Performance Evaluation of MIMO Detectors over Impulsive Noise

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***Abstract*—Multiple-Input and Multiple-Output (MIMO) is expected to be one of the most crucial technologies towards the 5G mobile communication systems and beyond. The understanding of the performance and limits of MIMO detectors is essential in order to transmit signals at high rates and with high reliability. In this paper, we present an evaluation of MIMO detectors over non-gaussian impulsive noise. The traditional MIMO detectors are designed assuming noise modeled as gaussian second-order statistics. However, many works have presented non-gaussian impulsive noise in different MIMO scenarios degrading the detector performance. Also, we investigate an alternative to symmetric α-stable distribution to model impulsive noise called the gaussian mixture model. The simulation results show that the Symbol Error Rate (SER) performance depends on not only the quality of the signal but also the impulsiveness level of the noise.**

***Keywords*—Impulsive noise, non-gaussian model, alpha-stable distribution, GMM.**

I. Introduction

Multiple-Input and Multiple-Output (MIMO) technology has been receiving considerable attention recently from the wireless communication field. Nowadays, wireless systems are demanding higher data rates with reliability, being efficient in terms of bandwidth. In this context, MIMO plays a key role in achieving highly efficient spectrum usage with a relatively small number of antennas involving large amounts of data. Thus, MIMO techniques have been investigated by researchers and engineers in several contexts, such as 4G and 5G networks, distributed antennas, heterogeneous network, IEEE 802.11ac and millimeter-wave impacts due to its high frequency [1].

Authors argue that the performance of wireless communication systems is mainly governed by wireless channel characteristics [2]. Measurement and environmental conditions, such as multipath and noise create additional difficulty within already existing detection challenges faced by MIMO systems. Especially for classical MIMO detectors, which rely on second-order statistical noise assumptions, they may suffer severe impact via meaningful degradation in non-gaussian scenarios [3]. Thus, one way to improve the reliability of MIMO systems is by analyzing undesirable effects of channel and noise, thereby evaluating MIMO detectors while considering realistic models. Notably, characteristics of impulsive noise have been modelled accurately by non-gaussian processes [4], demonstrating better fitting than gaussian model in several scenarios due to man-made and electromagnetic interference noises. Also, studies have investigated the presence of non-gaussian noise components in millimeter wave scenarios [5]–[7] at high frequencies.

Several statistical models have been proposed to describe non-gaussian impulsive noise. In particular, stable distributions is one of the most used ones for this purpose [4]. They offer more freedom degrees than the gaussian model by adjusting free distribution parameters, allowing us to describe how impulsive the noise is. This model has been explored in many different communication scenarios, such as acoustic channels [8], wireless communication solutions [9], and satellite communications [10]. Moreover, the α-stable model presents relevant properties for noise modelling such as generalized central limit, stability property, and heavy tails [4].

Additionally, many approaches have been studied modelling impulsive noise by Gaussian Mixture Models (GMM) [8], [11]. They claim that the GMM is capable of representing heavy tailed impulsive noise by an arbitrary additive, independent and identically distributed (i.i.d.), symmetric, non-gaussian GMM noise. Moreover, the expectation-maximization (EM) algorithm for estimating the distribution parameters is a well-known tool based on maximum likelihood. Thus, we purpose the GMM as a beneficial complementary alternative to α-stable distribution to model noise in MIMO systems.

MIMO detectors based on exhaustive searching and channel estimation have been proposed with high performance if compared to traditional detectors in non-gaussian environments [3]. However, those detectors usually have too high computational complexity, making them infeasible in practical scenarios. On the other hand, the classical detectors have unknown performance in non-gaussian noise environments depending on the impulsiveness level. Therefore, the comprehension of the relationship between impulsiveness levels and the performance of detectors is crucial in the making decisions about the choice of methods.

In this article, we examine the performance of MIMO detectors in non-gaussian impulsive noise, highlighting the noise models in such technology and Monte Carlo analysis for relevant distribution parameters. This study also describes an alternative to model impulsive noise, called the gaussian mixture model, and its impact for MIMO detector evaluation. This work uses the Rayleigh fading model, which may represent realistic narrowband mmWave systems [12].

