

Hybrid Approach Analysis of Energy Detection and Eigenvalue Based Spectrum Sensing Algorithms with Noise Power Estimation

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Abstract—Two particular semi-blind spectrum sensing algorithms are taken into account in this paper: Energy Detection (ED) and Roy’s Largest Root Test (RLRT). Both algorithms require the knowledge of the noise power in order to achieve optimal performance. Since by its nature the noise power is unpredictable, noise variance estimation is needed in order to cope with the absence of prior knowledge of the noise power: this leads to a new hybrid approach for both considered detectors. Probability of detection and false alarm with this new approach are derived in closed-form expressions. The impact of noise estimation accuracy for ED and RLRT is evaluated in terms of Receiver Operating Characteristic (ROC) curves and performance curves, i.e., detection/mis-detection probability as a function of the Signal to Noise Ratio (SNR). Analytical results have been confirmed by numerical simulations under a flat-fading channel scenario. It is concluded that both hybrid approaches tend to their ideal cases when a large number of slots is used for noise variance estimation and that the impairment due to noise uncertainty is reduced on RLRT w.r.t. ED.

Keywords—Cognitive radio; spectrum sensing; hybrid detectors; noise estimation

I. INTRODUCTION

Among the functionality provided by Cognitive Radio [1], Opportunistic Spectrum Access (OSA) is devised as a dynamic method to increase the overall spectrum efficiency by allowing Secondary Users to utilize unused licensed spectrum. For this Cognitive Radio System requires the implementation of a spectrum sensing unit in order to gain awareness of available transmission opportunities. This unit must indicate whether a transmission is taking place in the considered channel. Among several spectrum sensing methods put forward for Cognitive Radio applications, techniques based on eigenvalues of the received covariance matrix evolved as a promising solution for spectrum sensing outperforming classical ED.

Semi-blind spectrum sensing algorithms, i.e., ED and RLRT, are the optimum spectrum sensing techniques in a known noise power level scenario. However, in real systems the detector does not have a prior knowledge of the noise level. In recent years, variation and unpredictability of the precise noise level at the sensing device came as a critical issue, which is also known as noise uncertainty. With the goal of reducing the impact of noise uncertainty on the signal detection performance of ED and Eigenvalue Based Detection

(EBD), several research has been proposed including [2], [3], [4] for ED and [7], [8] for EBD. Hybrid spectrum sensing algorithms based on the combination of ED and Feature Detection techniques have been put forward for the reduction of the effect of noise variance uncertainty [5], [6]. Similar hybrid spectrum sensing approach was discussed in [9] using the positive points of ED and Covariance Absolute Value detection methods while Sequeira et al. [10] used Akaike Information Criterion (AIC), Minimum Description Length (MDL) and Rank Order Filtering (ROF) methods for estimation of noise power in presence of signal for energy based sensing. In [7], the importance of accurate noise estimation has been shown for better performance of the EBD algorithms.

This paper presents an idea of auxiliary noise variance estimation and focuses on the performance evaluation of Hybrid Approach of semi-blind detection algorithms, namely ED and RLRT, using the same estimated noise variance. The rest of this paper is organized as follows: the system model is developed in Section II, test statistics of detection algorithms is noted in Section III, noise estimation approaches in relation to ED and RLRT are discussed in Section IV, Hybrid Energy Detection and Hybrid Roy’s Largest Root Test schemes based on the noise estimation approaches are discussed in Section V and Section VI respectively, the simulation results and the effect of noise variance estimation on considered detection algorithms are discussed in Section VII and finally, Section VIII concludes the paper.

II. SYSTEM MODEL

We consider K sensors (receivers or antennas) for the ED / EBD detector, which senses and decides the presence or absence of the single primary signal within a defined spectrum band \mathbf{W} . In a given sensing time interval T , the detector calculates its detection statistic T_D by collecting N samples from each one of the K sensors. The received samples are stored by the detector in the $K \times N$ matrix \mathbf{Y} .

