NC-OFDM Cognitive Radio Optimal Pilot Placement for the LS Estimator

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Abstract— **A theoretical contradiction between the areas of the optimal pilot-pattern design and the optimal power loading algorithms has been found to exist for proposed Non-Contiguous Orthogonal Frequency-Division Multiplexing (NC-OFDM) Cognitive Radio (CR) systems. It has been found that the proposed, optimal pilot-patterns specify that the Secondary User (SU) should convert the two sub-channels adjacent to a Primary User (PU) to pilot sub-channels in order to ensure the lowest estimator Mean Squared Error (MSE) attainable. This algorithm has been found to contradict with the optimal power loading algorithm for CR systems should the Pilot-to-Data Power Ratio (PDPR) be greater than unity. The contradiction arises in that the optimal power loading algorithms for CR systems require that, in order for interference to the PU to be kept below an acceptable threshold, the sub-channels of the SU should have less power assigned to them the closer they are to the PU. In this paper, a proof of concept is demonstrated and evaluated such that the lowest MSE possible is achieved while maintaining interference threshold constraints in a simplistic environment.**

Keywords-Cognitive Radio; Power Loading; Pilot Patterns; Orthogonal Frequency Division Multiplexing.

I. INTRODUCTION

Spectrum scarcity is an omnipresent and greatly impacting problem which needs to be overcome in order to allow new communications technologies to flourish. Due to the rapid pace of technological innovation, spectrum has become a very valuable and rare commodity. It has been noted that even though much of the practically usable spectrum has been occupied and licensed, it is only used anywhere from 15% to 85% of the time in a wide geographic and time dispersion [1]. This can be even lower in certain situations such as sub-urban environments where frequency utilisation from 100 MHz to 3 GHz can be utilised as little as 7% of the time [2]. This means that much of the usable spectrum is reserved for licensed operation but is only used by its licensees a very small percentage of the time or its actual licensed use is limited to a relatively small geographical area.

To address the problems of spectrum crowding, cognitive radio has been proposed as an attractive, viable solution [3]. Cognitive radio proposes to alleviate the problem of spectrum crowding by conducting communications in licensed bands during the time instances in which they are unused.

It is commonly proposed that a variation of OFDM, non-contiguous OFDM, be used to implement a CR system. This allows the sub-channels of an OFDM system which interfere with the primary user to be switched off. This means that the NC-OFDM system would comply with one of the principles of CR such that any CR-compliant communications are transparent to, and need not be considered by, non CR-compliant systems.

Much work has been done on power loading for the cognitive radio environment. In [4], a power loading algorithm was devised where the amount of interference to a PU was calculated for each sub-channel based on their power and spectral distance to the PU. It was found in [4] that a ‘step’ profile needs to be applied where the sub-channels closest to the PU need to be allocated the least amount of power so that the interference to the PU is kept below an acceptable threshold. This means that the closer a sub-channel is to a PU, the less power should be allocated to it.

Another aspect which has been investigated is the pilot-pattern algorithms needed for CR systems. Due to the fact that narrow and wideband interference from any PUs is not known prior to transmission, a PU could possibly take up one or several pilot sub-channels. This would greatly decrease channel interpolation accuracy due to the loss of one or more channel observations. It has been found that the optimal way to maximize channel estimation accuracy when one or more pilot sub-channels need to be disabled is converting the sub-channels adjacent to the interfering PU’s signal into pilot-bearing sub-channels [5].

If one considers these two aspects, they cannot be mutually ignored since it is necessary for the pilot-pattern of the system to adapt to changes in the utilised spectrum (such as intermittently appearing and disappearing PUs). This is because the effect on the bit-error rate (BER), and consequently the maximum channel capacity, is severe should the channel estimation accuracy (MSE) be degraded [8]. When also factoring the criterion for interference to the PU, indeed one on which the principles of CR is based, this would lead the implementation into placing pilots in the sub-channels closest to the PU while reducing the power of those sub-channels significantly so as not to cause any interference to the PU.

Another area of focus which has been noted is the pilot-to-data power ratio. In most applications, the pilot symbols or sub-channels need to be allocated higher power than the data sub-channels so that the instantaneous channel

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estimation at the pilot symbol is as accurate as possible by providing a relatively high signal-to-noise ratio (SNR). This holds true especially for conditions where the SNR is low and therefore the transmitted pilot symbols are plagued by relatively high amounts of noise. The PDPR therefore needs to be increased substantially such that the channel estimation accuracy remains at a desirable level.

