Spectrum Leasing Game for Underlay Cognitive Ratio Network with Primary System Using Adaptive Rate-Based Pricing Strategy

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***Abstract*—In this paper, we propose a low-complexity spectrum leasing game for the underlay CR network with PS using adaptive rate-based pricing strategy. In the proposed scenario, an SU can make a request for sharing a channel with multiple PUs and pay for the spectrum lease in proportional to his transmission power. In the meantime, the PUs can determine the leasing price to maximize his own revenue at the cost of ignorable throughput degradation; In other words, one can say that the PU can actively protect himself from using the lower transmission rate by adaptively rising the leasing prize, rather than passively imposing interference-limit rule as in the conventional methods. The simulation results show that the proposed spectrum leasing game can grant SUs transmission opportunities without causing PUs throughput degradation. Moreover, the convergency of the proposed scheme is numerically proved, which accounts for the existence of Nash Equilibrium.**

Keywords-Game theory, auction, cognitive radio, pricing, spectrum leasing.

I. INTRODUCTION

Due to the scarceness of spectrum and the increasing demands for radio resources, the issue of radio resource management (RRM) has attracted much attention for decades, especially when the truth of extremely low spectrum efficiency had been revealed by the US Federal Communications Commission (FCC) [1]. To improve the spectrum efficiency, the technology of cognitive radio (CR) has been proposed to opportunistically and temporarily utilize the so-called spectrum holes [2]. Accordingly, many important CR-related topics have been extensively explored, including spectrum leasing (trading), spectrum sensing, sub-channel allocation, power and interference control, cooperative communications etc [3]–[8].

Speaking of spectrum leasing (trading), the auction theory, one of the important applications of Game theory, can be utilized to design effective protocols for managing the aforementioned interactions between primary system (PS) and secondary system (SS) as well as the competitions between secondary users (SUs) [9], [10]. Moreover, according to CR's operation modes, different spectrum leasing protocols can be designed for the purpose of protecting PS and improving the overall spectrum efficiency (more discussions can be found in the following literature survey). One should note that the key to the success of CR networks lies in the harmless interactions between the PS and SS. Therefore, from the literature, one can find that whatever the spectrum leasing protocols are applied, the well-known interference-limit (IT) rule is still pivotal to regulate SS's behaviors in the underlay CR networks.

In this paper, PUs can actively play the leading characters in the spectrum leasing game in stead of passively imposing the IT rule on SS. That is to say, rather than the IT rule, each PU can adaptively set and announce the price for spectrum sharing based on the degradation of his transmission rate. Note that the acceptable degradation level depends on each PU's willingness and tolerance. Then, each SU can play a one-shot game to decide the amount of power he can purchased from each of PU. Afterwards, the PU may adjust the price based his attainable transmission rate. The bidding process between PU and SUs may go back and forth for several times until Nash equilibrium (NE) is reached.

It should be noticed that, in the proposed scheme, all the SUs who share the same channel (owned by a particular PU) do not need to know whether the aggregated amount of interference has gone beyond the IT threshold or not, which may cost some considerable amount of message exchanges. Via the simulation results, the proposed spectrum leasing game can grant SUs transmission opportunities without causing PUs significant throughput degradation. Moreover, the existence of a unique NE can also be proved numerically, which accounts for the convergence of the proposed scheme.

The rest of this paper is organized as follows. In Section II, some important related works are reviewed. Section III describes the system model and proposed spectrum leasing game. Simulation results are provided in Section IV. Section V concludes this work.

II. LITERATURE SURVEY

Here, we review several auction-based spectrum leasing (trading) schemes in the underlay and interweave CR networks.

A. Underlay CR Operation Mode

In the underlay CR networks, PS can be protected by imposing the IT rule on the SS's transmissions. Thus, in the following auction games, the SS's transmission power is regulated by this rule. In [11], two auction mechanisms were

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designed to allocating power for SUs, of which an SU can be charged according to the received signal-to-interference-and-noise ratio (SINR) or transmission power. The charging policies, in other words bidding strategies, were to reach the pre-defined balance, i.e. NE, between SUs. In [12], the pricing method was included into the design of channel and power allocations for the CR networks. Then, the distributed price-based iterative water-filling (PIWF) algorithm as well as the corresponding media access control (MAC) protocol were proposed to reach NE. In [13], the authors allowed the primary users (PUs) to join the auction game by actively adjusting the tolerable amount of interference (the interference cap). Then, a dynamic spectrum leasing strategy was designed to control the transmission power for both PUs and SUs such that the utility functions of SUs can be maximized under the limit of the so-called interference cap.