Let us introduce the $1 \times N$ signal matrix $\mathbf{S} \triangleq [s(1) \cdots s(n) \cdots s(N)]$ and the $K \times N$ noise matrix $\mathbf{V} \triangleq [v(1) \cdots v(n) \cdots v(N)]$ where,

- $s(n)$ is the transmitted signal sample at time n , modeled as Gaussian with zero mean and variance $\sigma_s^2 : s(n) \sim$

$\mathcal{N}_{\mathbb{C}}(0, \sigma_s^2)$

- $\mathbf{v}(n)$ is a noise vector at time n , modeled as Gaussian with mean zero and variance σ_v^2 : $\mathbf{v}(n) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{K \times 1}, \sigma_v^2 \mathbf{I}_{K \times K})$

As all the signal samples $s(n)$ of \mathbf{S} and the noise vectors $\mathbf{v}(n)$ of \mathbf{V} are assumed statistically independent, the detector must distinguish between Null and Alternate Hypothesis given by $\mathbf{Y}|_{H_0} = \mathbf{V}$ and $\mathbf{Y}|_{H_1} = \mathbf{h}\mathbf{S} + \mathbf{V}$ where, \mathbf{h} is the complex channel vector $\mathbf{h} = [h_1 \cdots h_K]^T$ assumed to be constant and memory-less during the sampling window.

Under \mathcal{H}_1 , the average SNR at the receiver is defined as,

$$\rho \triangleq \frac{\mathcal{E}\|\mathbf{x}(n)\|^2}{\mathcal{E}\|\mathbf{v}(n)\|^2} = \frac{\sigma_s^2 \|\mathbf{h}\|^2}{K\sigma_v^2} \quad (1)$$

where, $\|\cdot\|$ denotes the Euclidean norm and \mathcal{E} the mean operator. The sample covariance matrix is given by

$$\mathbf{R} \triangleq \frac{1}{N} \mathbf{Y}\mathbf{Y}^H \quad (2)$$

and $\lambda_1 \geq \cdots \geq \lambda_K$ its eigenvalues sorted in decreasing order.

III. TEST STATISTICS

The test statistic of ED and RLRT algorithms based on the scenario developed in Section II can be noted in the following subsections.

A. Energy Detection (ED)

ED computes the average energy of the received signal matrix \mathbf{Y} normalized by the noise variance σ_v^2 and compares it against a predefined threshold t_{ed} .

$$T_{ED} = \frac{1}{KN\sigma_v^2} \sum_{k=1}^K \sum_{n=1}^N |y_k(n)|^2. \quad (3)$$

If $T_{ED} < t_{ed}$ it decides in favor of Null Hypothesis \mathcal{H}_0 otherwise in favor of Alternate Hypothesis \mathcal{H}_1 . The detection probability $P_d = \text{Prob}\{T_{ED} > t_{ed} | \mathcal{H}_1\}$ and false alarm $P_{fa} = \text{Prob}\{T_{ED} > t_{ed} | \mathcal{H}_0\}$ probabilities for this detector are well-known in the literature (e.g., [11]).

B. Roy's Largest Root Test

Using the information of the received signal matrix \mathbf{Y} and assuming a perfect knowledge of the noise variance σ_v^2 and the channel parameter \mathbf{h} , test statistic for RLRT [12] is given by

$$T_{RLRT} = \frac{\lambda_1}{\sigma_v^2}. \quad (4)$$

RLRT is the optimum test algorithm under the "semi-blind" class of EBD algorithms, which is considered as the reference test in this class whose Detection and False Alarm Probability could be noted as [7],

$$P_{fa} = 1 - F_{TW2} \left(\frac{t_{rlrt} - \mu}{\xi} \right) \quad P_d = Q \left(\frac{t_{rlrt} - \mu_x}{\sigma_x} \right) \quad (5)$$

where, $F_{TW2}(\cdot)$ is the CDF of Tracy Widom Distribution of order 2. μ and ξ are centering and scaling parameter of a Tracy Widom Distribution given by,

$$\mu = \left[\left(\frac{K}{N} \right)^{\frac{1}{2}} + 1 \right]^2 \quad (6)$$

$$\xi = N^{-2/3} \left[\left(\frac{K}{N} \right)^{\frac{1}{2}} + 1 \right] \left[\left(\frac{K}{N} \right)^{-\frac{1}{2}} + 1 \right]^{1/3} \quad (7)$$

and finally, μ_x and σ_x^2 are mean and variance parameters of a Normal Distribution given by expression,

$$\mu_x = (1 + K\rho) \left(1 + \frac{K-1}{NK\rho} \right) \quad (8)$$

$$\sigma_x^2 = \frac{1}{N} (K\rho + 1)^2 \left(1 - \frac{K-1}{NK^2\rho^2} \right) \quad (9)$$