This paper is organized as follows. In Section II, we describe the MIMO system, presenting the channel and noise model. MIMO detectors are presented in Section III. In Section IV, the main results are presented and discussed, comparing the performance of the tradition MIMO detectors in non-gaussian scenarios by simulations. In Section V, we present our final

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remarks.

II. MIMO System

Consider a MIMO digital system with N_R antennas at the receiver and N_T antennas at the transmitter. The N_R antennas are spaced, such that the received signals may be considered independent of each other. The k-th symbol received by the m-th antennas is given by:

$$y_m(t) = \sum_{n=1}^{N_T} s_n(t)h_{mn}(t)p(t) + w_m(t), \quad (1)$$

where $s_n(t)$ represents the transmitted symbol from the n-th antenna, originated from a modulation scheme, $h_{mn}(t)$ represents the channel model between the n-th transmitting antenna and m-th receiving antenna, $w_m(t)$ corresponds to the channel noise, and $p(t)$ is a rectangular pulse.

We assume the time-domain channel model coefficients $h_m(t)$ as a Rayleigh distribution, being defined by

$$h_{mn}(t) = h_{mn,r}(t) + jh_{mn,q}(t), \quad (2)$$

where $h_{mn,r}(t)$ and $h_{mn,q}(t)$ are gaussian processes with mean zero and variance equal to $1/2$. We also assume that the differences in propagation times of the signals from the transmitters to the receivers are small relative to the symbol duration.

III. Impulsive Noise Model

The α-stable and gaussian mixture model are the most frequently used distributions to model impulsive noise. Those models have different characteristics presented in this section.

A. Symmetric α-Stable Model

Reasons for statistical modelling using α-stable distributions are based on crucial properties, such as generalized central limit theorem and stability. According to the generalized central limit, if the sum of independent and identically distributed random variables with or without finite variance converge, then the limit distribution must be α-stable. Another relevant property states that the sum of two independent random variables with the same characteristic exponent (α value) is also α-stable, known as stability property. Finally, we consider that the signal exhibits heavy tails and skewness, which is well represented by α-stable model.

There are different parametrizations of α-stable distribution of the characteristic function. We assume the parameters $\boldsymbol{\theta}_\alpha = (\alpha, \beta, \gamma, \delta)$ and the following characteristic function [4]:

$$\varphi(\omega; \boldsymbol{\theta}_\alpha) = \exp(-\gamma^\alpha|\omega|^\alpha[1 - j\Theta(\omega;\alpha,\beta)] + j\delta\omega), \quad (3)$$

with

$$\Theta = \begin{cases} \beta(\tan\frac{\pi\alpha}{2})(\text{sign}\,\omega), & \alpha \neq 1 \\ -\beta\frac{2}{\pi}(\ln|\omega|), & \alpha = 1, \end{cases} \quad (4)$$

where

α is the characteristic exponent such that $0 < \alpha < 2$,
β is the symmetry parameter such that $-1 \leq \beta \leq 1$,
γ is the dispersion parameter such that $\gamma > 0$,
δ is the location parameter such that $-\infty < \delta < \infty$.

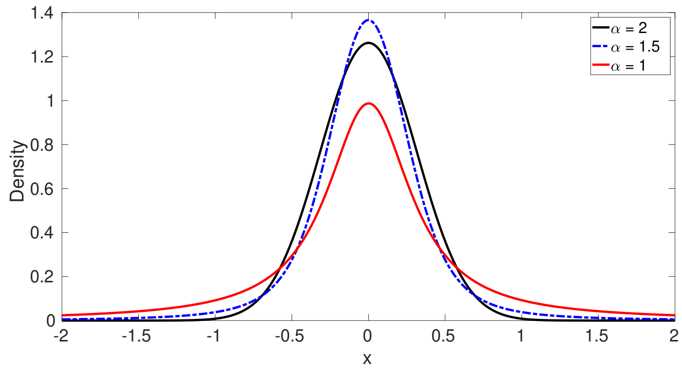

Figure 1. Probability distribution function of symmetrical α-stable with $\beta = \delta = 0$ and $\gamma = 1$.

