

# Accuracy and Predictability Analysis of a Highly Sensitive Liquid Level Prediction Setup

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**Abstract**— Liquid level measurements play a major role in many industrial applications, where different physical principles have been used and tested by many researchers. Whispering Gallery Mode (WGM) theory indicates major accomplishments in a diverse area of interest where sensitivity is of utmost importance. This study is focusing on the accuracy and predictability analysis of such liquid level prediction experiments, using WGM sensor technology, indicating a dependence of WGM shifts on liquid levels, which have been assumed as perfectly linear in all the measurement ranges.

**Keywords**- accuracy; error; uncertainty; sensor; laser optics; liquid level sensor; Whispering Gallery Mode (WGM).

## I. INTRODUCTION

Level sensors are used in automotive [8] and industrial applications [13] to gather or send valuable information to ensure that the level (of the engine oil in our case) does not become dangerously low or high without being noticed. The sensor monitors the oil level continuously during the entire engine operation. Secondary influences, such as the slope of the vehicle's lateral and longitudinal accelerations are compensated by the vehicle control unit calculating a mean value [2][8].

While searching for methods to detect the liquid levels accurately, a new laser optics sensor technology [3]-[5], based on the previously demonstrated Whispering Gallery Mode phenomenon [1] has been observed, pointing superb sensitivity over the experimental range, and finally used in our experiments for the detection of liquid levels. The experimental outcomes obtained are further processed, and the accuracy and uncertainty analysis of these liquid level prediction measurements are presented in Section 3. The results showed a linear dependence within the experimental range, and uncertainty calculations are shown in this section as well. Finally, in Section 4, general conclusions are given according to the experimental results, for the benefit of future studies.

## II. EXPERIMENTAL DESIGN FOR THE LEVEL MEASUREMENTS

In the Whispering Gallery Mode theory, developed by John William Strutt (Lord Rayleigh) [11], where light undergoes total internal reflection, and because it is trapped inside the sphere, WGMs are observed under certain conditions. The details of the theory, and verification of the existence of a strong relation between changes in WGM resonance shifts and force applied to a micro optical-sphere was presented by Ioppolo et al. [3]-[5]. The theory is based on a phase delay occurring during light travel inside a microsphere [11].

Figure 1 presents the experiment setup in the laboratory. The light coming from the laser goes inside the microsphere, where the measurements are exactly taken [3]-[5].

The microsphere inside the sensor, is directly in contact with the fiber optics, and works as an input and output channel for the information, while facilitating a light coupling between fiber and resonator (see Figures 2-4). This light coupling creates resonances in return signal according to the Whispering Gallery phenomenon [1]. Tracking the changes and shifts of these resonances demonstrates the basics of this new laser optics sensing technology [3]-[5] we have used in our measurements.

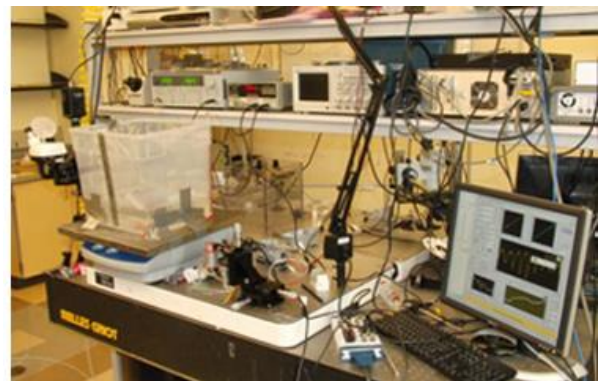


Figure 1. The experimental setup used having the sensor located inside the water container, and a pressure transducer for reference measurements [2]. WGM measurements are obtained by an inhouse software [3]-[5].

A computer controlled software both the tunes the frequency of the laser and records the shifts in the return signal resonances [3]-[5]. This technology has been used to detect the liquid levels accurately.