IV. NOISE ESTIMATION

It is evident that the knowledge of the noise power is imperative for the optimum performance of both ED and RLRT. Unfortunately, the variation and the unpredictability of noise power is unavoidable. Thus, the knowledge of the noise power is one of the critical limitations especially of semi-blind detection algorithms for their operation in low SNR.

A. Offline noise estimation: Hybrid approach 1

In the first type of hybrid approaches (HED1 and HRLRT1), noise variance is estimated from S auxiliary noise-only slots in which we are sure that the primary signal is absent.

Consider a sampling window of length M prior and adjacent to the detection window containing noise-only samples for sure. Then, the estimated noise variance from the noise-only samples using a Maximum Likelihood noise power estimation can be written as,

$$\hat{\sigma}_{v1}^2 = \frac{1}{KM} \sum_{k=1}^K \sum_{m=1}^M |v_k(m)|^2 \quad (10)$$

If the noise variance is constant, the estimation can be averaged over S successive noise-only slots and (10) can be modified by averaging over S successive noise-only slots as,

$$\hat{\sigma}_{v1}^2(S) = \frac{1}{KSM} \sum_{s=1}^S \sum_{k=1}^K \sum_{m=1}^M |v_k(m)|^2 \quad (11)$$

A possible scheme of RLRT/ED detection algorithm using offline noise estimation approach is shown in Fig. 1: t_{tot} represents a periodic time interval divided into a training phase (noise estimation) and a runtime phase (detection). The runtime interval can be much longer than the training one, however the noise estimation needs to be updated after t_{tot} .

B. Online noise estimation: Hybrid approach 2

In a real time scenario, it is difficult to guarantee the availability of signal free samples so as to estimate the noise variance. Some literature analyzed the performance of ED using estimated noise variance setting aside a separate frequency channel for the measurement of the noise power [13]. However, it is not always suitable to assume uniformly distributed noise in all the frequency bands of concern.

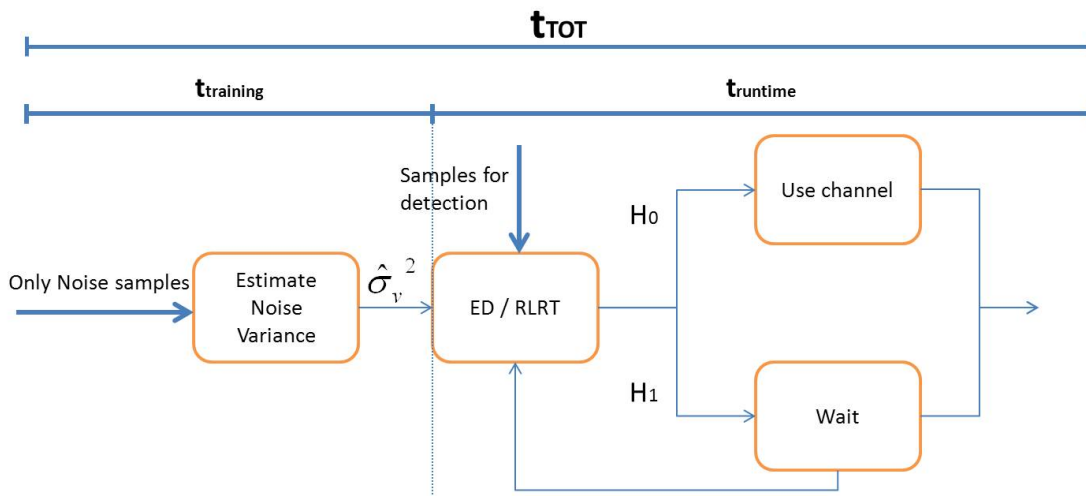


Fig. 1. HED1 / HRLRT1 with offline noise estimation approach

The second hybrid approach does not resort to the existence of auxiliary noise-only slots, but estimates the noise variance information from the previous slots declared as \mathcal{H}_0 by the algorithm. Now, the noise variance estimated from those S auxiliary noise-only slots (previously declared \mathcal{H}_0) is used in the following detection interval to get the decision about the presence or absence of the primary signal.