We also assume a symmetric α-stable (SαS) class because it has proved to be very useful in modelling impulsive noise [13]. For such distribution class, $\beta = 0$ and $\delta = 0$ [14]. Figure 1 shows the α value variation representing the impulsiveness level of the distribution, where a low value of α suggests high impulsiveness and a non-gaussian behavior, and a high value of α means that the distribution is close to the gaussian behavior, which $\alpha = 2$ is the gaussian case.

B. Gaussian Mixture Model

The GMM is a linear combination of gaussian functions where the sum of all weight coefficients is equal to one. Thus, a random variable y with GMM distribution is defined by its probability density function as

$$p(y) = \sum_{i=1}^{M} c_i N(x_i|\mu_i,\sigma_i), \text{ with } \sum_{i}^{M} c_i = 1, \quad (5)$$

where c_i is the weight of the i-th Gaussian distribution function, M represents the number of Gaussian distributions in the mixture, and $\mathcal{N}(x_i|\mu_i,\sigma_i)$ is a Gaussian distribution function given by

$$N(x_i|\mu_i,\sigma_i) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{(x_i-\mu_i)}{2\sigma_i^2}}, \quad (6)$$

where μ_i and σ_i represents mean and variance, respectively, of the i-th Gaussian. Figure 2 illustrates the gaussian mixture model representing the impulsive noise, which results in a heavy tail distribution.

IV. MIMO Detectors

We consider three different detectors based on frequency nonselective MIMO channel and Rayleigh fading. Those methods are designed for recovering the data symbols with additive gaussian noise assumptions. However, in practical scenarios, those assumptions can mislead the real performance of MIMO systems making them unfeasible depending on channel estimation.

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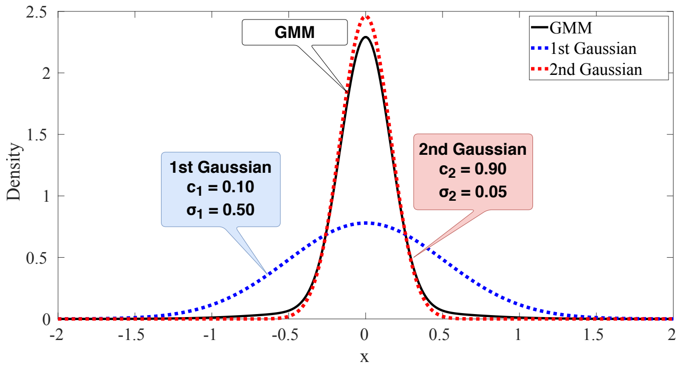

Figure 2. Probability distribution function of gaussian mixture model with two gaussians with parameters $\mu_1 = \mu_2 = 0$, $\sigma_1 = 10$, and $\sigma_2 = 1$.

A. Maximum Likelihood Detector

The maximum likelihood detection is optimum in terms of performance assuming gaussian noise model. This detector minimizes the average error probability by finding the minimum Euclidean distance. This technique requires high computational complexity due to the searching algorithm.

$$\hat{\boldsymbol{s}}_{\mathrm{MLD}} = \arg\min_{\boldsymbol{s}} \left| y_m - \sum_{n=1}^{N_T} h_{mn} s_n \right|^2, \tag{7}$$

where s_n is a symbol among a set of possible constellation symbols used in the transmission.

B. Minimum Mean-Square-Error Detector

The MMSE detector estimates the transmitted symbols based on the linear combination of the received signals. The linear combination is given by

$$\hat{\boldsymbol{s}}_{\mathrm{MMSE}} = \boldsymbol{W}^H \boldsymbol{y}_m, \tag{8}$$

where $\boldsymbol{W}$ is a weighting matrix. In order to minimize the mean square error, the weighting matrix is represented by

$$J(\boldsymbol{W}) = E\big[||\boldsymbol{s}_{\mathrm{MMSE}} - \boldsymbol{W}^H \boldsymbol{y}_m||^2\big]. \tag{9}$$

The weight vectors inside the matrix can be obtained by

$$\boldsymbol{w}_n = \boldsymbol{R}^{-1} \boldsymbol{r}_{s_n y}, \tag{10}$$

where $\boldsymbol{R}$ is the autocorrelation matrix of the received signal y_m, and $\boldsymbol{r}_{s_n y} = E[s_n \boldsymbol{y}_m]$.