For the liquid level detection setup, a new sensing cell has been developed to measure the pressure changes of the medium filled with water. The initial design of the sensing cell can be seen in Figure 2, where the microsphere [6][7] sensing element is completely separated from the test medium by using an elastic membrane. The membrane is able to transfer the pressure changes precisely, while keeping the liquid outside. Actual sensing cell design used in the experiments having the micro resonator [6][7] in the form of a sphere can be seen in Figures 3 and 4.

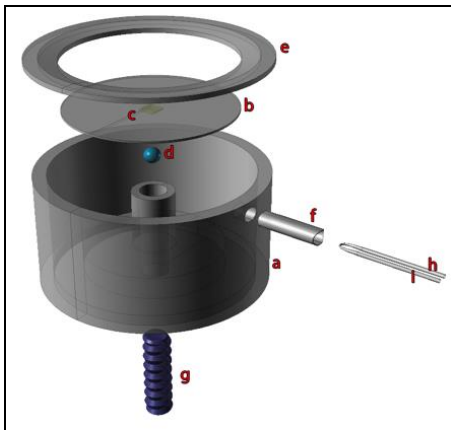


Figure 2. Initial design of the sensing cell is shown. Sensor Case(a); Latex membrane(b); Metal Boss(c); PDMS Microsphere(d); Upper cover(e); Fiber Protector(f); Screw Mechanism for leveling the microsphere(g); Optical Fiber Input(h); Optical Fiber Output(i) [2]

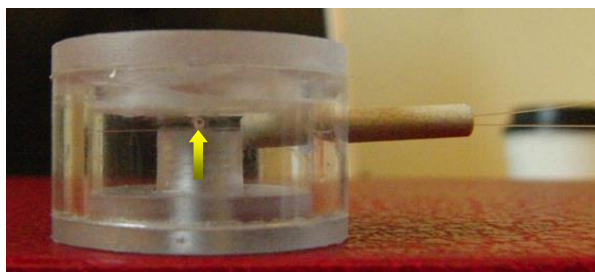


Figure 3. Actual Sensing cell includes: a membrane for force detection, Microsphere[3]-[5] being the sensing element inside, and Optical Fibers for data transfer[3]-[5].

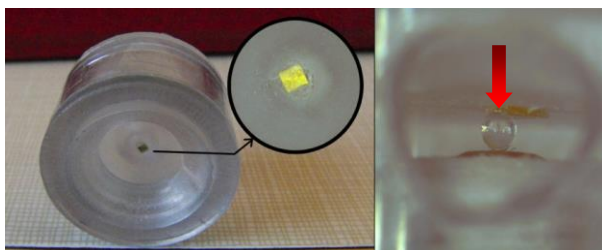


Figure 4. Metal boss added to the membrane for greater sensitivity (a) The microsphere touching metal boss (b).

As shown in Figure 3, the microsphere is in contact with the fiber outlets facilitating a light coupling, while still being able to sense the pressure changes of the medium which is being transferred by the elastic membrane. Figure 4 shows fine tuning over the actual design, where a tiny metal boss touching the microsphere, added to the membrane for greater sensitivity to transfer the pressure changes more rigidly over the microsphere.

This sensing cell is designed to be used inside liquid, therefore the experiments have been able to performed under water, shown in Figure 5. The target is to detect and track the liquid level changes, as it increases or decreases the liquid pressures according to Bernoulli's principle. A pressure transducer has also been used as the reference device, connected to the container through a U tube, tracking the changes of the pressure, in other terms the changes of the liquid levels.

### III. EXPERIMENTAL RESULTS

An experimental setup is built to measure the pressure changes of the medium, detecting the changes of the liquid height, as stated previously (see Figures 1-5) [2].

#### A. Liquid Level Measurement Results

The liquid level measurements have been performed while filling and draining water from the container – this will be called the experiment setup from now on. The red line in Figures 6 and 8 indicates the measurements taken from a reference pressure transducer [2].