Given P_S the probability of receiving primary signal plus noise, P_d is probability of detection, and S is the number of slots, the Maximum Likelihood noise variance estimate $\hat{\sigma}_{v_2}^2(S)$ using M received signal samples declared noise samples by the detector from K receivers is given by,

$$\frac{\left[\sum_{s=1}^{S_S} \sum_{k=1}^K \sum_{m=1}^M |h_k s(m) + v(m)|^2 + \sum_{s=1}^{S_N} \sum_{k=1}^K \sum_{m=1}^M |v_k(m)|^2 \right]}{KMS} \quad (12)$$

where, $S_S = SP_S(1 - P_d)$ is the number of primary signal slots missed by the detector and $S_N = S - S_S$ is the number of noise samples successfully detected.

Fig. 2 shows a possible scheme of RLRT/ED detection algorithm using online noise estimation approach; after a transient stage (offline noise estimation), the detector automatically updates the noise estimation after S slots declared \mathcal{H}_0 (sliding window). Unlike the first approach, no further training offline phases are required.

V. HYBRID ENERGY DETECTION

Incorporating the *offline noise estimation* and *online noise estimation* described in Section IV in ED, hybrid approaches of ED are developed and their performance parameters are derived in the following subsections.

A. Hybrid ED approach 1 (HED1)

The Energy Detection Test Statistic in (3) can be modified to HED1 test statistic using (11) as,

$$T_{HED1} = \frac{1}{KN\hat{\sigma}_{v_1}^2(S)} \sum_{k=1}^K \sum_{n=1}^N |y_k(n)|^2 \quad (13)$$

Moreover, (13) can be considered as the parametric likelihood ratio test when the signal to be detected is assumed to be Gaussian with zero mean and variance σ_s^2 .

Under Null Hypothesis, after rigorous simplification, the test statistic in (13) could be approximated with a Normal Random Variable whose Probability of False Alarm P_{fa}^{HED1} for number of sensors K , number of samples N , number of auxiliary slots S and threshold t_{hed1} is given by,

$$P_{fa}^{HED1} = Q \left[\frac{t_{hed1} - 1}{\sqrt{\frac{MS + Nt_{hed1}^2}{KMS}}} \right] \quad (14)$$

Similarly, under Alternate Hypothesis, the test statistic in (13) also approximates to Normal Random Variable with different mean and variance parameters whose Probability of Detection P_d^{HED1} could be written as,

$$P_d^{HED1} = Q \left[\frac{(t_{hed1} - 1 - \rho)}{\sqrt{\frac{t_{hed1}^2}{KMS} + \frac{K\rho^2 + 2\rho + 1}{KN}}} \right] \quad (15)$$

B. Hybrid ED approach 2 (HED2)

Using (12), decision statistic of HED2 can be written as,

$$T_{HED2} = \frac{1}{KN\hat{\sigma}_{v_2}^2(S)} \sum_{k=1}^K \sum_{n=1}^N |y_k(n)|^2 \quad (16)$$

Under Null Hypothesis, after rigorous simplification, the test statistic in (16) could be approximated with a Normal Random Variable whose False Alarm Probability P_{fa}^{HED2} for number of sensors K , number of samples N , number of auxiliary slots S for noise estimation using (12) and threshold t_{hed2} is given by,