C. Inverse Channel Detector

The ICD detector is similar to MMSE, where the estimation is designed using a linear combination of the received signal. However, in ICD the interchannel interference is eliminated due to the weighting matrix with $N_R = N_T$. Therefore, the estimation is given by

$$\hat{s}_{\mathrm{ICD}} = \boldsymbol{H}^{-1} \boldsymbol{y}_m, \tag{11}$$

where $\boldsymbol{W}^H = \boldsymbol{H}^{-1}$.

V. Results

This section presents computer simulation results for the performance evaluation of MIMO detectors. We examined the Symbol Error Rates (SER) for different levels of impulsiveness and quality of signal considering 2x2 MIMO systems. The simulations assessed the error rate performance based on the Monte Carlo method where each point of the SER curves employed at least 50 errors in the estimation. All simulations were performed considering baseband with BPSK modulated signal and unity energy, being the antennas statistically independent of one each other. In addition, Rayleigh flat fading was assumed as the multipath propagation model in the wireless channel.

The SER metric is usually computed versus the signal-to-noise ratio (SNR). However, the infinite variance of non-Gaussian SαS processes prevents to compute the signal-to-noise ratio as a measurement of signal quality. In this work, we use the geometric signal-to-noise ratio (GSNR) [R] instead of the SNR. The GSNR is given by

$$\mathrm{GSNR} = \frac{1}{2C_g}\left(\frac{A}{S_0}\right)^2, \tag{12}$$

where the normalization constant $C_g = e^{C_e} \approx 1.78$ is the exponential of the Euler constant (C_e), used to ensure that GSNR corresponds to SNR when the channel is Gaussian ($\alpha = 2$); S_0 is the geometric power of a SαS random variable; and A is the root-mean-square value of the signal.

For the GMM, we use two gaussians, i.e., $M = 2$, where one gaussian has much higher variance than the other one in order to represent the impulsiveness of the noise. Thus, the variances are given by

$$\sigma_1^2 = \xi \cdot \sigma_2^2. \tag{13}$$

where σ_i are the variances of the i-th gaussian, and ξ is the relationship between them, describing how different they are. We assume that the first variance has higher value than the second one, i.e., $\xi > 1$, and their occurrences are described by $c_1 = 0.1$ and $c_2 = 0.9$. In this case, the total variance is given by the weighted sum of the variances as

$$\sigma_T = c1 \cdot \sigma_1 + c2 \cdot \sigma_2. \tag{14}$$

A. Noise Model Analysis

First, we show in Figure 3 the MIMO performance in the gaussian scenario as a reference scenario indicating no presence of impulsiveness. This behavior of the detectors is expected in environments where the gaussian model describes well the noise model.

Figure 4 presents the MLD, MMSE, and ICD detectors over SαS noise with parameter $\alpha = 1.9$, a low impulsive noise scenario. The SER of detectors are clearly higher than in the gaussian case, since the impulsiveness degrades them. However, the ML detector has low SER values at high GSNR.

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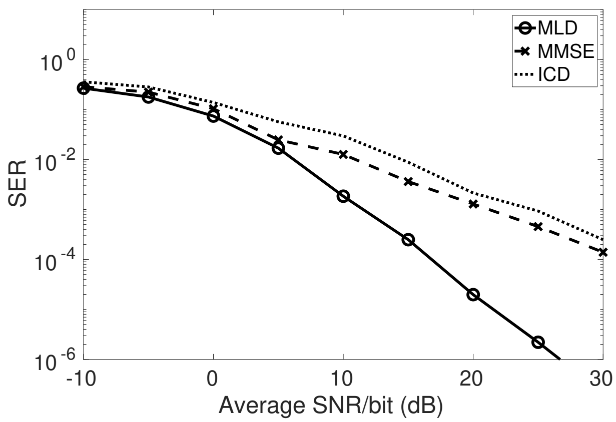

Figure 3. MIMO 2x2 over gaussian noise.