In [14], the authors proposed a sub-optimal pricing strategy for PS to own better revenue, and SUs can also adjust their uplink transmission power to maximize their utility functions. The price was set according to the amount of each SU's transmission power. Via numerical proof, the proposed method was claimed to achieve fairness of power allocation between SUs. In [15], the Stackelberg model was applied to deal with the spectrum leasing problem between PS and SS. In additional to the payment by SUs (i.e. revenue of PUs), a shutdown mechanism was developed to prevent SUs from using unacceptable transmission power such that the IT rule can be satisfied. In [16], the problem of power control and relay selection in the multi-hop CR relay-network was investigated. An SU can pay prices to PUs who share the spectrum and SUs who relays for him to obtain optimal performance. The effects of difference pricing functions of PUs were investigated and several distributed power control algorithms were developed for the scenarios with single and multiple CR transmission pairs.

B. Interweave CR Operation Mode

In the interweave CR networks, PS can be protected by separating the access time slots or spectrum bands from those of SS, and (or) by the helpful cooperative transmissions from SS. In [17], for the purpose of maximizing quality-of-service (QoS), a PU can allure SUs for cooperative transmissions by offering a fracture of time slots in return. To own the offered fracture of time, SUs compete between each others following the distributed power control mechanism. The Stackelberg game was applied to model leader-follower relationship between PU and SUs. In [18], a PU can set a price for time sharing with a properly selected SUs and each SU can decide the amount of time to purchase according to his own QoS requirement. Note that the more time purchased, the more power should be used to forward packets for the PU. Moreover, an admission control mechanism was also develop to protect the SUs who participate in the cooperation transmissions. In [19], similar scenario was extended to multiple PUs, i.e. multiple sellers, by adopting the framework of generalized NE, of which these PUs competed with each other for the cooperative transmissions from SUs. And SUs can become willing to cooperate when the their QoS requirements can be achieved.

In [20], during the spectrum leasing period, each PU can preserve a required bandwidth for himself to satisfy his own QoS. Then, SUs can bid for the extra bandwidth, rather than the aforementioned a fracture of time, to maximize their utility function and achieve the fair allocations, i.e. NE. Similar scenario was also extended to multiple PUs. In [21], a generalized Branco's mechanism was proposed to tackle with the spectrum trading between primary service providers (PSPs) and secondary service providers. In the proposed model, the PSPs (seller) can cooperate to gain maximal profits and further share these profits. In [22], the bandwidth-sharing problem in the multi-hop relaying cellular network (MCN) was modeled by the reversed Stackelberg game. With the aid of a trust model, the base station (BS) can encourage relay station (RS) to cooperate and also discourage their misbehavior. Owing the allocated bandwidth, the RSs can serve the nearby mobile terminals. Via simulation results, the well-developed MCN cooperation can maximize the overall network performance.

III. SYSTEM MODEL AND SPECTRUM LEASING GAME

A. System Model

Now, we consider an underlay CR network (SS) imbedded in the incumbent cellular system (PS), which is assumed to be in the uplink transmissions using adaptive modulation and coding scheme (ACM) with fixed transmission power. Each PU exclusively own a subchannel and he may share this subchannel with multiple SUs. A CR transmitting end and a CR receiving end form a CR transmission pair, which means the CR network operates in the ad hoc mode. For simplicity, a CR transmission pair is named SU in the following context.

In the proposed spectrum auction game, SUs can issue spectrum sharing requests to the neighbor PUs. After receiving these requests, each of PU can decide whether or not to join this auction and then announce the price per unit of generated interference if the requests are accepted. Note that a higher price can resist SU from producing intolerable amount of interference. According to the prices, each SU can decide the amount of transmission power he can afford to allocate to each of the subchannels. Several rounds of the bidding process (i.e., the price setting and power allocations) can give the balanced portfolio (the so-called NE). In the following context, the whole period of bidding process is named bidding phase.