In Figure 5, the experiment setup for measuring the liquid levels can be seen in detail, where the sensing cell is located inside the bottom of the liquid container design. The sensing cell is specifically designed to have a deflectable membrane to transfer the force/pressure changes into the sensing software [3]-[5] without letting the liquid enter inside the cell (see Figures 2-4). Figure 5 shows the experiment setup for liquid level detection in greater detail.

When the liquid level increases, it also increases the pressure in the U tube connected to the pressure transducer, which results and increase in its signal. Also, when draining water, the pressure in connection tube decreases, resulting in a decrease in the Pressure Transducer signal.

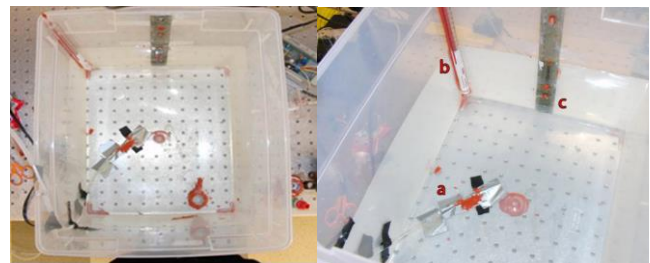


Figure 5. Water pressure change experiments (left), (a) Liquid inlet and outlet, (b) Pressure Transducer connection, (c) Scaled Ruler for instant Level Readings (right).

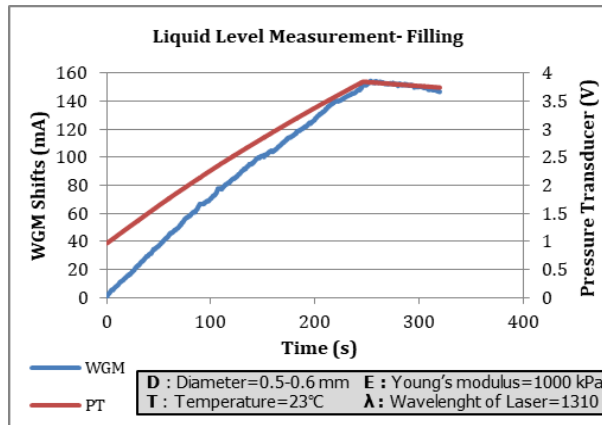


Figure 6. Liquid Level Measurements-Filling

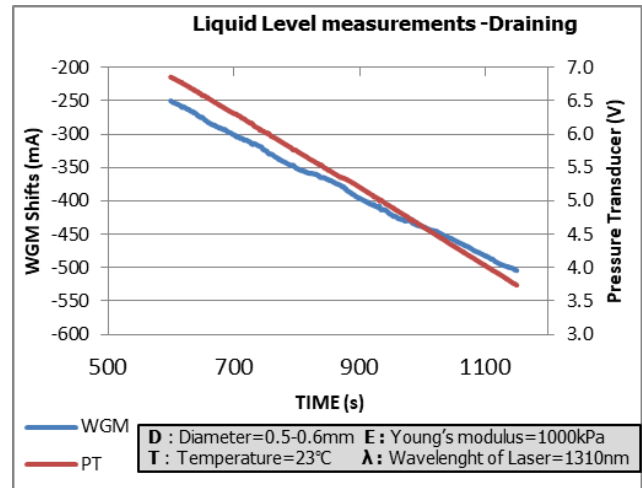


Figure 8. Liquid Level measurements while draining water from the setup

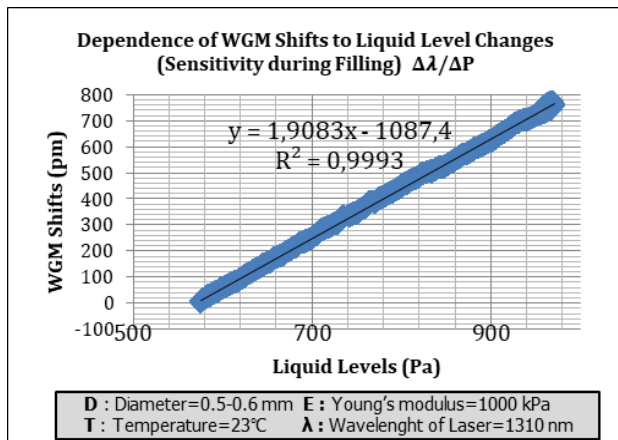


Figure 7. The Dependence of WGM shifts to Liquid Pressure changes (Pa)

