# Improving Spectrum Sensing Performance by using Eigenvectors

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***Abstract*—In this paper we present a method to improve the performance of eigenvalue-based detection, facilitated with eigenvectors of the sample covariance matrix. We focus on the multi-sensor detection of a single source case. If the channel is constant over adjacent sensing slots, it can be blindly estimated by using the eigenvector associated to the largest eigenvalue on condition of the source's presence. We introduce a new test where the eigenvector value, computed over some previous auxiliary slots, is properly used by the detection algorithm. The ROC curves show that the new test is able to outperform popular algorithms like the Roy Largest Root Test and the Energy Detection for both PSK and Gaussian sources, and to approach the optimal Neyman-Pearson performance with a very small number of auxiliary slots.**

***Keywords*-largest eigenvector; spectrum sensing; cognitive radio; signal detection**

## I. Introduction

In Cognitive Radio (CR) scenario, secondary users can use multiple spectrum sensing techniques to enhance the detection performance of primary user's absence or presence, i.e., making a decision between the alternative hypothesis of pure noise ($\mathcal{H}_0$) or signal plus noise ($\mathcal{H}_1$). The new CR systems and standard will impose tight constraints on the detection algorithm performance. As an example, the IEEE 802.22 WRAN (Wireless Regional Area Networks) standard [1] requires very low values for the false alarm ($P_{fa}$ lower than 0.1) and high detection probability ($P_d$ no less than 0.9). These requirements may be difficult to be obtained with the popular energy detection (ED) [2], which simply compares the energy of the received signal against the noise level. ED performs very well in normal situations, but it can be unable to match stringent requirements in some extreme (but important) cases, like for example hidden nodes, where the signal-to-noise ratio (SNR) may be -10 dB or lower.

For this reason, several eigenvalue-based detection algorithms have been proposed in recent years, aiming to improve the performance of ED. Non-parametric methods, i.e., without a complete knowledge of all the system (signal, noise and channel) parameters can be generally categorized as (i) blind detectors, without knowledge of any parameter - to name a few, the Generalized Likelihood Ratio Test (GLRT) [3] and the Eigenvalue Ratio Test (ERT) [4]; (ii) semi-blind detectors, where some parameters are known - such as the Roy Largest Root Test (RLRT) [5] which resorts to the noise level knowledge; (iii) trained detectors, where some parameters are estimated outside the sensing slots. These algorithms can be compared by studying the ROC (Receiver Operating Characteristics) curves, plotting the $P_d$ as the function of $P_{fa}$. Most systems compute the test threshold as a function of $P_{fa}$ (to guarantee Constant False Alarm Rate - CFAR). Then, a system outperforms another if it achieves a higher $P_d$ at the parity of a certain $P_{fa}$.

It is known that, among all possible detection tests, an optimal solution exists. The likelihood ratio test derived according to the Neyman-Pearson (NP) lemma [6] for an alternative hypothesis, is the uniformly most powerful test. Unfortunately, the NP test is a parametric method requiring strong knowledge of the signal, noise, and channel parameters. For the case of semi-blind detectors with known noise level but unknown channel, the RLRT test is proved to be the best algorithm [7]. However, as can be observed by comparing their ROC curves, the gap between the NP and the RLRT is rather large. This penalty is essentially due to the gain/lack of the channel knowledge.

In single source case, if the channel is constant within adjacent sensing slots, it can be blindly estimated (i.e., without any use of pilot symbols) by using the eigenvector associated to the largest eigenvalue of the sample covariance matrix of the signal slots. The idea of this paper is to exploit this to enhance the detection performance of eigenvalue-based tests. To do this we propose a new test, called EigenVEctor (EVE) test, that explicitly employs the eigenvector in the test statistic. Simulation results shows the EVE test performs at least as good as the RLRT and with increased prior available auxiliary signal slots, is able to significantly cover the gap between the NP and the RLRT test. The paper is arranged as follows: The adopted model is presented in Section II. NP and RLRT tests are reviewed in Section III. The new EVE test is introduced in Section IV and its performance is analyzed in Section V. The practical consideration and future works are described in Section VI and Section VII.

## II. Model

Denote by $K$ the number of antennas or cooperative sensors and by $N$ the number of samples per sensing slot.