These three aspects, namely the pilot-pattern, the power loading and the pilot-to-data power ratio are then seen to be contradictory. While the optimal pilot-patterns for CR systems imply that the pilot symbols or sub-channels need to be placed adjacent to the PU, the optimal power loading algorithms state that the sub-channels need to have their assigned power levels reduced such that they do not interfere with the PU but the principles of OFDM and PDPR research states that the pilot sub-channels should usually be assigned more power in order to achieve as high as possible channel estimation accuracy.

These contradictions can then be modelled and solved by expressing them in the form of a constrained optimisation problem. In this paper, an optimal solution is derived for the case of a least squares (LS) estimator using linear interpolation. The research demonstrated in this paper is a continuation from [11], where an LS-based approach is investigated due to its practicality in terms of low-complexity estimators.

This paper is organised as follows. Section II describes the system model used and Section III derives and explains the optimal solution to the outlined problem. In Section IV, the simulation parameters are given as well as results of the simulations themselves. The results are discussed in this section and a conclusion is derived from the findings. This is elaborated upon in Section V.

II. System Model

The CR system model considered is that of having a contiguous OFDM system interrupted by a PU of a fixed bandwidth, this means that the sub-channels of the SU which conflict with the PU's used frequency band are disabled by the SU. This allows the spectrum to be fully utilised in that there are no guard bands between the PU's and the SU's signal.

The CR system is then seen as an OFDM system of N sub-channels with certain sub-channels dedicated to transmitting pilot symbols meaning that, for simplicity, 1-dimensional channel estimation is used to obtain the instantaneous channel gains.

As prescribed in [6], the interference in the system is differentiated into PU-to-SU and SU-to-PU interference.

A. Power Density Spectrum of Signals

The transmitted signals in the system model are assumed, for the sake of simplicity, to be shaped by a rectangular pulse shaping function. The power density spectrum of the rectangular pulse shaping function can be represented as [4]

$$\phi_i(f) = P_i T_s \left(\frac{\sin(f \cdot \pi \cdot T_s)}{f \cdot \pi \cdot T_s} \right)^2 . \qquad (1)$$

In (1), P_i represents the transmit power of the i^{th} sub-carrier and T_s represents the symbol duration of that same sub-carrier. It should be noted that this equation is only applicable for a rectangular pulse-shaping function. Equations for other pulse-shaping functions can be used as well but the problem will remain unchanged since every pulse-shaping function will have some form of spectral roll-off (leakage), and therefore, present interference to non-orthogonal frequencies.

B. Interference from PU to SU

The signals between the PU and the SU are assumed to be non-orthogonal, and therefore, the interference imposed by the PU on the SU is effectively 'smeared' due to the Fast Fourier Transform (FFT) processing performed by the SU [6]. The expected value of the power density spectrum of the PU's signal after an FFT of size M is performed can be described as [6]

$$E\{I_M(\omega)\} = \frac{1}{2\pi M} \int_{-\pi}^{\pi} \phi_{PU}(e^{j\omega}) \left(\frac{\sin(\omega-\psi)M/2}{\sin(\omega-\psi)/2} \right)^2 d\psi , \qquad (2)$$

where ω represents the angular frequency which has been normalised to the sampling frequency, M is the number of samples (FFT size in this case) and $\phi_{PU}(e^{j\omega})$ represents the power density spectrum of the PU's pulse-shaping filter. The interference from the PU to the SU can then be described as the integral of the expected value of the power spectral density, which may be expressed as

$$I_{PU}(d_i, P_i) = \int_{d_i - \Delta f/2}^{d_i + \Delta f/2} E\{I_M(\omega)\} d\omega . \qquad (3)$$

In (3), d_i represents the spectral distance between the considered sub-carrier and the PU, and Δf represents the width of one sub-channel of the SU (equivalent to the inverse of the OFDM symbol duration).

C. Interference from SU to PU

The interference from the secondary user to the primary user is modelled using simpler mathematics due to the assumption that we do not have any information about the PU's modulation scheme and other transmission properties, only the bandwidth and signal power. The interference caused by spectral roll-off from the SU can then be simply modelled as the integration of the power density spectrum of the signal, represented as (1) for the rectangular pulse shaping filter case. The interference from the SU can be modelled as [4]

$$I_{SU}(d_i, P_i) = \int_{d_i - B/2}^{d_i + B/2} \phi_i(f) df . \qquad (4)$$

It should be noted that B denotes the bandwidth occupied by the PU's signal such that the integration is performed over the PU's bandwidth with an added frequency 'offset' introduced by the spectral distance between the considered sub-channel and the PU's signal.