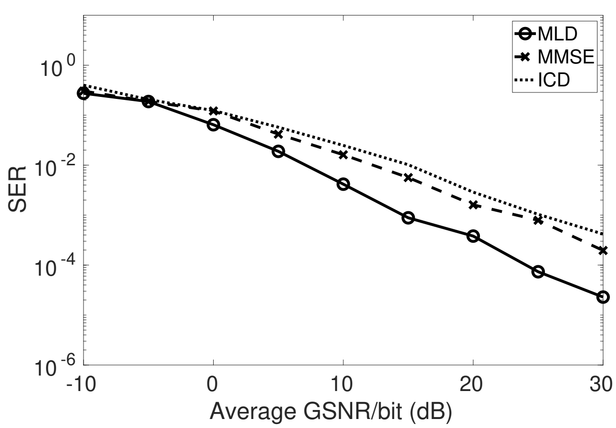

Figure 4. MIMO 2x2 over alpha-stable noise with $\alpha = 1.9$.

Figure 5 shows the detectors over SαS noise with parameter $\alpha = 1.3$. This scenario represents a severe impulsive noise where all detectors are degraded. We also visualize that MLD has a similar performance to other detectors in this scenario even for high GSNR values.

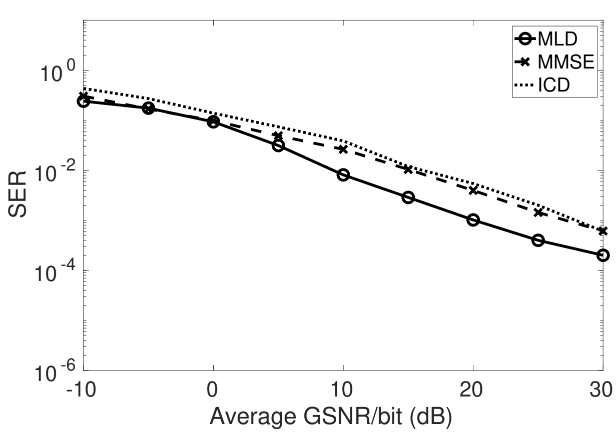

Figure 5. MIMO 2x2 over alpha-stable noise with $\alpha = 1.3$.

Figures 6 and 7 present the same detectors over GMM noise with two gaussians and means equal to zero. They have different variances, which one represents an usual class of noise with weighting of $c_1 = 0.1$, and another one represents the impulsive component with higher variance and weighting of $c_2 = 0.9$. In this scenario, the impulsiveness level is given by the relation between the variances σ_1 and σ_2. Figure 6 presents the detectors over GMM with low impulsiveness level, given by $\xi = 2$.

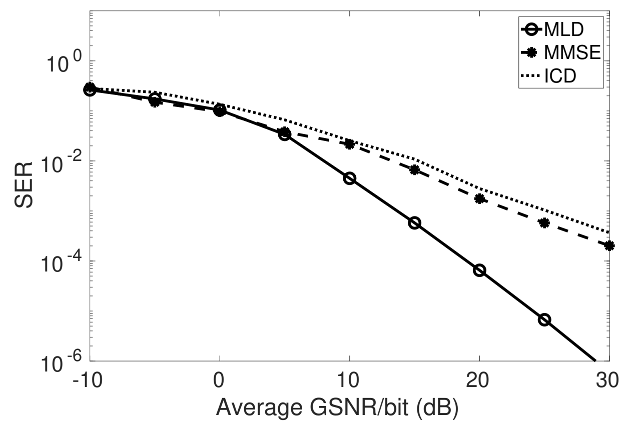

Figure 6. MIMO 2x2 over GMM noise with $\xi = 2$.

Figure 7 shows the performance of detectors over GMM with impulsiveness level given by $\xi = 10$. In this scenario, the detectors have higher SER if compared to the scenario with $\xi = 2$ due to the high impulsiveness level.