The liquid levels have been converted to Pa with the basic conversion ratio of 1mm H<sub>2</sub>O=9.8065 Pa.

The WGM resonance shifts as a function of applied pressure is plotted in Figure 7, where these resonance shifts demonstrate a linear response with nearly no hysteresis in the pressure range tested, thus, providing a reliable pressure reading essential for sensor performance.

The best sensitivity dependence has been observed while draining water from the setup, as expected, due to the smoother level change characteristics of draining. Therefore, the dependence of shifts to liquid levels changes is better, in other words, higher sensitivities have been obtained while draining water from the experiment setup.

The red line in Figure 6 and Figure 8 represents the pressure transducer output, while the blue line represents the WGM resonance shifts developed in the setup [2][4].

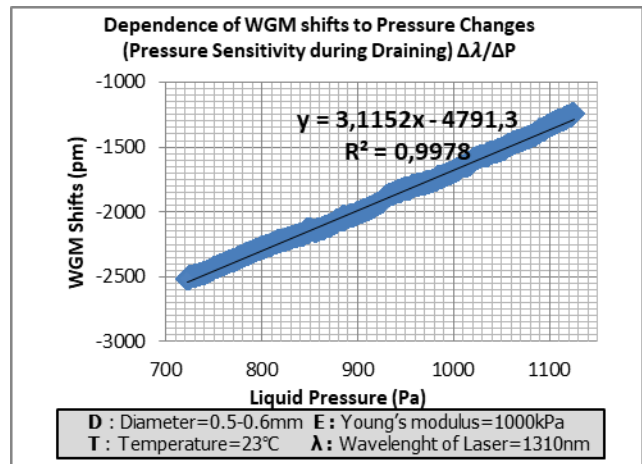


Figure 9. The Dependence of WGM shifts to Liquid Pressure changes (Pa)

In Figures 7 and 9, the Y axis represents the resonance shifts in pm scale, while the X axis represents the liquid levels in the container in mm scale [2][4]. A nearly perfect ( $R^2=0.9978$  (correlation coefficient)) linear dependence of WGM shifts due to liquid levels can be observed while draining water from the experiment setup. Therefore, it can be seen that for every Pa change, experimenters observe 3.2 pm shift in resonances [2].

The pressure values seen in Figure 9, can also be converted to liquid levels (1mm H<sub>2</sub>O=9.8065 Pa), whereas we are still going to use the resonance shifts over the pressure changes (pm/pa) as the sensitivity dependence definition and the formulation in our case [3]:

$$\frac{\Delta\lambda}{\Delta P} = S \quad (1)$$

Results indicate that, there is a strong correlation between WGM shifts [3]-[5] and the liquid levels. The resonance shifts appear as a function of liquid levels with a linear response with no discernible hysteresis in the pressure range tested.

### B. Accuracy and Uncertainty

In engineering and science, the accuracy of a measurement system has been defined as the degree of closeness of measurements of a quantity to that quantity's true value [12][13].

It is considered that measurements are made by calibrated instruments for which all known systematic errors have been removed. However, even the most carefully calibrated instruments will have errors associated with the measurements [12][13].

