this algorithm was developed for solving the ME tasks, which is based on a model of the mechanical principles of the particle movement, and complemented by the mechanisms borrowed from the biological laws, as well as, the method of adaptation mechanisms, as property of the ME task.

The Mechanical Movement Model (MMM) of particles [20] in MSP was significantly modified and refined:

$$X_{ti} = X_{(t-\Delta t)i} + \vec{V}_{(t-\Delta t)i} \cdot \Delta t\,, \qquad (2)$$

$$\vec{V}_{ti} = \vec{V}_{(t-\Delta t)i} + \vec{A}_{(t-\Delta t)i} \cdot \Delta t\,, \qquad (3)$$

$$\vec{A}_i = \vec{A}_{pi} + \vec{A}_{tri}, \qquad (4)$$

where: $X_{(t-\Delta t)i}$ – i-th particle previous position; X_{ti} – i-th particle current position; V_{ti} – i-th particle velocity at the current time; $V_{(t-\Delta t)i}$ – i-th particle current velocity; $A_{(t-\Delta t)i}$ – particle previous acceleration in previous time; Δt – integration interval; A_{pi} – acceleration caused by the particles biologically action attractive forces; A_{tri} – slowing under the action of friction forces.

To improve the searching properties, the stochastic blur parameter was introduced:

$$\lambda^{\xi}(\varepsilon) = \lambda \cdot (1 + 2 \cdot \varepsilon(rnd(1) - 0.5))\,, \qquad (5)$$

where: $\lambda^{\xi}(\varepsilon)$ – fluctuating parameter value by tact; ε – distorted relative deviation parameter from nominal value; $rnd(1)$ – random number in the range [0, 1].

For particles, the natural clustering mechanisms were proposed and tested:

- gradient, based on particles sensitivity to change the velocity changing sign [9][10][11];
- potential, based on the introduction in MMM attractive forces at all local extrema, which detected by swarming and scanning the search space:

$$\vec{A}_{pi} = \vec{A}^{G}_{pi} + \vec{A}^{L}_{pi} + \vec{A}^{C}_{pi}, \qquad (6)$$

where: A^{G}_{pi} - particles attraction to global extremum; A^{L}_{pi} - particles attraction to the local extremum; A^{C}_{pi} - particles attraction to the cluster center.

The ME MSP algorithm showed good selectivity by localization extremaareas, but the clustering and clusters localization of mechanisms (finding values of extremaand their coordinates) require substantial structural and parametric improvements.

This mechanism does not break the swarm into multiple clusters. It only saves the particles position that entered into the cluster. This allows the particles to be attracted, at all times to the global, local and cluster extrema, and does not stop them within their cluster.

In this research, we have introduced and verified the following modifications:

- mechanism for dropping out "bad" clusters was introduced by certain criteria (the worst for a given number of iterations);
- mechanism for combining similar clusters was introduced at each step;
- parametrical settings were introduced for conditional attraction to the nearest cluster center;
- clusters areas localization mechanism and a mechanism finding local extremaparameters in them were added.

The modification of the dynamic clustering mechanism allows reducing the time and increasing the search accuracy. However, in the follow up studies the authors suggest a modification, which might reduce the processing time to drop out the "bad" cluster's members and to combine the clusters by several criteria.

The testing modifications effectiveness was carried out for RF in coordinate range $(x,y) \in [-1.5, 1.5]$. In this area, RF has 9 local minimums, including one global. Fig. 1(a), 1(b) and 1(c) show extremaareas localization process and creation corresponding clusters.

Fig. 1(a), 1(b) and 1(c) show that the particles are initially attracted to the resulting cluster, which is located in the global extremum area. This is due to the overall prevalence of the global attraction power over the local forces of attraction. Some peripheral particles might find the