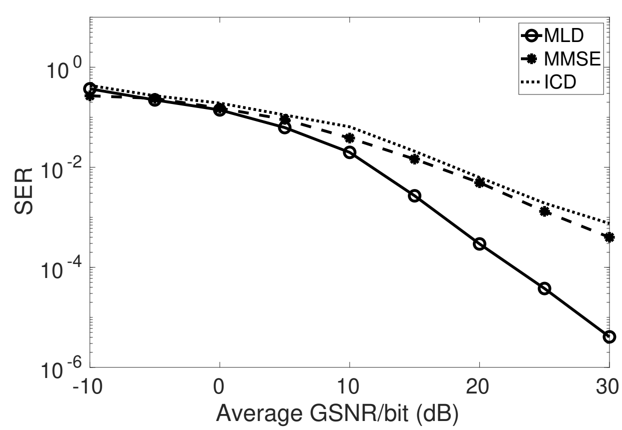

Figure 7. MIMO 2x2 over GMM noise with $\xi = 10$.

B. Impulsiveness Analysis

A crucial analysis of detectors over impulsive noise is the impulsiveness level. As the detectors are operating not respecting the gaussian assumption, then we can not affirm the exactly behavior of the system. However, we expect that less impulsive is the noise better is the performance of the detectors. So, we evaluate all detectors over the two models, SαS and GMM, evaluating their impulsiveness level. Each model has a different parameter associated, being α for SαS and ξ for GMM. We adopt a constant GSNR for each noise model and compute the SER versus different values of α and ξ.

Figure 8 presents the SER of MIMO detectors for GSNR of 10 and values of α from 1.1 (more impulsive) to 2 (gaussian case). In high impulsiveness scenario, i.e., low value of α, higher is the SER of the detectors, as expected. However, the MLD is more sensitive to the impulsiveness level than the other detectors. In addition, we can affirm that the SαS model may represent higher impulsiveness level than the GMM, in terms of the detectors performance.

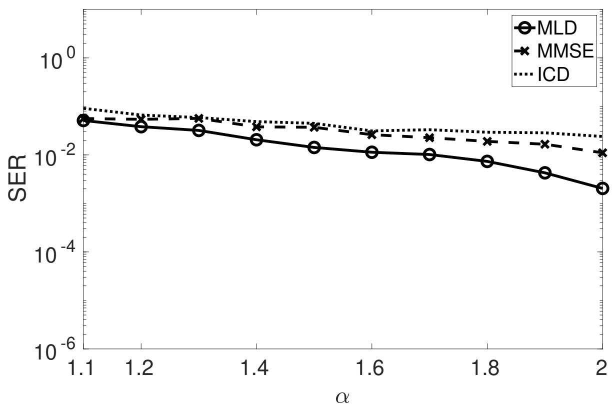

Figure 8. MIMO 2x2 over SαS with different impulsiveness levels.

Figure 9 shows the SER of MIMO detectors for GSNR of 10 and values of ξ from 2 to 20, representing the relation between the variances σ_1 and σ_2. In high impulsiveness scenario, all detectors have higher SER performance, where they are more sensitive for ξ values from 2 to 10. Also, we can note that the detectors performance degrade smoother over GMM than over SαS model.

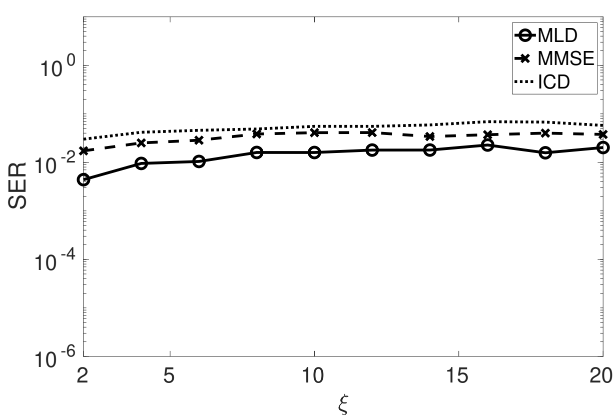

Figure 9. MIMO 2x2 over GMM with different impulsiveness levels.