 

We focus on a single source scenario, which is of interest for many detection problems, including cognitive radio. The $K \times 1$ received vector at time $n$ collects the baseband complex (I/Q) samples from the $K$ antennas:

$$\boldsymbol{y}(n) = [y_1(n) \dots y_k(n) \dots y_K(n)]^T.$$

We have:

$$\boldsymbol{y}(n) = \begin{cases} \boldsymbol{v}(n) & \mathcal{H}_0 \\ \boldsymbol{h}s(n) + \boldsymbol{v}(n) & \mathcal{H}_1 \end{cases}$$

where $\boldsymbol{v}$ is a $K \times 1$ circularly symmetric complex Gaussian (CSCG) vector of noise samples with zero mean and variance $\sigma_v^2$, i.e., $\boldsymbol{v}(n) \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}_{K\times 1}, \sigma_v^2 \boldsymbol{I}_{K\times K})$. All the noise vectors $\boldsymbol{v}(n)$ are assumed to be statistically independent.

The channel complex vector $\boldsymbol{h} = [h_1 \dots h_K]^T$ is assumed constant and memoryless within all the sensing window.

The signal samples $s(n)$ are assumed to have constant probability density function, zero mean and variance $\sigma_s^2$. In the following, we will focus on two case studies, where signal samples $s(n)$ are independent Gaussian or PSK samples.

Under $\mathcal{H}_1$, we define the average SNR at the receiver as

$$\rho \triangleq \frac{\mathrm{E}\,\|\boldsymbol{h}s(n)\|^2}{\mathrm{E}\,\|\boldsymbol{v}(n)\|^2} = \frac{\sigma_s^2 \|\boldsymbol{h}\|^2}{K\sigma_v^2} \tag{1}$$

Given the $K \times N$ received matrix

$$\boldsymbol{Y} \triangleq [\boldsymbol{y}(1) \dots \boldsymbol{y}(n) \dots \boldsymbol{y}(N)]. \tag{2}$$

and the $K \times K$ sample covariance matrix

$$\boldsymbol{R} \triangleq \frac{1}{N}\boldsymbol{Y}\boldsymbol{Y}^H \tag{3}$$

we will denote by $\lambda_1 \geq \dots \geq \lambda_K$ the eigenvalues of $\boldsymbol{R}$ sorted in decreasing order and by $\boldsymbol{e}_1, \cdots, e_K$ the corresponding normalized eigenvectors.

## III. Known test statistics

To make the decision between $\mathcal{H}_0$ and $\mathcal{H}_1$, a test statistic compares a quantity $T$ against a pre-defined threshold $t$: if $T > t$ $\mathcal{H}_1$ is selected, otherwise $\mathcal{H}_0$ is chosen. The test performance is evaluated by the false alarm probability $P_{fa} = \Pr(T > t|\mathcal{H}_0)$ and the detection probability $P_d = \Pr(T > t|\mathcal{H}_1)$. In practical, the decision threshold $t$ is typically computed as a function of the target $P_{fa}$, to guarantee the aforementioned CFAR property.

The Neyman Pearson (NP) test is given by the following likelihood ratio:

$$T_{NP} = \frac{p_1(\boldsymbol{Y}; \boldsymbol{h}, \sigma_s^2, \sigma_v^2)}{p_0(\boldsymbol{Y}; \sigma_v^2)}. \tag{4}$$

and is known to be optimal, i.e., to achieve the maximum possible $P_d$ for any given value of $P_{fa}$.

As an example, under the considered model, if the signal samples are independent Gaussian samples, the NP test is obtained by using [3]:

$$p_0(\boldsymbol{Y}; \sigma_v^2) = \frac{1}{(\pi\sigma_v^2)^{NK}} \exp\left(-\frac{N\mathrm{tr}\boldsymbol{R}}{\sigma_v^2}\right)$$

and

$$p_1(\boldsymbol{Y}; \boldsymbol{h}, \sigma_s^2, \sigma_v^2) = \frac{1}{(\pi^K \det\boldsymbol{\Sigma})^N} \exp\left[-N\mathrm{tr}\left(\boldsymbol{R}\boldsymbol{\Sigma}^{-1}\right)\right].$$

where $\boldsymbol{\Sigma} = \sigma_v^2 I_K + \sigma_s^2 \boldsymbol{h}\boldsymbol{h}^H$.