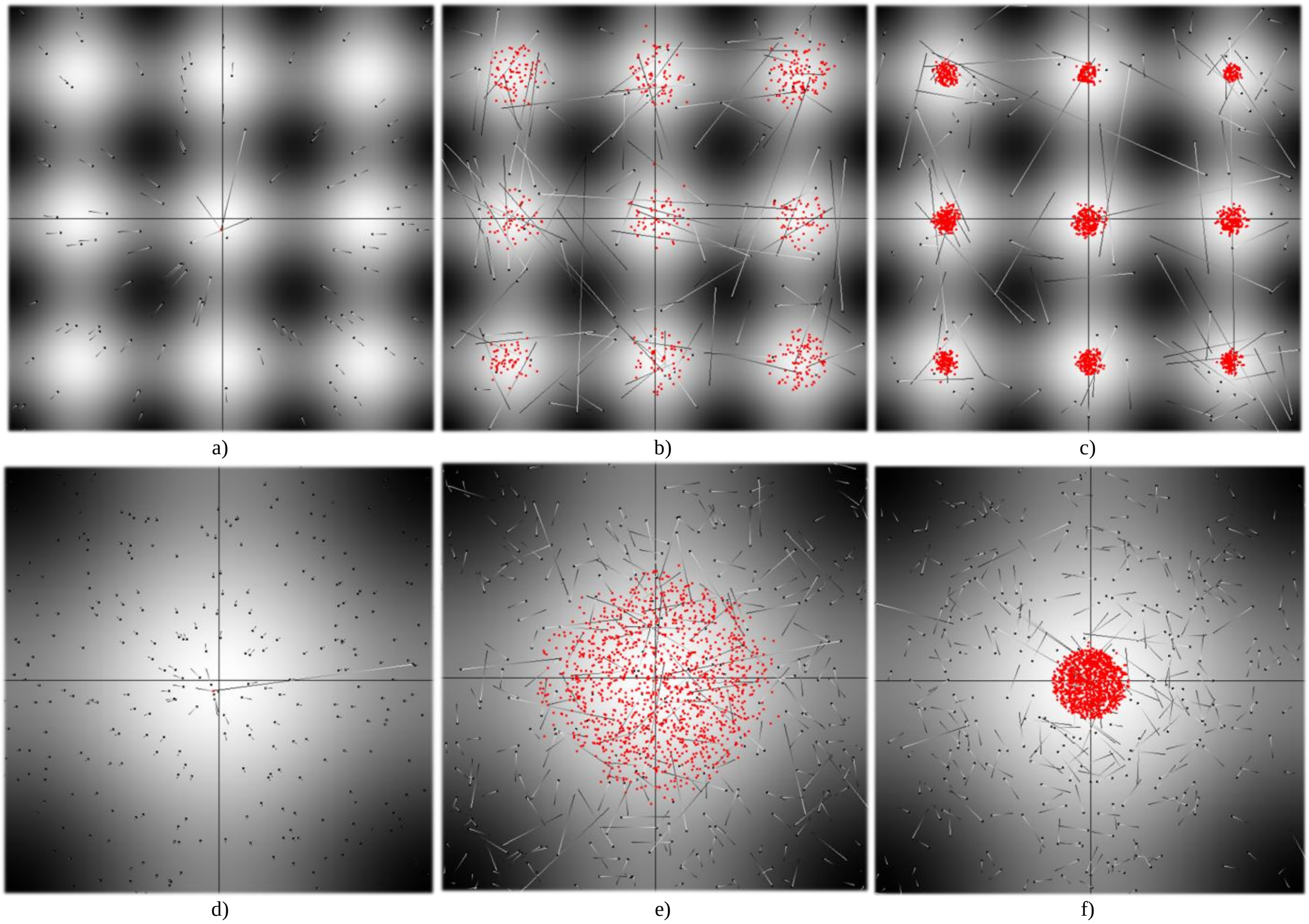


Figure 1. Extremaareas localization of (a – the 1st iteration, b – the 15th iteration, c – the 50th iteration). Local identification of one local extremum of (d – the 1st iteration, e – the 33rd iteration, f – the 50th iteration)

local extrema, which are attracted to them, and gathered in clusters. In strict clusters areas, the ME MSP algorithm (in case of having less isolated and significant extrema) is repeated. This process is iteratively repeated until the desired accuracy of the local and global extremaparameters is achieved.

Within an unlimited time for fulfilling the algorithm of each cluster, a quite stable dynamic equilibrium of particles is set. The calculations, for the modelling activity, make obvious, that the average number of the particles is correlated with the value of the extremum. The degree of correlation depends on the ME MSP algorithm settings.

In order to improve the accuracy of any extremum parameters estimation, the repetition of ME MSP algorithm, for the contracted areas of the defined clusters, is applied. This process can be iteratively repeated until the desired accuracy is achieved in respect to all the local and global extrema.