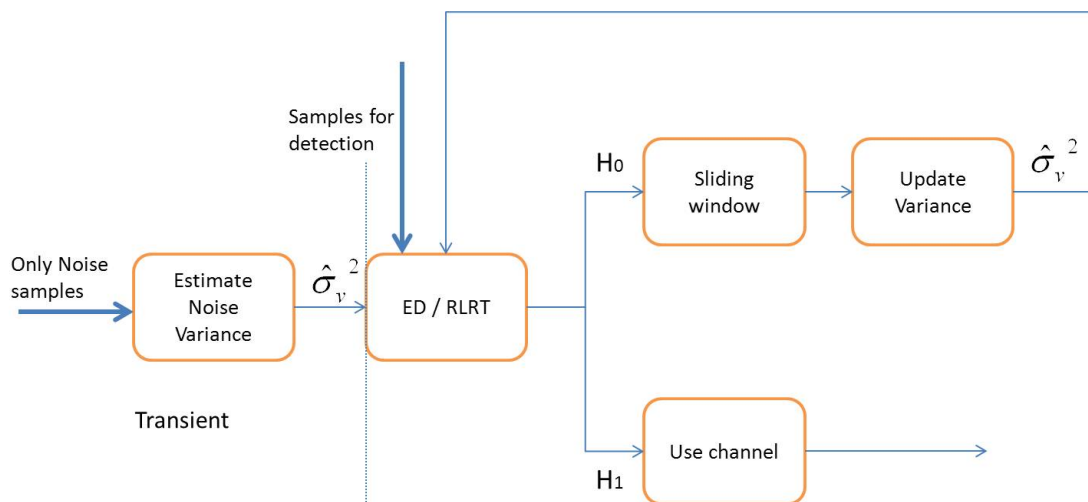


Fig. 2. HED2 / HRLRT2 with online noise estimation approach

$$P_{fa}^{HED2} = Q \left[\frac{t_{hed2} - \frac{S}{S + \rho S_S}}{\sqrt{\frac{t_{hed2}^2 NC + MS^2}{KMN(S + \rho S_S)^2}}} \right] \quad (17)$$

where, $C = (S_S K \rho^2 + \rho S_S + S)$.

Similarly, under Alternate Hypothesis, the test statistic in (16) also approximates to Normal Random Variable with different mean and variance parameters whose Probability of Detection P_d^{HED2} in a similar scenario could be written as,

$$P_d^{HED2} = Q \left[\frac{t_{hed2} - \frac{S(\rho + 1)}{S + \rho S_S}}{\sqrt{\frac{t_{hed2}^2 NC + MS^2(K\rho^2 + 2\rho + 1)}{KMN(S + \rho S_S)^2}}} \right] \quad (18)$$

VI. HYBRID ROY'S LARGEST ROOT TEST

In a similar way as for ED, if we incorporate *offline noise estimation* and *online noise estimation* in RLRT, hybrid approaches of RLRT are developed and their performance parameters are derived in the following subsections.

A. Hybrid RLRT approach 1 (HRLRT1)

HRLRT1 is a similar approach as HED1, which deals with the study of detection performance of the RLRT algorithm using estimated noise variance. Noise variance is estimated from S auxiliary noise-only slots where we are sure that the primary signal is absent. Using the ML estimate of the noise variance (11), the decision statistic of HRLRT1 can be expressed as,

$$T_{HRLRT1} = \frac{\lambda_1}{\hat{\sigma}_{v_1}^2(S)} \quad (19)$$

Under Null Hypothesis, after rigorous simplification, the test statistic in (19) could be approximated to the ratio of a Tracy

Widom Random Variable of order 2 and a Normal Random Variable. Hence, the False Alarm Probability P_{fa}^{HRLRT1} for number of sensors K , number of samples N , number of auxiliary slots S for noise estimation using (11) and threshold t_1 is given by,

$$P_{fa}^{HRLRT1} = 1 - F_0^{H1}(t_1) \quad (20)$$

where $F_0^{H1}(t_1)$ is the Cumulative Density Function CDF of the Probability Density Function shown below,

$$f_0^{H1}(t_1) = C_1 \int_{-\infty}^{+\infty} |x| f_{TW2} \left(\frac{xt_1 - \mu}{\xi} \right) e^{-\frac{D(x-1)^2}{4}} dx \quad (21)$$

with $f_{TW2}(\cdot)$ being the pdf of Tracy Widom Distribution and $C_1 = \frac{1}{2\xi} \sqrt{\frac{D}{\pi}}$.