The NP test requires the exact knowledge of both the channel vector $\boldsymbol{h}$ and the noise variance $\sigma_v^2$. As pointed out in [7], if only the noise variance is known and the SNR is above the identifiability threshold:

$$\rho_{\text{crit}} = \frac{1}{\sqrt{KN}} \tag{5}$$

the best statistical test is the RLRT [5], which compares the largest eigenvalue of the sample covariance matrix against $\sigma_v^2$:

$$T_R = \frac{\lambda_1}{\sigma_v^2}. \tag{6}$$

The RLRT outperforms all the other algorithms belonging to the class of semi-blind algorithms [8], i.e., where the noise level is assumed to be known, including the popular ED, which is given by:

$$T_{ED} = \frac{1}{KN\sigma_v^2} \sum_{k=1}^{K} \sum_{n=1}^{N} |y_k(n)|^2 \tag{7}$$

Despite of the superiority in its class, the gap between the RLRT and the NP test is not negligible, as it can be seen by observing the ROC curves reported in Figure 2. The scope of this paper is to cover the gap between RLRT and NP. To do this, we intend to use the eigenvector of the sample covariance matrix.

## IV. The new EVE test

Since the RLRT is the best test within the class of semi-blind algorithms, the performance differences with respect to the NP test must be due to the lack of channel knowledge. The method proposed in this paper works for a static channel, or at least a channel that can be considered as flat fading not only for the sensing slot but for some adjacent sensing slots. Clearly, this prevents its application to any kind of mobility. Despite of this, the results are extremely interesting for all the cases where the channel can be considered as static for some time because, as we will see, the new method is indeed able to approach the NP optimal performance.

The starting idea of the new test is that, given a $\mathcal{H}_1$ slot, the eigenvector $\mathbf{e_1}$ associated to the largest eigenvalue $\lambda_1$ provides an estimation of the channel vector $\mathbf{h}$. Given $N_{aux}$ signal slots available before the current sensing slot, let us denote by $e_{aux}$ the normalized (i.e., with unitary energy) eigenvector computed by using $N_{aux} \cdot N$ samples. Now, the problem is how to properly use $e_{aux}$ during the current sensing slot. To do this we want to introduce a new statistical test that (i) uses $e_{aux}$, (ii) is at least as strong as the RLRT, (iii) is able to approach the NP test when $N_{aux}$ increases. Given the matrix $Y$ received in the current sensing slot, let

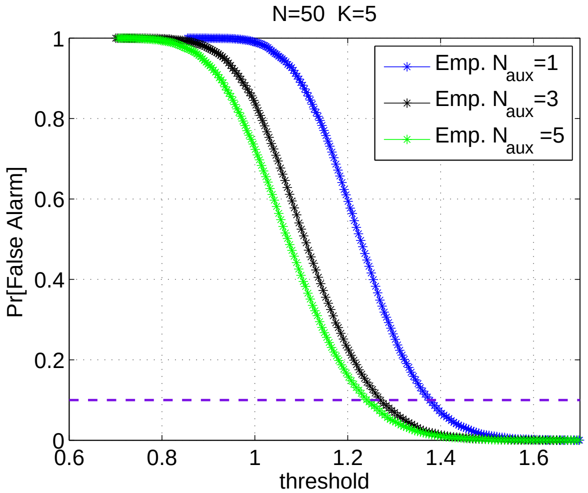


Figure 1. False Alarm Probability

us compute the sample covariance matrix $R$ and the largest eigenvector $\mathbf{e}$. Suppose that the quantity $\mathbf{e_{aux}}$ computed over $N_{aux}$ previous signal slots is available, the proposed statistical test, called the EigenVEctor (EVE) test, is defined as:

$$T_{EVE} = \frac{N_{aux}\left[e_{aux}^H R\, e_{aux}\right] + \left[e^H R\, e\right]}{\sigma_v^2 (N_{aux}+1)} \tag{8}$$

Note that if $N_{aux} = 0$, the test reduces to

$$T_{EVE} = \frac{e^H R\, e}{\sigma_v^2} = \frac{\|\mathbf{e}\|^2 \lambda_1}{\sigma_v^2} = \frac{\lambda_1}{\sigma_v^2} \tag{9}$$

and has the same statistical power of the RLRT. We will now show that, for increasing $N_{aux}$, the test outperforms the RLRT and is able to cover the gap with respect to the NP test.

## V. The performance of the EVE test

Fixed a given threshold $\gamma$, the behavior of the false alarm probability is shown in Figure 1 for $N = 50$, $K = 5$ and $N_{aux} = 1, 3, 5$. These curves can be used to compute the threshold necessary to achieve a given false alarm rate. For example, the threshold values corresponding to $P_{fa} = 0.1$ are identified in the figure.