The examples in Fig. 1(d), 1(e) and 1(f) demonstrate the fragments of the iterative identification of the local extremum, which is located at the point [-1, 1]. TABLE I shows the results obtained by the localization in all areas. The table presents the coordinates $x=x_1$ and $y=x_2$, and the RF values obtained by applying the equation (2). The increase of number of iterations (and the search time) increases the estimation accuracy.

TABLE I. RESULTS OF THE EXPERIMENT

Standard			**Extremum evaluation item**		
x	*y*	*f (x, y)*	*Coordinates*		*Value*
			x	*y*	*f (x, y)*
-1	1	2	-0,9957	0,9953	1,9901
-1	0	1	-0,9949	0,0001	0,995
-1	-1	2	-0,9951	-0,9947	1,9899
0	1	1	$-3{,}20 \cdot 10^{-5}$	0,9948	0,995
0	0	0	$9{,}85 \cdot 10^{-5}$	$-6{,}49 \cdot 10^{-6}$	$1{,}94 \cdot 10^{-6}$
0	-1	1	0,00017	-0,995	0,995
1	1	2	0,9949	0,995	1,9899
1	0	1	0,995	0,00011	0,995
1	-1	2	0,9952	-0,9951	1,9899

Thus, we can conclude that ME MSP is an effective tool for solving the ME tasks.

B. *RF using evolutionary-genetic algorithm*

The Evolutionary-Genetic Algorithm (EGA) is one of the most popular tools for solving optimization tasks [21][22][23][24]. The structure and basic operators of EGA are well known, and the specific parametric features depend on the application. In particular, the use of EGA for solving ME tasks [25][26][27][28] requires the addition of the classic EGA with the tools of extremaselection by type (max or min), by the value and by the object's coordinates in the factor space. This paper develops the approach for the extremaselection, based on the use of one sample Student's t-test [27][28][29]. Its essence lies in the consistent use of EGA with further clustering values, which are obtained in its generations of the final results. Latter on, they are separated, as coordinate groups, in order to test the null hypothesis for each of them.

The arithmetic model of the present clustering method involves the consistent comparison of vectors with the middle value of vectors group $v=\{\Delta v_i=v_i-v_0 \mid i \in\{1;n\}\}$, where n – quantity of vectors. Regarding the specified probability, the decision on the set membership of vector v is taken into account. To clarify, for the set membership, it is necessary to calculate the average value of the group vectors lengths, used for their comparison:

$$\Delta v = \sum_{i=1}^{n} \Delta v / n . \quad (7)$$

Then, we calculate the standard deviation of the vectors lengths of already identified cluster:

$$S_{\{\Delta v\}} = \sqrt{\sum_{i=1}^{n} (\Delta \bar{v} - v)^2 / (n-1)} , \quad (8)$$

$$S_{\{\Delta v\}} = S_{\{\Delta v\}} / \sqrt{n} . \quad (9)$$

Using the calculated values, the experimental values of one-sample Student's t-test are counted:

$$t_0 = \left| \Delta \bar{v} - \Delta v \right| / S_{\{\Delta v\}} . \quad (10)$$

If the determined experimental t_0 value does not exceed the table t_r value [30] at n freedom degrees and a chosen level of confidence probability P, we can assume that t_0 belongs to this group of objects.

This method has proven to be well adapted to study the ME dependencies [27][28][29][30][31].

By following this approach, the algorithm and respective software tool (ST) were developed. By using ST, we studied RF in the same range, as we described in the previous section of this paper, while investigating RF by the algorithm of the swarming particles.

The structure of the EGA parameters input, which was used in the RF investigation, includes the generations of EGA individuals = 10, in each generation = 1000, the probability of crossover = 95% and probability of mutation = 30%. It should be noted that the search area parameter is the same, as in the previous section, and the accuracy of the research in this area = 7 digits, after the decimal point. Consequently, it becomes possible to allocate 9 clusters, whose minimums can correlate with those that came into the study area.

Fig. 2 shows the graphs of sequential detection of RF values and their different coordinates (X and Y), as well as the corresponding values of the objective function ($F(X, Y) \approx 2$), which are in descending order (for clusters formed around minimums with (-1,1), (1,-1), (1,1), (-1,-1) values).