D. Channel Model

The multipath channel model used can be described as [7]

$$h(n) = \sum_{l=0}^{L-1} \alpha_l \cdot \delta(n - \tau_l) \quad , \tag{5}$$

in the time domain where α_l and τ_l are the complex gain and delay for the lth path of a multipath propagation channel with a total of L resolvable paths. To model the channel in the frequency domain, the discrete Fourier transform (DFT) is applied to the time domain response in (5), resulting in

$$H(i) = \sum_{l=0}^{L-1} \alpha_l \cdot \exp\left(\frac{-2j\pi\tau_l i}{N_{fft}} \right). \tag{6}$$

In (6), i and N_{fft} represent the sub-channel index and the size of the DFT respectively.

The probability distribution functions of the parameters α_l and τ_l may vary for different types of channels. In this case, they are assumed to be Rayleigh distributed such that a Rayleigh fading channel is simulated. The channel fading model is also used per OFDM symbol and a new frequency-selective channel frequency response is calculated for each OFDM symbol. This allows the simulation of a worst-case, fast-fading channel where there is no correlation between one OFDM symbol and the next. As such, the coherence time of the channel compared to the OFDM frame is 1 OFDM symbol.

E. Pilot error

The pilot error for the least squares estimator can be effectively modelled as dependent on the noise to pilot power ratio for pilot symbols, namely [7]

$$\hat{\mathbf{H}}_p = \mathbf{H}_p + \mathbf{P}^{-1}\mathbf{n}_p, \tag{7}$$

therefore the error can simply be represented as

$$\varepsilon_p = \hat{\mathbf{H}}_p - \mathbf{H}_p = \mathbf{P}^{-1}\mathbf{n}_p. \tag{8}$$

where $\mathbf{H}$ represents the vector form of the channel frequency response as derived in (6) and $\mathbf{H}_p$ is the vector subset of $\mathbf{H}$ at the pilot positions such that $p \subseteq i$.

F. Linear interpolation error bound

The instantaneous channel gain at the data sub-channels needs to be interpolated in either the time or frequency direction. Since, for simplicity, it was assumed that sub-channels were dedicated for pilot symbols, the interpolation was therefore done only in the frequency dimension. As the interpolation error cannot be known exactly unless the full channel frequency response is also known (which renders the need for interpolation moot), an error bound is used such that a worst-case interpolation error is used.

The interpolation error bound for a linear interpolator is dependent on the second derivative of the function being interpolated and the distance between the two interpolation points, thus, the more a function varies on a given interval, the higher the linear interpolation error will be. The linear interpolation error bound can be described as [10]

$$\varepsilon_{\text{int}} \le \frac{d_i^{\,2}}{8} \cdot \max\left| \frac{\partial^2 H(i)}{\partial i^2} \right|. \tag{9}$$

G. Optimal power loading

The optimal power loading algorithm is specified in [4]. It is important to note that the same power loading algorithm is derived at the boundary level where the interference to the PU is equal to the interference threshold parameter such that transmission power is maximized and, consequently, so is channel capacity. This also then allows us to effectively ignore the interference to the PU when placing the pilot as the power we may use at each sub-channel index complies with the optimal power loading requirements.

The interference equation at the threshold was therefore used such that the equation is formulated as

$$P_i^* = \frac{1}{\lambda \cdot \frac{\partial I_{SU}}{\partial P_i}} - \frac{\sigma^2 + I_{PU}}{|H(i)|^2} \tag{10}$$

where λ is the Lagrangian multiplier used to find the optimal power level for each sub-channel.