The ROC curves of the EVE test are plotted in Figure 2, 3, and 4 for $N = 50$, $K = 5$, $SNR = -10dB$, $N_{aux} = 1, 3, 5$, and compared against those of of the Neyman-Pearson, the RLRT, and the Energy Detection. We can observe that (i) the new EVE test significantly outperforms the RLRT test and (ii) the EVE test approaches the Neyman-Pearson test even with a limited number of auxiliary slots.

Given a false alarm probability value $P_{fa} = 0.1$ and chosen the threshold $\gamma$ according to it, fixed the SNR value, the behavior of the detection probability as a function of the number of antennas is reported in Figure 5, 6, and 7. It can

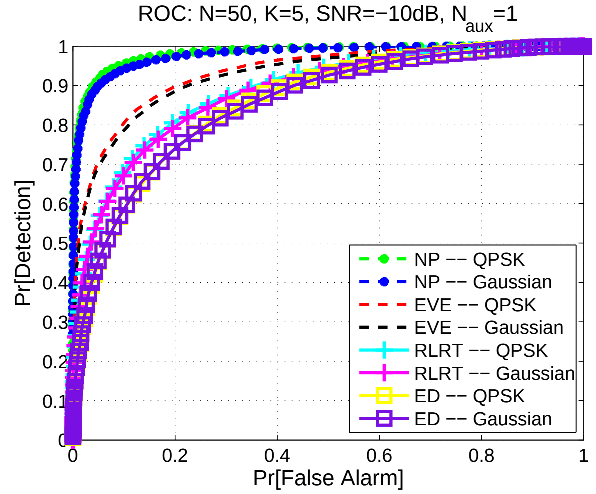


Figure 2. ROC curve

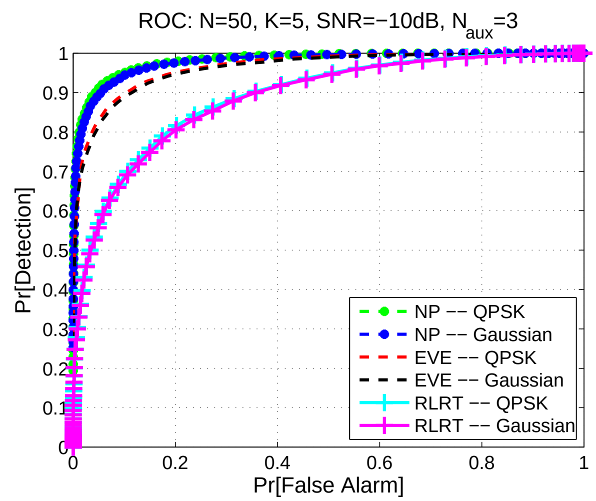


Figure 3. ROC curve

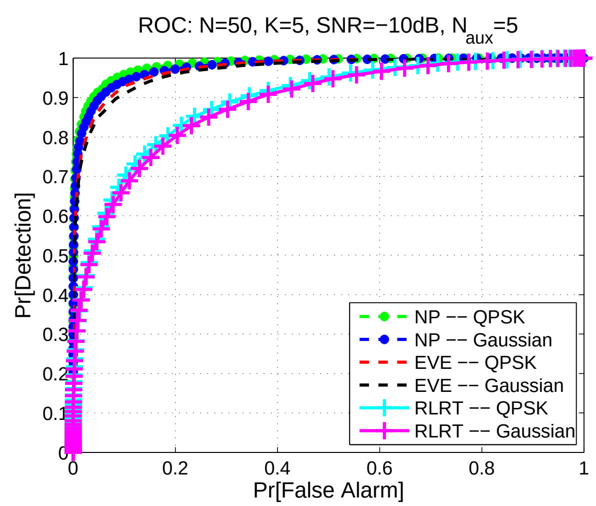


Figure 4. ROC curve

be observe that, at the parity of $P_{fa}$ and $P_d$ values, the EVE test requires a much lower number of antennas to achieve the same performance.

Given a false alarm probability value $P_{fa} = 0.1$ and chosen the threshold $\gamma$ according to it, the behavior of the detection probability as a function of the signal-to-noise ratio is reported in Figure 8, 9, 10, and 11. By fixing a $P_d$ value (for example 0.9 in the figures) it is possible to appreciate the improvement achieved by the EVE test in terms of received SNR at the parity of $P_{fa}$ and $P_d$ values. In the figures, a gain up to 2 dB is observed with respect to the RLRT, and up to 4 dB with respect to ED.