Eight peripheral clusters characterize local minimums, and the central cluster contains the results approaching the global minimum of the function (see Fig. 3(a). It clearly shows, that close (and in some cases equal) to the value of the function objectives, which contain significant differences in the coordinate parameters (i.e., parameters of the objective function, that provide values close to the minimum are unlike). The ME of the studied object confirms this fact.

The results of the function study in terms of global and local minimums are shown in TABLE II. Their actual values are written down under the data in the table. The values of the objective function, adjusted for the second iteration, as well as their corresponding coordinates are shown in TABLE III.

Based on the data presented in TABLE II, it can be mentioned that extremavalues and their coordinates are not very accurate.

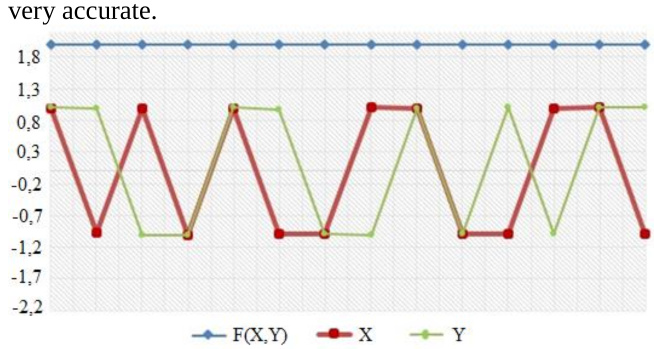


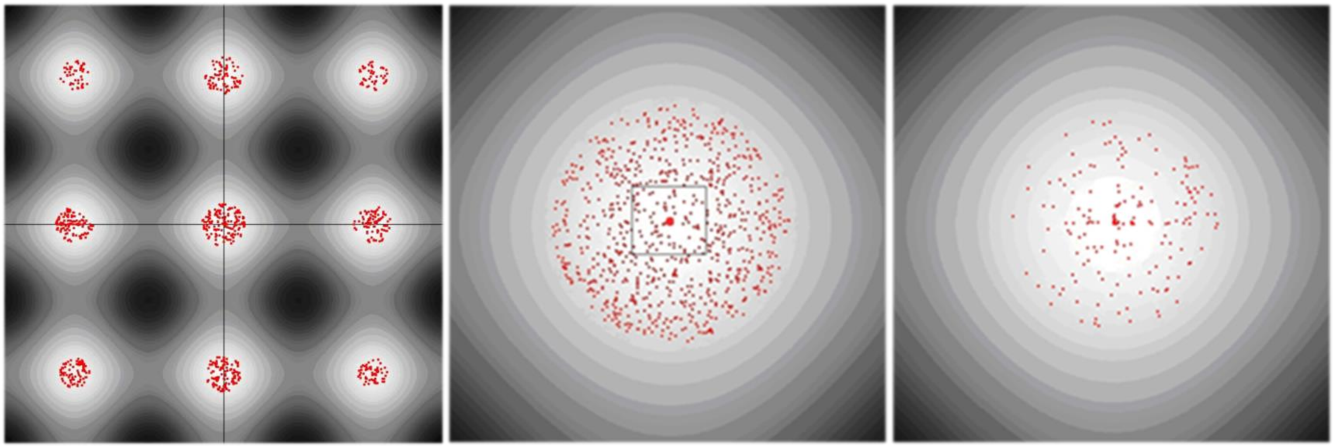
Figure 2. Clusters allocation in the experiment.

TABLE II. FOUNDED PARAMETERS OF THE OBJECTIVE FUNCTION AT THE FIRST ITERATION (IN CLUSTERS)

Standard			**Extremum evaluation item**		
x	*y*	*f(x,y)*	*Coordinates*		*Value*
			x	*y*	*f(x,y)*
0	0	0	0,00188	0,00015	0,00071
-1	0	1	-0,98932	0,00073	1,00137
0	-1	1	0,00824	-1,00043	1,01436
1	0	1	0,99948	-0,01328	1,03398
0	1	1	0,01403	1,00665	1,0611
1	-1	2	0,99146	-0,99314	1,993
1	1	2	0,99702	1,00467	2,00947
-1	-1	2	-0,9897	-1,01397	2,06707
-1	1	2	-1,00528	1,00077	2,01775

TABLE III. FOUNDED PARAMETERS VALUES OF THE OBJECTIVE FUNCTION AT THE SECOND ITERATION

Values	**2nd cluster**	**4th cluster**	**8th cluster**
x	-0,999996	0,999992	-0,999996
y	0,000005	-0,000008	-1,000005
f(x,y)	0,999997	0,999989	1,999991



a) b) c)

Figure 3. Extremalocalization areas of (a - the selected RF clusters). Forming clusters in the localized region of (b - the 100th generation, c - the 110th generation).