Error is the difference between the experimentally determined value and the true value, therefore, accuracy increases as error approaches zero. In practice, the true values of measured quantities are rarely known, thus, an error should be estimated, and this estimation is called uncertainty [12][13].

$$w_R = \left[ \left( \frac{\partial R}{\partial x_1} w_1 \right)^2 + \left( \frac{\partial R}{\partial x_2} w_2 \right)^2 + \dots + \left( \frac{\partial R}{\partial x_n} w_n \right)^2 \right]^{1/2} \quad (2)$$

According to equation (2), error in the resulting value is the result of errors in the variables. When neglecting the errors of the devices such as: laser source, laser diode controller, function generator, data acquisition card (DAQ), computer, photodiodes, and the fibers, the accuracy measurement can be derived by using the sensitivity value obtained at the end of the experiments. The sensitivity dependence has been already defined as  $\Delta\lambda/\Delta P = S$ , therefore, the uncertainty error value of  $S$  becomes:

$$w_S = \left[ \left( \frac{\partial S}{\partial \Delta\lambda} w_{\Delta\lambda} \right)^2 + \left( \frac{\partial S}{\partial \Delta P} w_{\Delta P} \right)^2 \right]^{1/2} \quad (3)$$

The  $\Delta\lambda$  term is according to the resonance shifts, while the  $\Delta P$  term represents the pressure levels. By deriving each term:

$$\delta S / \delta \Delta\lambda = 1 / \Delta P \text{ and } \delta S / \delta \Delta P = -(\Delta\lambda) / (\Delta P)^2 \quad (4)$$

as a result:

$$w_S = \left[ \left( \frac{1}{\Delta P} w_{\Delta\lambda} \right)^2 + \left( \frac{-\Delta\lambda}{(\Delta P)^2} w_{\Delta P} \right)^2 \right]^{1/2} \quad (5)$$

can be evaluated.

Thus, the uncertainty term of the resonance shifts have been obtained by 14 bit data acquisition card where the seven step software tracking algorithm takes place, resulting in a certainty of  $2^{14}$ ; this means an error of [3]-[5]:

$$\sum_{n=7} \frac{1}{2^{14}} = \frac{7}{16384} = \pm 0,043\%, \quad (6)$$

where the uncertainty value becomes:

$$w_{\Delta\lambda\_Filling} = (761,85 - 0,228) \times \frac{7}{16384} = 0,325 \text{ pm} \quad (7)$$

$$w_{\Delta\lambda\_Draining} = (2515,55 - 1249,50) \times \frac{7}{16384} = 0,541 \text{ pm} \quad (8)$$

While filling water into the experiment setup, data shows  $\Delta\lambda/\Delta P = 1,9083 \text{ pm/Pa}$  conversion, seen in Figure 7, while draining water from the setup, the data has a  $\Delta\lambda/\Delta P = 3,115 \text{ pm/Pa}$  value, as shown in Figure 9.

For finding the  $w_{\Delta P}$ , or in other terms, the uncertainty value of the pressure component, we need the general pressure to liquid height formula  $P = \rho h$ , and, when we differentiate both sides, the resulting formula becomes:  $\Delta P = (\rho \cdot g) \cdot \Delta h$ , where  $\rho$  is the Density, and  $g$  is the Acceleration of Gravity, which are both constants.

$$\Delta P = \rho \cdot g \cdot \Delta h \quad (9)$$

$$w_{\Delta P} = \left[ (\rho \cdot g \cdot w_{\Delta h})^2 \right]^{1/2} \quad (10)$$

The pressure value comes from the pressure transducer, and liquid level readings, thus the uncertainty calculation should include both. The reading and pressure transducer errors combined having a liquid height vs. voltage relationship seen in Figures 6 and 8. Thus, for representing the uncertainty, the conversion formula becomes:

$$\delta \Delta h = S_c \cdot \Delta V \quad (11)$$

Therefore:

$$w_{\Delta h} = \left[ (\Delta V \cdot w_{S_c})^2 + (S_c \cdot w_{\Delta V})^2 \right]^{1/2} \quad (12)$$

where  $S_c$  represents the liquid height reading errors, and  $\Delta V$  represents the voltage uncertainty, or accuracy of the pressure transducer.