B. Power Allocation of SS

The utility function of the ith SU can be defined as

$$\begin{aligned} U_i^S &= \sum_{k\in\Omega_M} u_i^{Sk} \\ &= \sum_{k\in\Omega_M}\Bigg[\alpha_i \log_2\left(1+\frac{P_i^{Sk}G_{ii}^{Sk}}{\sum_{j\neq i, j\in\Omega_N} P_j^{Sk}G_{ji}^{Sk}+Q_{ki}^{PS}+N_o}\right) \\ &\quad -\lambda_k(P_i^{Sk}G_{ik}^{SP})\Bigg] \ , \end{aligned} \tag{1}$$

where u_i^{sk} represents the utility of the ith SU sharing kth subchannel; $\Omega_M = \{1,\cdots,M\}$ and $\Omega_N = \{1,\cdots,N\}$ respectively stand for the sets of PUs and SUs who join this auction; P_i^{Sk} is the ith SU's allocated power to the kth subchannel; G_{ij}^{Sk} means the channel gain between the ith SU's transmitter and jth SU's receiver over the kth subchannel, while G_{ik}^{SP} is that between the ith SU's transmitter and kth

PU's receiver, i.e. the base station (BS); Q_{ki}^{PS} denotes the ith SU's received interference from the kth PU; N_o accounts for the additive white Gaussian noise (AWGN); λ_k is the announced price for sharing the kth subchannel. Note that α_i is the adjusting weight factor which can personalize ith SU's characteristic. Moreover, observing (1), the first term and second term account for the capacity reward and spectrum sharing cost (in other words, interference penalty). Also, the summation explains the spectrum sharing property, i.e. an SU can share multiple subchannels with PUs.

The goal of SU is to maximize the utility function via a proper power allocation. Thus, we can now form an optimization problem as what follows.

$$\begin{aligned} &\max_{\mathbf{P}_i^S} \; U_i^S(\mathbf{P}_i^S, \mathbf{P}_{-i}^S) \\ &subject\ to \\ &0 \le P_i^{Sk} \le P_{max} \\ &0 \le \sum_{k \in \Omega_M} P_i^{Sk} \le P_{max} \ , \end{aligned} \tag{2}$$

where $\mathbf{P}_i^S = \{P_i^{S1}, ..., P_i^{SM}\}$ is the ith SU's power allocation vector, while $\mathbf{P}_{-i}^S$ describes that of all the other SUs belonging to Ω_N. Prior to solving this optimization problem, it should be noted that the concavity of $U_i^S(\mathbf{P}_i^S, \mathbf{P}_{-i}^S)$ with respect to $\mathbf{P}_i^S$ can be proved by calculating $\partial^2 U_i^S / \partial {P_i^{Sk}}^2$, of which the negativity renders the proofs of its concavity and existence of the maximum.

First, we relax the constraint of $0 \le P_i^{Sk} \le P_{max}$ and then solve the following KKT problem, which gives a power allocation scheme to satisfy this constraint. The KKT problem can be defined as

$$\begin{aligned} &\max_{\mathbf{P}_i^S} \; L(\mathbf{P}_i^S, \mu_i) \\ &subject\ to \\ &0 \le \sum_{k \in \Omega_M} P_i^{Sk} \le P_{max} \ , \end{aligned} \tag{3}$$

where

$$\begin{aligned} L(\mathbf{P}_i^S, \mu_i) = &\sum_{k \in \Omega_M} \left[\alpha_i \log_2 \left(1 + \frac{P_i^{Sk} G_{ii}^{Sk}}{\sum\limits_{j \ne i, j \in \Omega_N} P_j^{Sk} G_{ji}^{Sk} + Q_{ki}^{PS} + N_o} \right) \right. \\ &\left. - \lambda_k (P_i^{Sk} G_{ik}^{SP}) \right] + \mu_i \left[(P_{max} - \left(\sum_{k \in \Omega_M} P_i^{Sk} \right) \right] \ ; \end{aligned} \tag{4}$$

And μ_i is the Lagrangian multiplier. Solving $\partial L(\mathbf{P}_i^S, \mu_i) / \partial P_i^{Sk} = 0$ gives

$$P_i^{Sk^*} = \left[\frac{\alpha_i}{\ln 2 (\mu_i + \lambda_k G_{ik}^{SP})} - \frac{\Gamma_i^{Sk}}{G_{ii}^{Sk}} \right]_0^{P_{max}} \ , \tag{5}$$

where $\Gamma_i^{Sk} = \sum_{j \ne i, j \in \Omega_N} P_j^{Sk} G_{ji}^{Sk} + Q_{ki}^{PS} + N_o$. Note that μ_i can be adjusted to satisfy $\sum_{k \in \Omega_M} P_i^{Sk} = P_{max}$ such that higher capacity can be achieved.