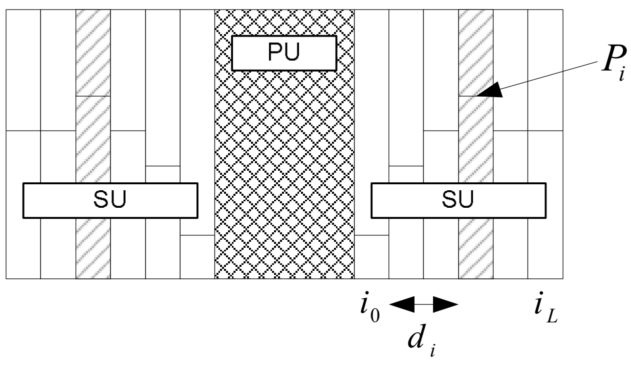

Figure 1. Pilot and data sub-channels for an NC-OFDM, CR system involving one PU and one SU. The pilots concerned for optimal placement is shaded on one diagonal.

III. Optimal Solution

In order to derive an optimal solution, the problem is formulated such that the estimation error between the concerned sub-channels, namely $i_0 \le i \le i_L$, is minimized. The constrained optimization problem is therefore modelled as

$$\varepsilon = \min_i \left|\varepsilon_p\right| + \varepsilon_{\text{int}} \tag{12}$$

where

$$\varepsilon_p = \mathbf{P}^{-1}\mathbf{n}_\mathbf{p} = \frac{\sigma^2 + I_{PU}(i)}{P_i^*} \tag{13}$$

and

$$\varepsilon_{\text{int}} \le \frac{d_i^{\,2}}{8} \cdot \max\left|\frac{\partial^2 H(i)}{\partial i^2}\right|$$
$$= \frac{(i - i_L)^2}{8} \max\left|\sum_{l=0}^{L-1} \frac{-4\pi^2\tau_l^2}{N_{fft}^2} \alpha_l \exp\left(\frac{-2j\pi\tau_l i}{N_{fft}}\right)\right| \tag{14}$$

subject to,

$$i \le i_L, \tag{15}$$

and $P_i \ge 0$, (16)

where $\forall i = 0,1,\ldots i_L$.

In the context of the optimisation problem, i_L is used to represent the upper limit (i.e. adjacent to the nearest, original pilot sub-channel) of the possible pilot sub-channel placement position and i_0 represents the lower limit (i.e. adjacent to the PU).

The interpolation error only considers the decrease in error as the pilot sub-channel approaches the PU since it is specified in [5] that a new pilot sub-channel is created instead of shifting an existing one and therefore MSE can only be decreased, assuming that the power allocated to the already existing pilot symbols remains the same.

The derivative of the optimal power loading function in (10) was found to be a transcendental function and therefore the error function cannot be optimised using traditional, algebraic methods such as the Karush-Kuhn-Tucker (KKT) conditions and as such the solution may only be computed numerically. The optimal was therefore computed numerically by searching for the value of i where the error function is lowest.

In practice, the value of i_L may not be bigger than the pilot spacing and therefore the optimisation problem only considers the sub-channels between the PU and the nearest pilot sub-channel (before insertion of the extra pilot sub-channel).

The provided solution is for a single side of the PU, this can be identically applied to the other side of the PU's transmission power remains uniform throughout the PU's bandwidth.

IV. Simulation Parameters and Results

A simulation was conducted by setting up an NC-OFDM system with parameters as listed in Table I.

TABLE I
SYSTEM SIMULATION PARAMETERS

Parameter	Value
PU bandwidth	768 kHz
Channel path gain means (dB)	[0 -15 -20]
Path delay time means (µs)	[0 0.4 0.9]
OFDM symbol length	333.3 µs
SU sub-channel bandwidth	3 kHz
FFT size	1024
Pilot spacing (frequency, time)	(12,12)
Maximum Doppler shift	24 Hz
PU signal power	20 dBm
Noise floor	-90 dBm
Interference thresholds (mW)	[1, 3, 5, 7, 9, 10]

The simulation was run using a Monte Carlo method with 10000 sample runs such that a statistically significant result was obtained and the results were noted. This allows for most noise factors to be factored out (specifically from AWGN) and an averaged result to be obtained.

It was found that one of the most significant factors contributing to the error function's variance was the interference threshold parameter as specified in the simulation. This meant that the interference threshold parameter is critical in determining the performance of the channel estimator and the optimal placement of the new pilot sub-channel.

This problem was found to be exacerbated for a least squares estimator due to the estimation error at the pilot symbols being only a product of the inverse of the sub-channel SNR. Since the LS estimator, unlike the Minimum Mean Squared Error (MMSE) estimator, is not dependent on the knowledge of noise statistics, therefore the optimal positioning for the LS estimator, without considering interpolation error, would indeed be where the SNR is highest.