## VI. Practical considerations: adjacent slots and unknown noise variance

For practical applications, it is interesting to note that the test does not really require dedicated training slots. The eigenvectors can be computed by using the samples from the slots marked as $\mathcal{H}_1$ by the running sensing algorithm with a high reliability. Moreover, the EVE test can be modified to cover the case of unknown noise variance. As an example, the following modified test

$$T'_{EVE} = \frac{N_{aux}\left[e^H_{aux} R\, e_{aux}\right] + \left[e^H R\, e\right]}{\left(\frac{1}{K-1}\sum_{i=2}^{K}\lambda_i\right)(N_{aux}+1)} \tag{10}$$

Since $\frac{1}{K-1}\sum_{i=2}^{K}\lambda_i$ represents the Maximum Likelihood estimation of the noise variance [9], the test is equivalent to the GLRT for $N_{aux} = 0$ and is able to improve it for increasing $N_{aux}$.

## VII. Conclusions and future works

A new test using the eigenvector of the sample covariance matrix has been presented and evaluated in this paper. The test requires the channel to be constant over a number of adjacent slots, so it can be used only for constant or slowly changing channels. The improvement obtained with respect to the popular RLRT and ED tests is significant and the performances rapidly approach the optimal NP curves. Future research will focus on the computation of closed-form analytical formulas for the false alarm and detection probability.

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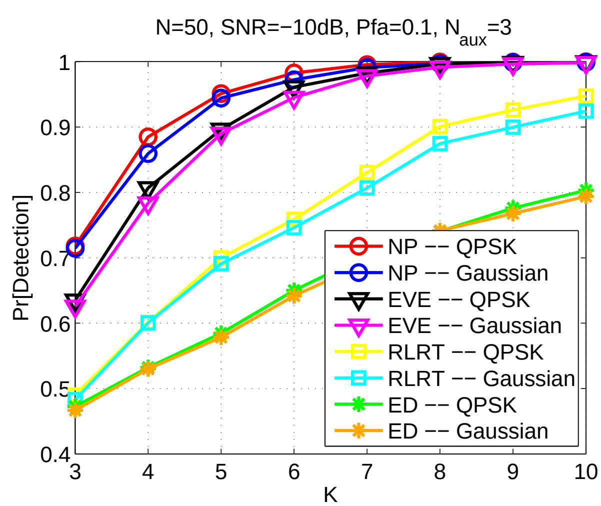