C. Pricing Strategy of PS

Here, we define the utility function for PU.

$$U_k^P = \left[\lambda_k - C_k \cdot e^{\beta_k (1 - R_k^P / R_k^o)} \right] I_k^P \ , \tag{6}$$

where C_k is kth PU's reserved price (or the cost in other words) for spectrum leasing; β_k (similar to α_i) is the price weighting factor, which can adjust the sensitivity to the change of transmission rate; R_k^o and R_k^P denotes the original and current transmission rate of kth PU, respectively; $I_k^P = \sum_{i \in \Omega_N} (P_i^{Sk} G_{ik}^{SP})$ is the amount of experienced interference. Similar to the SS's case, calculating $\partial^2 U_k^P / \partial \lambda_k^2$ can prove the concavity of U_k^P with respect to λ_k as well as the existence of maximum. Also, observing (6), one can find that the lower the current transmission rate R_k^P, the higher the cost of spectrum sharing, which can result in a higher price (as proved by the following equation).

Solving $\partial U_k^P / \partial \lambda_k = 0$ gives the optimal pricing strategy of PU as

$$\lambda_k = C_k \cdot e^{\beta_k (1 - R_k^P / R_k^o)} - \frac{I_k^P}{(\partial I_k^P / \partial \lambda_k)} \ . \tag{7}$$

Note that whenever the price is changed, the variation of experienced amount of interference with respect to price, i.e. $\partial I_k^P / \partial \lambda_k$, can be estimated by the kth PU itself.

D. Discussions on Information Exchanges

To put the proposed spectrum auction game into practice, only two additional information exchanges over one control channel are needed: one for SU to broadcast the spectrum sharing request and one for PU to announce the spectrum sharing price (λ_k). Observing (5), Γ_i^{Sk} and G_{ii}^{Sk} can be estimated by some existing techniques (which are beyond the scope of this paper). Also, thanks to the channel reciprocality, G_{ik}^{SP} can be attained by listening to λ_k over the kth subchannel in the time-division duplex (TDD) mode. Furthermore, observing (7), I_k^P and $\partial I_k^P / \partial \lambda_k$ can surely be estimated by PU itself.

It should be noticed that conventionally, an IT rule should be imposed on SS in the underlay CR network. However, in the considered scenario that a PU can share his subchannel with multiple SUs, some massive information exchanges are required. For example, each of SUs should know the percentage of the amount of interference he has produced. Then the SU can actively control the transmission power such that the IT rule is not violated. Or, alternatively, a group of SUs can sequentially adjust their transmission power, which may additionally cause some difficult problems, e.g. how to form the group of SUs and inform each of them, the fairness issue, scheduling protocol design etc. Certainly, PU can also inform each of SUs to adjust transmission power. However, in either case, massive information exchanges as well as more additional control signals are required.

IV. SIMULATION RESULTS

The simulation environment is built up based on the system model described in Section III(A). In addition, PUs are uniformly distributed over the primary incumbent cell of radius 2000 (m). The transmitting end of each SU is randomly located at where the distance to BS is uniformly distributed over the

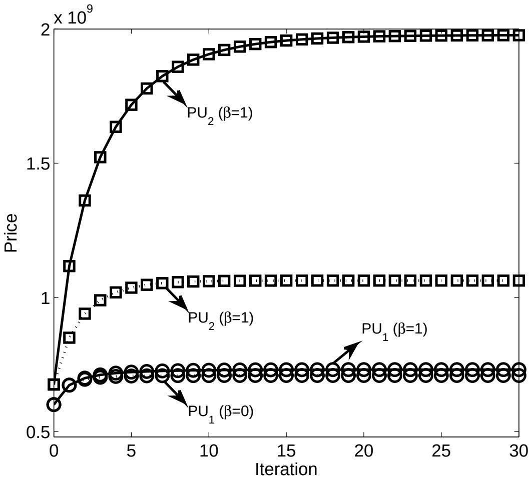


Figure 1: Convergence of PUs' prices (λ_1 and λ_2) with $\beta_1 = \beta_2 = 1$.