The accuracy of the pressure transducer used during the experiments is known as  $\pm 1,0\%$  [2], so we calculate the uncertainty value of the pressure transducer for this experiment as follows:

$$w_{PT} = w_{DV\_Filling} = 2,845 \text{ V} \times 0,01 = 0,0285 \text{ V} \quad (13)$$

and

$$w_{PT} = w_{DV\_Draining} = 3,079 \text{ V} \times 0,01 = 0,03079 \text{ V} \quad (14)$$

The uncertainty value coming from the liquid height readings consists of the scale of the ruler, and total filling or draining limits:

$$0,5\text{mm}/300\text{mm}=\pm 0,166\%, \quad (15)$$

$$w_{\text{RulerReading}} = w_{SC} = w_{\text{SlopeOfConversion\_Filling}} = \frac{39,668\text{mm} \times 0,00166}{0,065\text{mm}}, \quad (16)$$

and

$$w_{\text{RulerReading}} = w_{SC} = w_{\text{SlopeOfConversion\_Draining}} = \frac{40,94\text{mm} \times 0,00166}{0,0679\text{mm}}. \quad (17)$$

For finding the exact uncertainty  $w_{\Delta P}$  value, errors occurred during the liquid level readings, and the pressure transducer uncertainty should be included:

$$w_{\Delta P} = \left[ (\rho g \cdot \Delta V \cdot w_{SC})^2 + (\rho g \cdot SC \cdot w_{\Delta V})^2 \right]^{1/2} \quad (18)$$

$$w_{\Delta P\_Filling} = \left[ (9,80 \cdot 2,845 \cdot 0,065)^2 + (9,80 \cdot 14,013 \cdot 0,0285)^2 \right]^{1/2} \\ = 4,31\text{Pa} \quad (19)$$

$$w_{\Delta P\_Draining} = \left[ (9,80 \cdot 3,079 \cdot 0,0679)^2 + (9,80 \cdot 13,204 \cdot 0,03079)^2 \right]^{1/2} \\ = 4,48\text{Pa} \quad (20)$$

In other words, we have an uncertainty level of  $\pm 1,10\%$  for filling, and  $\pm 1,11\%$  for draining. When these results are used in equation (8):

$$w_{S\_Filling} = \left[ \left( \frac{1}{389,047} \cdot 0,325 \right)^2 + \left( \frac{761,622}{(389,047)^2} \cdot 4,31 \right)^2 \right]^{1/2} \\ = 0,0217 \frac{\text{pm}}{\text{Pa}} \quad (21)$$

$$w_{S\_Draining} = \left[ \left( \frac{1}{401,882} \cdot 0,541 \right)^2 + \left( \frac{-1266,05}{(401,882)^2} \cdot 4,48 \right)^2 \right]^{1/2} \\ = 0,0351 \frac{\text{pm}}{\text{Pa}} \quad (22)$$

Therefore, the experiment performed during draining water from the setup showing  $\Delta\lambda/\Delta P$  sensitivity dependence value of 3,115 pm/Pa, having a calculated uncertainty of  $\pm 1,126\%$ .

The experiment performed during filling water to the setup showing  $\Delta\lambda/\Delta P$  sensitivity dependence value of 1,9083 pm/Pa, similarly has a calculated uncertainty of  $\pm 1,137\%$ .

#### IV. CONCLUSIONS

After performing the liquid level measurement experiments, it has been observed that sensitivities of 1.9083 pm/Pa during the liquid level measurements have been reached, while having a linear dependence within the experimental range.

As the uncertainty calculations has been performed for liquid level measurements, a total uncertainty value of  $\pm 1,126\%$  for draining, and  $\pm 1,137\%$  for filling have been obtained. The pressure transducer, which acts as the reference device used in the experiments, has  $\pm 1,0\%$  accuracy, which limits the total value. Any alternative device with better accuracies would definitely help future researchers to decrease the uncertainties.

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