Figure 5. Antenna Gain

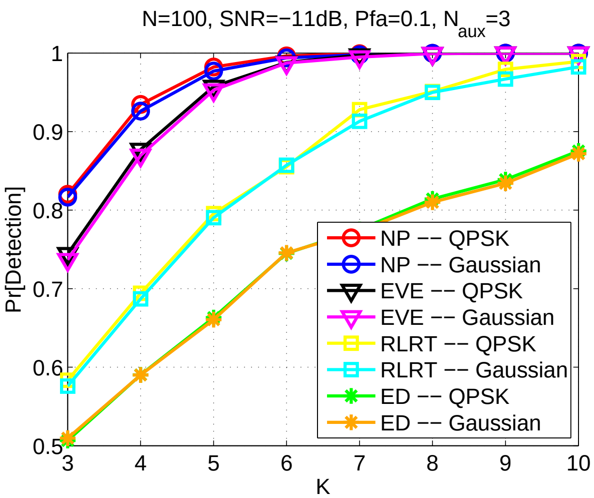


Figure 6. Antenna Gain

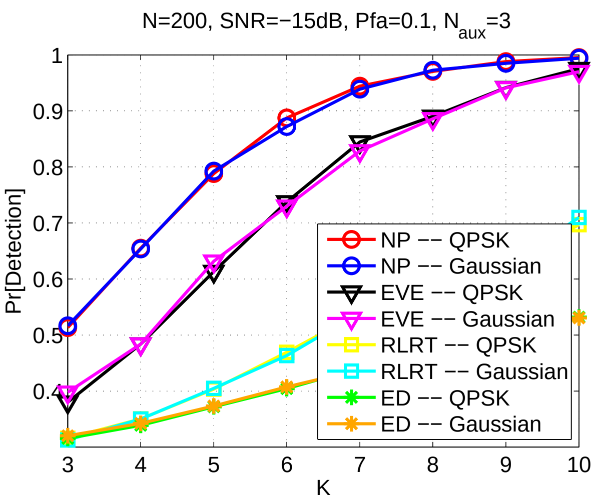


Figure 7. Antenna Gain

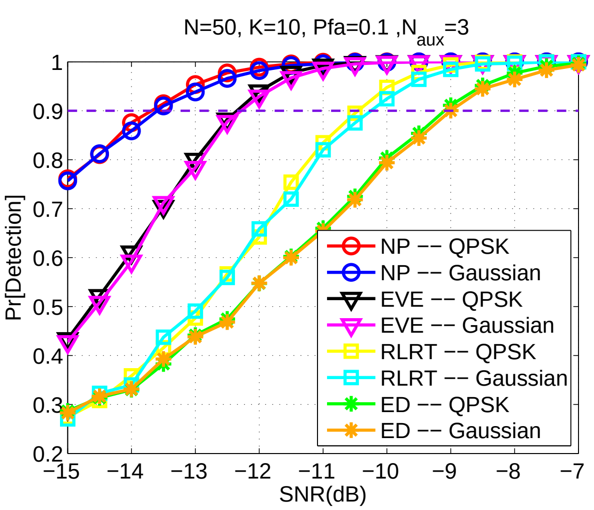


Figure 8. SNR Gain

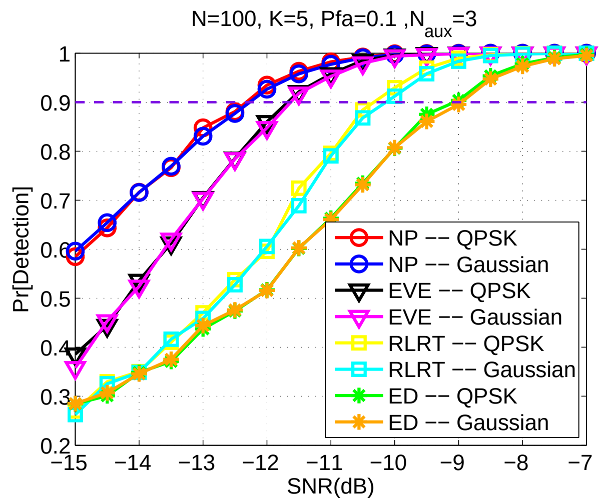


Figure 9. SNR Gain

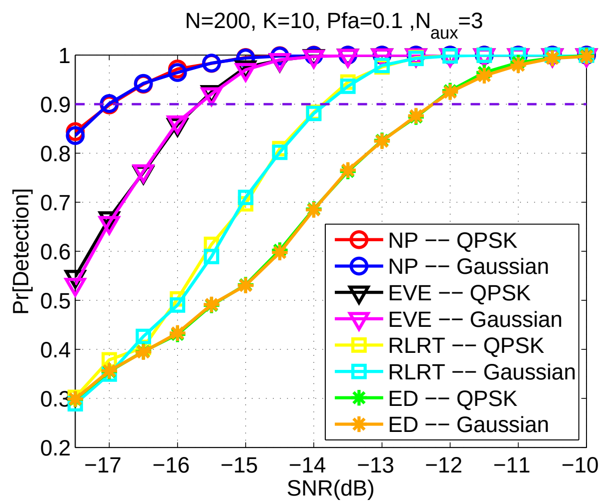


Figure 10. SNR Gain

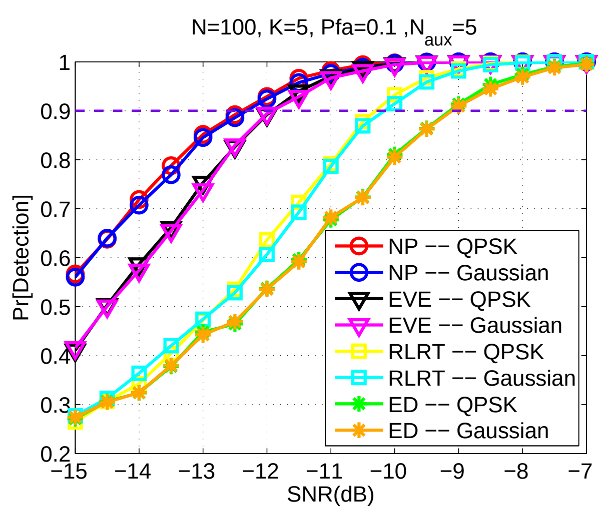


Figure 11. SNR Gain