TABLE I: Adaptive modulation scheme of PU

Required SINR (dB)	6	10	18	24
Throughput (bps/Hz)	1	2	4	6

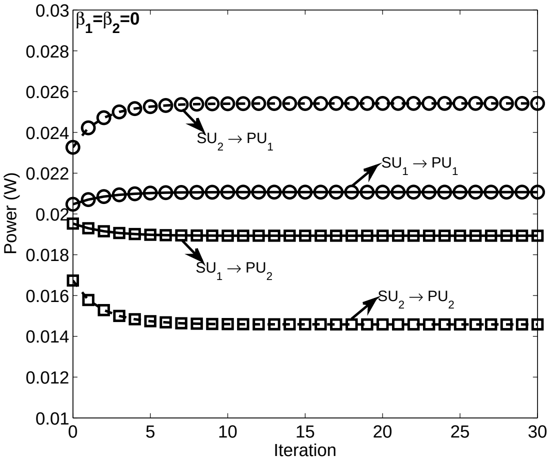
interval of [500 1000] (m); And, its corresponding receiving end is positioned randomly at where it is 50-150 meters away. The transmission power of PU and SU's maximum transmission power P_{max} are 1 and 0.1 watt, respectively. The power spectrum density of AWGN is -174 dBm/Hz. The log-distance path-loss model with exponent of three and flat Rayleigh fading are assumed. Furthermore, the stairwise effective throughput of ACM for PU are listed in Table I [23]. In the simulations, it is assumed that there are two PUs providing spectrum sharing opportunities to two SUs; And the results are averaged over 2×10^7 simulation rounds.

A. Convergency

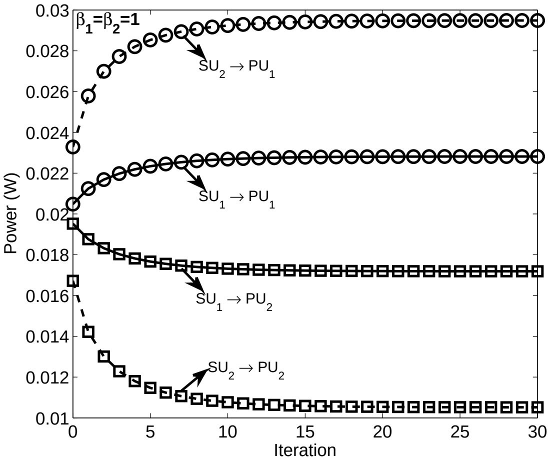
Here, we prove the convergence of the spectrum leasing game by showing the convergence of the PUs' price (λ_1 and λ_2) and SUs' transmission power. Figures 1 and 2 respectively show the snapshots of PU's price for $\beta_1 = \beta_2 = 1$, and SU's transmission power for $\beta_1 = \beta_2 = 0$ and 1. As shown in the figures, the bidding phase can be completed within ten iterations.

B. Impacts on PUs

In advance of showing the benefit SU can obtain through the spectrum auction, we first present its impact on PU. Figure 3 shows the impact of SU's activity on PU's transmission mode, i.e. ACM mode with (a) $\beta_1 = \beta_2 = 1$, and (b) $\beta_1 = \beta_2 = 4$, respectively. It can be observed that when the price weighting factor β equals to zero, PU can be severely affected, and consequently the probability of using lower ACM mode significantly rises. One should note that the case of $\beta = 0$ can be regarded as conventional method, i.e. simply maximizing the utility function without adaptively adjusting the cost C_k. In this situation, P_{max} in (2) can be regarded as the IT rule to regulate SU's transmission power. Fortunately, this unfavorable situation can be avoided by increasing the weighting factor β, i.e. sensitivity to the change of throughput. As shown in Figure 3(b), when β increases from zero to four, PU can almost maintain the same ACM mode.