If the obtained values are not satisfying the required accuracy, we can find a value that is more accurate in the next RF minimum, and which is found within each cluster. The evidence of this, as example is presented in the research of 2nd, 4th and 8th cluster, according to Table II. The authors have developed the approach to localize search in the extreme areas [29]. This approach is based on EGA using [27][28], where close minimum values of second cluster can be observed in Fig. 3(b)), with the best extreme estimation highlighted by a red circle. By using the localized search, another search was carried out in the area around the highlighted extreme (see Fig. 3(c)).

C. RF using ant colony optimization

The Ant Colony Optimization (ACO) is another group of methods used in solving different optimization tasks. The ACO distinctive feature is that the key behavioral characteristics of real ants are simulated [32]. Commonly, ACO is mostly applied to minimize the path in graph tasks [33], but the given algorithms also show good results in other domains [34][35]. In this paper, the classical ACO is applied to optimize ME RF benchmark task [12].

The described method, based on the classic ACO realization, is applied for solving graph problems [35], however with some additions.

Similar to the classic ACO, in this modification, we can distinguish such steps as “initialization and arrangement”, “moving of ants”, “updating of pheromone” and “breakpoint checking”.

For example, a RF fragment for the range $(x,y) \in [-1.5, 1.5]$ was examined. It is divided into nxn fragments, each of them associated with function value in the center and some pheromone level. The specified amount of ants is placed on each fragment. As all the fragments are equal, the size of the fragment can be calculated by the following formula:

$$m = (X_{\max} - X_{\min})/n\,. \tag{11}$$

Thus, the set of fragments is defined by the matrix $B=(I_{i,j})^{n,n}_{i=q,j=1}$.

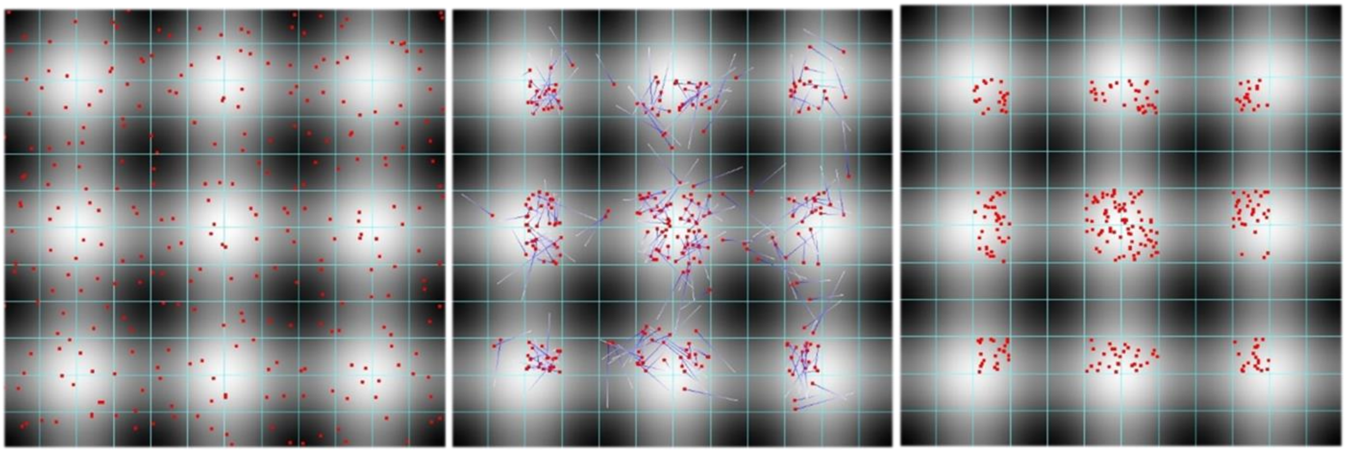
When an ant is moving from fragment $I_{i,j}$, it is calculating the moving probabilities towards the adjacent fragments, by using the following formula:

$$P_{ij,k}(t) = \\ = \begin{cases} \forall f(x_{i,j}, y_{i,j}) > f(x_{i+1,j+1}, y_{i+1,j+1}) \to Q^*, \\ \forall f(x_{i,j}, y_{i,j}) \le f(x_{i+1,j+1}, y_{i+1,j+1}) \to 0. \end{cases} \tag{12}$$

where: $Q^*=Q(\tau_{i+1,j+1}, \tau_{ij}, \eta_{i+1,j+1}, \eta_{ij}, \alpha, \beta, t)$ - dependence function of pheromone number in fragments $\tau_{i+1,j+1}$, τ_{ij} on the algorithm parameters within the task. In the quality of the function algorithm, we consider: $\eta_{i+1,j+1}$, $\eta_{ij} = |f(x_{ij}, y_{ij}) - f(x_{i+1,j+1}, y_{i+1,j+1})|^{-1}$ - weight (virtual distance) between 2 fragments; α - pheromone influence changeable coefficient; β - weight influence changeable coefficient; t - iteration number.

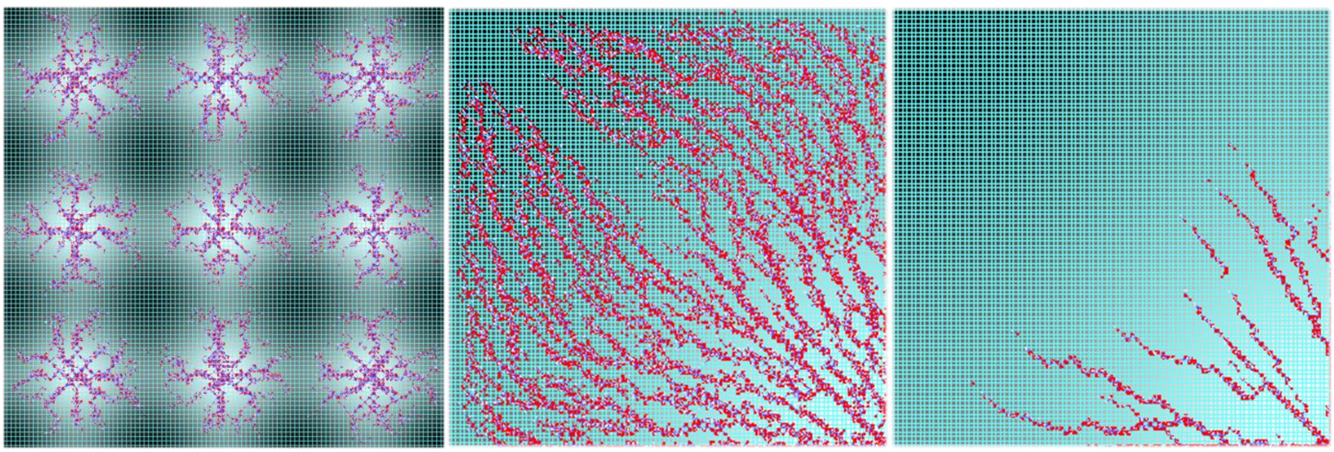
On the basis of the described algorithm and the model (see (11) and (12)) a software tool (ST) that implements the search of local and global extremawas developed. As an example, Fig. 4(a), 4(b) and 4(c) show the search results of RF global and local minimums. To solve the task, RF search area borders similar to the ones accepted in the previous sections were selected. The selected area was initially divided with a step of 0.25, and 2 ants were placed on each fragment. Coefficients are α=1, β=0.5, ρ=0.5, K=1 and τ=1. Fig. 4(a), 4(b) and 4(c) shows separate stages of work for ST.

For example, let us consider the higher area of subdivisions with the smaller fragments. This algorithm is iteratively applied to the localized fragments until the required accuracy is achieved. The operation results of ACO are presented by Fig. 5(a), which shows the obtained results of localization and two clarifications of global extremum with division into 100x100 fragments situated at point (0, 0).



a) b) c)

Figure 4. Visualization of software work stages of (a – initialization, b – the 3^{rd} iteration, c - the final result).



a) b) c)

Figure 5. The results of the location and clarification in the 100x100 division area of (a - selection of all local extrema, b - the selection of the best local extremum, c - clarification of the global extremum).