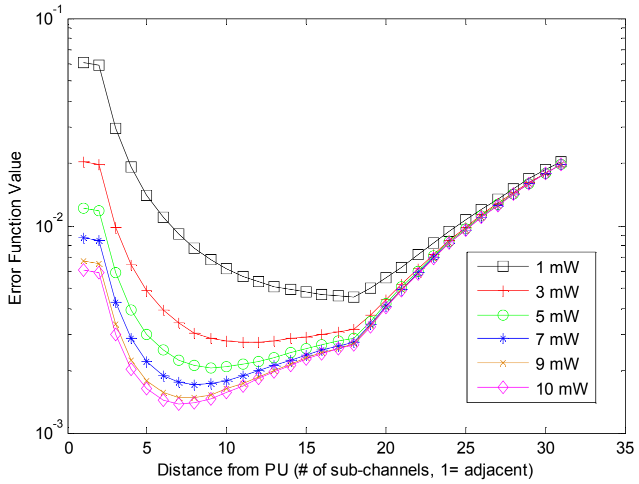

Figure 2. Error function values of the simulated system for parameters as set in table 1. Each curve represents the error function for a different threshold value where the lowest point of the curve is the optimal placement position for the pilot sub-channel.

In Figure 2, the error function is shown for the applied simulation parameters of table 1. The curves shown indicate the interference threshold power, this parameter is pre-set to define what the maximum amount of interference power may be transmitted to the PU by the SU. The first curve (squares) therefore shows the highest error function value but with a trade-off in that the interference threshold is as low as 1 mW. This also means that the pilot is indeed placed the farthest for the highest interference threshold (at 18 sub-channels away). The opposite can be observed for a high interference threshold, placing the new pilot sub-channel as close as 7 sub-channels away from the PU. An abrupt change is noticed for the error function values at sub-channel distance of 19, this is attributed to the fact that the distance component in the interpolation error begins to dominate the 2nd order derivative of the channel gain component. This is unlike what is noticed in Figure 3 where due to the rapidly varying channel gains, the effects are not noticed as abruptly.

In Figure 3, the error function is shown for the same simulation parameters as Figure 2 with the exception that a fast fading channel was used. This results in a decreased channel coherence bandwidth and as such makes the channel frequency response represent a more variant function. This in turn increases the error contribution of the interpolation error to the optimization error function.

It can be seen that the optimal pilot position has therefore moved closer to the PU (such as being placed as low as 5 sub-channels away for a 10 mW interference threshold).

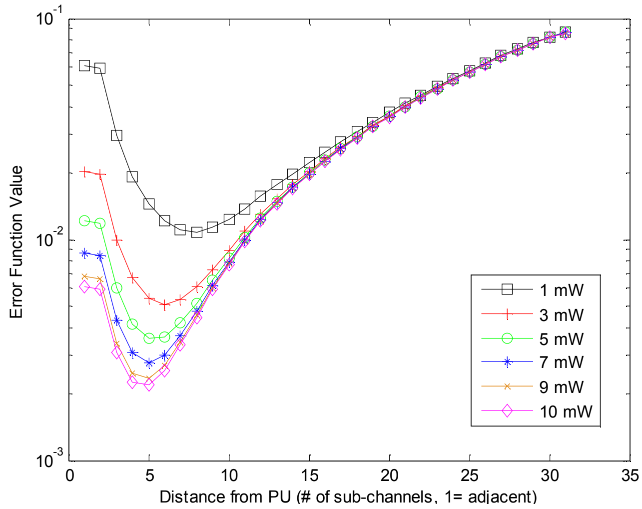

Figure 3. Error function values of the simulated system for parameters as set in table 1 but using a fast fading channel instead. Each curve represents the error function for a different threshold value where the lowest point of the curve is the optimal placement position for the new pilot sub-channel.

In Figure 4, the optimal pilot placement is shown for the given interference threshold parameters for both the fast fading and slow fading channel cases.