Similarly, under Alternate Hypothesis, the test statistic in (19) approximates to Normal Random Variable whose Probability of Detection P_d^{HRLRT1} under a similar scenario is given by,

$$P_d^{HRLRT1} = Q \left(\frac{t_1 - \mu_x}{\sqrt{\frac{2t_1^2}{D} + \sigma_x^2}} \right) \quad (22)$$

where, μ_x (8) and σ_x^2 (9) are mean and variance of a Normal Random Variable.

B. Hybrid RLRT approach 2 (HRLRT2)

HRLRT2 is an alternate hybrid approach of RLRT where noise variance given by (12) is estimated from the previously received signal slots declared as \mathcal{H}_0 by the algorithm. The decision statistic of HRLRT2 can be written as,

$$T_{HRLRT2} = \frac{\lambda_1}{\hat{\sigma}_{v_2}^2(S)} \quad (23)$$

Under Null Hypothesis, after rigorous simplification, the test statistic in (23) could be approximated to the ratio of a Tracy

Widom Random Variable of order 2 and a Normal Random Variable. Hence, the False Alarm Probability P_{fa}^{HRLRT2} for number of sensors K , number of samples N , number of auxiliary slots S for noise estimation using (12) and threshold t_2 is given by,

$$P_{fa}^{HRLRT2} = 1 - F_0^{H2}(t_2) \quad (24)$$

where, $F_0^{H2}(t_2)$ is the Cumulative Density Function *CDF* of the Probability Density Functions shown below,

$$f_0^{H2}(t_2) = C_2 \int_{-\infty}^{+\infty} |x| f_{TW2} \left(\frac{xt_2 - \mu}{\xi} \right) e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} dx \quad (25)$$

with $C_2 = \frac{1}{\xi\sigma_1^2\sqrt{2\pi}}$.

Similarly, under Alternate Hypothesis, the test statistic in (23) approximates to Normal Random Variable whose Probability of Detection P_d^{HRLRT2} under a similar scenario is given by,

$$P_d^{HRLRT2} = Q \left(\frac{t_2 - \mu_x / \mu_1}{\sqrt{\frac{t_2^2 \sigma_1^2 + \sigma_x^2}{\mu_1^2}}} \right) \quad (26)$$

where, μ_x (8), μ_1 (27) and σ_x^2 (9), σ_1^2 (28) are mean and variance parameters with,

$$\mu_1 = \frac{S + S_S}{S} \quad (27)$$

$$\sigma_1^2 = \frac{S + 2\rho S_S + \rho^2 K S_S}{K M S^2} \quad (28)$$

VII. SIMULATION RESULTS

This section shows the simulation of the ROC curves and performance curves of hybrid approaches of ED and RLRT spectrum sensing algorithms. The accuracy of the the closed-form expressions is confirmed by the results presented in Fig. 3 and Fig. 4, respectively, where the theoretical formulas are compared against the simulated detection performance over S auxiliary noise-only slots (S ranges from 1 to 8). Perfect match of the theoretical and the numerical curve validates the considered model. As it can be noticed, with the increase in the number of auxiliary slots used for the estimation of the noise variance, the probability of detection increases for both hybrid approaches.

Fig. 5 illustrates the comparison of ED, HED1 and HED2 performance as a function of the SNR. Performance of HED1 and HED2 varies typically around 0 dB SNR but no visible difference can be noted in extreme high or low SNR values. Since there is a chance of mis-interpretation of noise plus primary signal as only-noise samples (used to estimate the noise variance) by ED in case of HED2, performance of HED2 is slightly lower than HED1 near 0 dB of SNR. By increasing the number of slots used for the estimation of the noise variance, the gap between HED1 and HED2 decreases and both approaches approximate the known-variance ED curve.