The localization resulted in 4 extremain 4 central fragments (see Fig. 5(a)). In their centers, distant 0.015 apart from the point (0,0), the value of the function is equal to 0.089210707938399. This enables to suggest that the extremum is located at the joining point of these fragments. For more accurate solutions ACO is applied for one of the fragments $x \in [-0.03, 0]$, $y \in [0, 0.03]$. Fig. 5(b) shows that the ants tend to move to the lower right corner and accumulate in the fragment $x \in [-0.0003, 0]$, with assessed value $8{,}92764330373552*10^{-6}$. The ACO application (see Fig. 5(c)) specifies the value of the extremum to be $8{,}92761420345778*10^{-10}$, and its coordinates to be $1{,}5*10^{-6}$.

D. Computational resources and performance

Searching the extremaby swarming particles, evolutionary-genetic and ant colony algorithms on 2-dimensional Rastrigin function is carried out on a PC with processor AMD Phenom II P960 with 6 GB of RAM.

To achieve the accuracy 10^{-3}, the time was in range 20-100 sec. Additional search within each area was required in range 20-110 sec. for one area.

III. Related Work

In the design optimization process, we are often confronted with problems facing the multiextremal conditions. Such situation requires decisions, which take into consideration several identical or close extrema, and the best choice in-between them has to be made. The classical theory of schedules gives examples, where several identical optimums and identical suboptimums, close to them exist [2][3][4][7][8][36]. The majority of discrete, integer and combinatory programming problems differs in such property [37][38][39], in particular, when finding solution for graphs [40][41][42][43]. The finite number (though very big) of admissible decisions requires considering the multiextremal solutions for the discrete environment optimization. It is important to have a complete solution of the multiextremal task, because the criterion is usually a numerical expression related to the optimized object. However, there are many additional conditions, which can help to choose the extrema, equivalent or close in size, and satisfy both, the numerical criteria estimates and the heuristic ideas. Therefore, the choice, of the most effective methods and algorithms, is an extremely important step to find such solution of the multiextremal task.

However, not all the search methods provide the successful solution for the multiextremal task. It is well known that the determinate methods are sensitive to the sign-variable, so-called "gullied" surfaces, which define the real variables in the factor space. The solution of discrete tasks by such methods leads to the nondeterministic polynomial, in order to define for the complete problem in time. The methods of the accidental search are poorly predictable, since it is impossible to control the time expenditure, and even the basic decision, which heuristic method to apply, when having a real search optimization problem. In particular, in Russia, in the last years, the quite intensive research is conducted, to find appropriate solutions for the many optimization problems. Among these methods, it is important to mention the swarming particles algorithm [13][14][15][16][17][18][19][20] the ant colony algorithm [31][32][33][34][35] and the evolutionarily genetic algorithm [21][22][23][24][25][26][27][28]. These algorithms were investigated, as the traditional optimization tasks, and in relation to find the solution of the multiextremal tasks. For the last case, they have been significantly modified, by experimenting with different heuristic methods, which research was conducted earlier by the authors [9][10][11]. Therefore, the presented work brings forward a peculiar theoretical result, and trace the roadmap for the future research in this direction.

IV. Conclusion

The analysis of the application of the 3 heuristic algorithms for solving the ME tasks showed that these methods are efficient, effective, and bring some essential features to the described solutions.

The specific approaches to solve the task for each of these particular cases is determined through the analysis of the algorithms features; the detection and identification of local extrema, clustering methods and subsequent operations for the results analysis. However, in all these cases the modifications of algorithms is connected with the data clustering necessity, which was proved to be essential. Also, all the methods showed adequate performance.

To conclude, all the 3 methods, studied in this paper, are considered to be relevant and promising for future applications. The specific choice of the algorithm tool for solving ME tasks depends on the experience and personal researcher preferences, as well as on the special features of the subject research area.

In this paper, the task of finding the set of extremafor 2-dimensional Rastrigin test function was examined. In future research, it is advisable to study the problem of higher dimension (3 or more) in order to assess the impact of algorithms' parameters on time and search accuracy, and to enable algorithms modifications for the mathematical models of any problem dimension.

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