VI. Conclusion

In this paper, we evaluated traditional MIMO detectors over non-gaussian scenarios for different impulsiveness levels. Indeed, the traditional MIMO detectors have high error rates in impulsive noise scenarios making them infeasible for current wireless systems. On the other hand, depending on the noise power (GSNR), the detectors work well for impulsiveness levels that are not severe. Also, depending on the model used, the detectors can be more sensitive in relation to the impulsiveness level represented by their parameters. Therefore, studies in impulsive noise scenarios must pay attention for not only the GSNR value, but also for the impulsiveness level considered and how it impacts the detectors.

Future works may investigate the Gaussian mixture model including the number of Gaussian components and its effect in impulsive noise fitting. Also, future studies may use these results to produce adaptive detectors based on impulsiveness parameters, reaching better performance than the traditional detectors.

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References

[1] K. Maruta and F. Falcone, "Massive MIMO Systems: Present and Future," *Electronics*, vol. 9, no. 3, p. 385, feb 2020.

[2] Y. S. Cho, J. Kim, W. Y. Yang, and C. G. Kang, *MIMO-OFDM Wireless Communications with MATLAB®*. Chichester, UK: John Wiley & Sons, Ltd, aug 2010.

[3] P. T. V. De Souza, A. I. R. Fontes, V. S. V. De Souza, and L. F. Silveira, "A Novel Signal Detector in MIMO Systems Based on Complex Correntropy," *IEEE Access*, vol. 7, pp. 137 517–137 527, 2019.

[4] M. Shao and C. L. C. Nikias, "Signal Processing with Fractional Lower Order Moments: Stable Processes and Their Applications," *Proceedings of the IEEE*, vol. 81, no. 7, pp. 986–1010, jul 1993.

[5] L. Shhab, A. Rizaner, A. H. Ulusoy, and H. Amca, "Suppressing the Effect of Impulsive Noise on Millimeter-Wave Communications Systems," *Radioengineering*, vol. 29, no. 2, pp. 376–385, jun 2020.

[6] H. Amca, "The Presence of Thermal, Shot, Impulsive and Flicker Noise at Millimeter-Wave Frequencies," in *2019 International Symposium on Networks, Computers and Communications (ISNCC)*. IEEE, jun 2019, pp. 1–6.

[7] O. S. Badarneh and F. S. Almehmadi, "The effects of different noise types and mobility on error rate of digital modulation schemes over millimeter-wave Weibull fading channels," *Wireless Networks*, vol. 25, no. 5, pp. 2259–2268, jul 2019.

[8] D. Pena and et al., "Acoustic impulsive noise based on non-Gaussian models: An experimental evaluation," *Sensors (Switzerland)*, vol. 19, no. 12, p. 2827, jun 2019.

[9] H. Hamlili, S. Kameche, and A. Abdelmalek, "Convolutional Code Performance for OFDM System in an Alpha-Stable Noise Environment," in *2018 International Conference on Communications and Electrical Engineering (ICCEE)*. IEEE, dec 2018, pp. 1–5.

[10] M. D. Button, J. G. Gardiner, and I. A. Glover, "Measurement of the impulsive noise environment for satellite-mobile radio systems at 1.5 GHz," *IEEE Transactions on Vehicular Technology*, pp. 551–560, 2002.

[11] R. J. Kozick and B. M. Sadler, "Maximum-likelihood array processing in non-Gaussian noise with Gaussian mixtures," *IEEE Transactions on Signal Processing*, vol. 48, no. 12, pp. 3520–3535, 2000.

[12] N. Iqbal and et al., "Multipath Cluster Fading Statistics and Modeling in Millimeter-Wave Radio Channels," *IEEE Transactions on Antennas and Propagation*, vol. 67, no. 4, pp. 2622–2632, apr 2019.

[13] C. L. Nikias and M. Shao, *Signal Processing with Alpha-stable Distributions and Applications*. New York, NY, USA: Wiley-Interscience, 1995.
[14] G. Samorodnitsky and M. S. Taqqu, *Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance*, ser. Stochastic Modeling Series. Taylor & Francis, 1994.