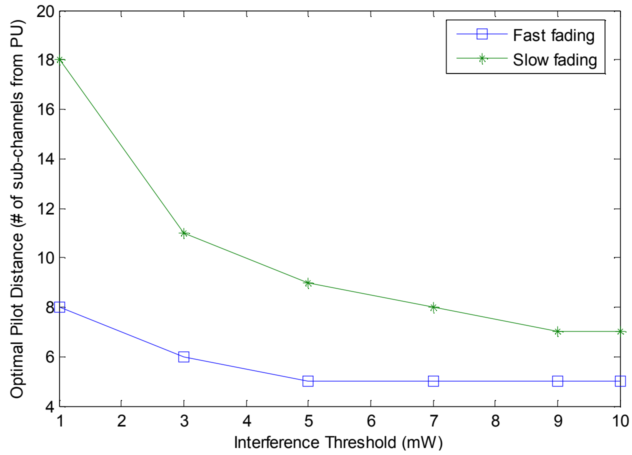

Figure 4. Optimal pilot placement (expressed as the separation distance between the pilot and the PU in number of sub-channels) for the fast-fading and slow fading channels.

V. CONCLUSION

A hypothesis of contradiction was noted between the optimal power loading and the optimal pilot-pattern algorithms for NC-OFDM cognitive radio systems. This meant that a compromise needed to be found such that the two contradictory ideas are implemented in the optimal way possible. An optimal solution for the simplified case was proposed in this paper as a proof of concept.

It was found that the interference threshold parameter greatly influences the pilot placement and hence the estimation error. This means that there is a trade-off where the desired interference threshold from the SU inversely affects the estimation error.

It was also discovered that a great dependency exists between the error function and the form of the channel frequency response. This was noted from the fact that the more variant the channel frequency response is (i.e. the less linear it is), the closer the new pilot sub-channels should be placed to the PU due to the greater interpolation error caused by having them move away.

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REFERENCES

[1] I. F. Akyildiz, W. Lee, M. C. Vuran and S. Mohanty, "NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey," Computer Networks, vol. 50, pp. 2127-2159, 9/15, 2006 [Nov 12, 2011].

[2] V. Valenta, Z. Fedra, R. Marsalek, G. Baudoin and M. Villegas, "Towards cognitive radio networks: Spectrum utilization measurements in suburb environment," in Radio and Wireless Symposium, 2009. RWS '09. IEEE, 2009, pp. 352-355 [Nov 12, 2011].

[3] J. Mitola III and G. Q. Maguire Jr., "Cognitive radio: making software radios more personal," Personal Communications, IEEE, vol. 6, pp. 13-18, 1999 [Nov 12, 2011].

[4] G. Bansal, M. J. Hossain and V. K. Bhargava, "Adaptive power loading for OFDM-based cognitive radio systems," in Communications, 2007. ICC '07. IEEE International Conference on, 2007, pp. 5137-5142 [Dec 4, 2011].

[5] I. Rashad, I. Budiarjo and H. Nikookar, "Efficient pilot pattern for OFDM-based cognitive radio channel estimation - part 1," in Communications and Vehicular Technology in the Benelux, 2007 14th IEEE Symposium on, 2007, pp. 1-5 [Dec 4, 2011].

[6] T. Weiss, J. Hillenbrand, A. Krohn and F. K. Jondral, "Mutual interference in OFDM-based spectrum pooling systems," in Vehicular Technology Conference, 2004. VTC 2004-Spring. 2004 IEEE 59th, 2004, pp. 1873-1877 Vol.4 [Dec 4, 2011].

[7] Shichang Zhang, Jun Wang and Shaoqian Li, "A channel estimation method for NC-OFDM systems in cognitive radio context," in Communication Systems, 2008. ICCS 2008. 11th IEEE Singapore International Conference on, 2008, pp. 208-212 [Dec 4, 2011].

[8] Chia-Horng Liu, "Adaptive two-dimensional channel estimation scheme for OFDM systems," in Cognitive Radio Oriented Wireless Networks and Communications, 2008. CrownCom 2008. 3rd International Conference on, 2008, pp. 1-5 [Dec 4, 2011].

[9] IEEE 802.16 Broadband Wireless Access Working Group. (2003-06-27, Channel models for fixed wireless applications).

[10] S. D. Conte and C. de Boor, "Interpolation by polynomials," in Elementary Numerical Analysis, an Algorithmic Approach, 3rd ed. McGraw-Hill, 1980 [Nov 12, 2011].

[11] B. Soubachov, N. Ventura, "Optimal Pilot Placement in Cognitive Radio Systems for Wiener Filtered MMSE Channel Estimation", in The First International Conference on Advances in Cognitive Radio, 2011. IARIA COCORA 2011, pp 54-59 [Dec 8, 2011].

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