(a) $\beta_1 = \beta_2 = 0$

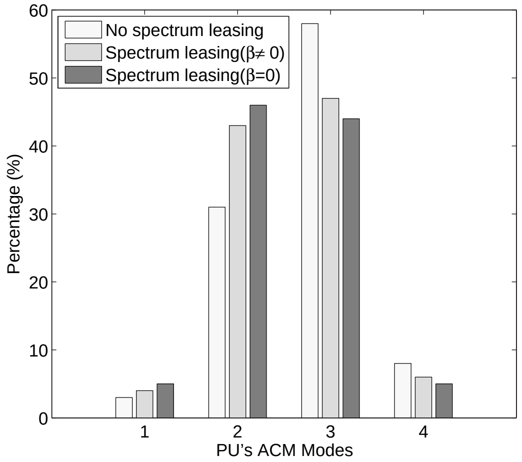


(b) $\beta_1 = \beta_2 = 1$

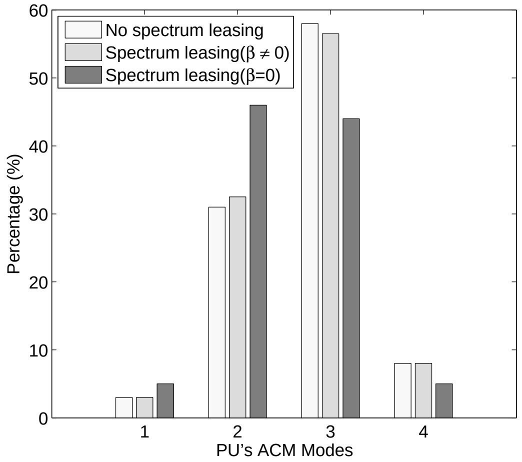
Figure 2: Convergence of SUs' transmission power with (a) $\beta_1 = \beta_2 = 0$, and (b) with $\beta_1 = \beta_2 = 1$.

C. Benefit to SUs

Figure 4 shows the average throughput of SU and PU with respect to various price weighting factors (β). One can find that higher throughput can be reached by SU with lower value of β. However, in this situation, it can cause larger loss of throughput to PU. As aforementioned, rising the β value can solve this dilemma. Therefore, using $\beta = 4$, both SU and PU can maintain at high throughput of 3.84 and 3.44 bps/Hz, respectively.

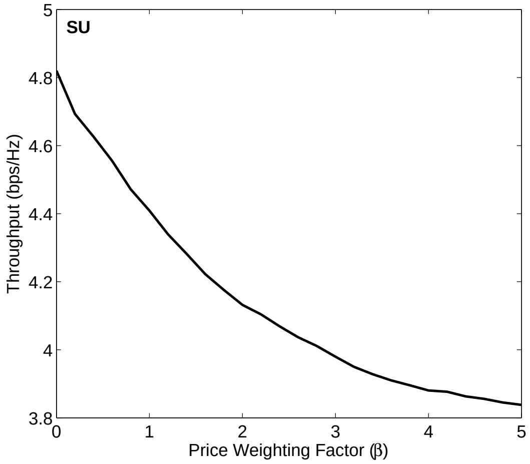


(a) $\beta_1 = \beta_2 = 1$

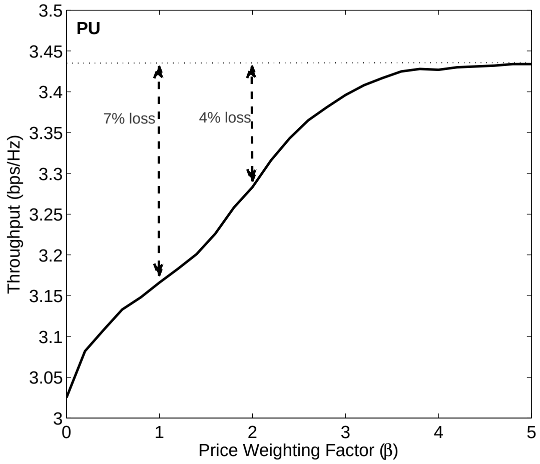


(b) $\beta_1 = \beta_2 = 4$

Figure 3: The impacts of SU's activity on PU's transmission mode, i.e. ACM mode, with (a) $\beta_1 = \beta_2 = 1$, and (b) $\beta_1 = \beta_2 = 4$.



(a) SU



(b) PU

Figure 4: The average throughput of SU and PU with respect to various price weighting factor (β).

V. CONCLUSION

In this paper, we have proposed a novel spectrum leasing game for the underlay CR network using rate-based pricing strategy for PU rather than the conventional interference-based method. By using the rate-based pricing strategy, several important advantages can be obtained. First, signaling overhead can be significantly decreased. Only two additional information exchanges (one for issuing SU's request and one for broadcasting the price of spectrum sharing) over one control channel are required. Second, PUs can actively join the auction and protect himself from using lower transmission rate by rising the price, instead of passively imposing the IT rule on SUs. Third, both PU and SU can simultaneously maintain at high transmission rates such that the overall spectrum efficiency can be largely improved. Many interesting future works are worth exploring, which could have potential impacts on the area of CR networks. For example: (1) apply the Stackelberg model to refine the spectrum auction game; (2) mathematically prove the existence and uniqueness of NE.

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