The convergence of the hybrid approach of RLRT to an ideal RLRT (known variance) is illustrated in Fig. 6. By increasing

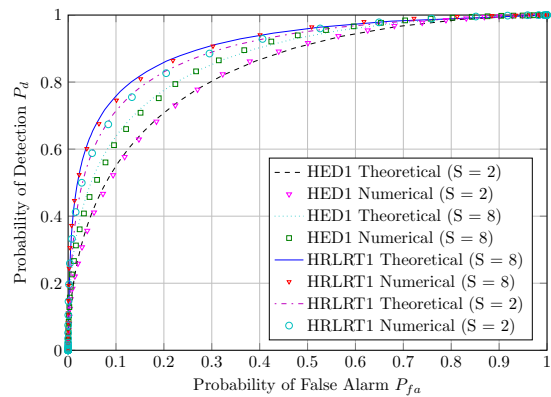


Fig. 3. Theoretical and numerical ROC plot of hybrid approach 1 of ED/RLRT. Parameters: $N = 80, M = 80, K = 4$ and $SNR = -10dB$

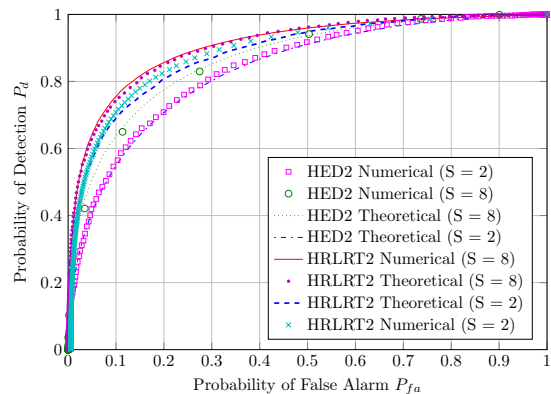


Fig. 4. Theoretical and numerical ROC plot of hybrid approach 2 of ED/RLRT. Parameters: $N = 80, M = 80, K = 4$ and $SNR = -10dB$

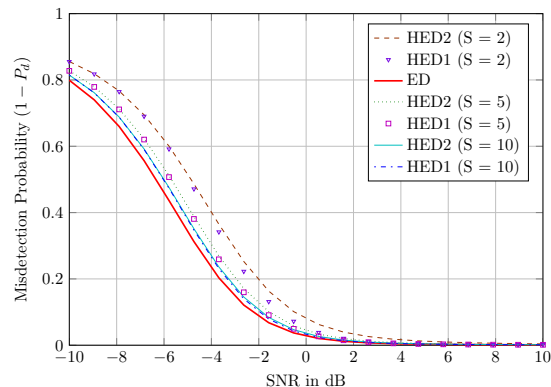


Fig. 5. Performance curves of ED and its hybrid approaches. Parameters: $N = 10, M = 10, K = 5$ and $P_{fa} = 0.05$

the number of auxiliary slots used for the estimation of noise variance, the performance of HRLRT1 and HRLRT2 converge at the ideal RLRT performance.

The performance of HED1 and HRLRT1 is compared in Fig. 7. The noise variance is estimated using (11) from S auxiliary sure noise-only slots. The curves approach the ideal ED and RLRT curves by increasing the number of auxiliary slots S , but the rate of convergence of HED1 is slower.

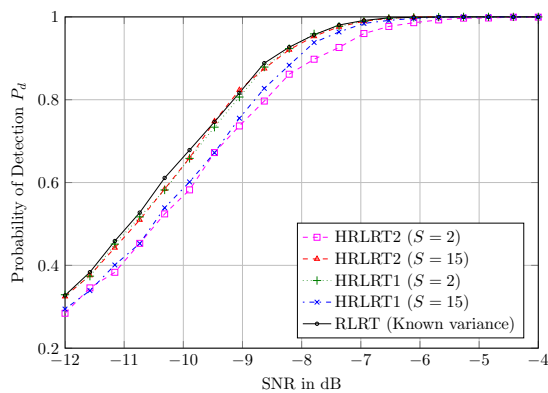


Fig. 6. Performance curves of RLRT and its hybrid approaches. Parameters: $N = 80, M = 80, K = 4$ and $P_{fa} = 0.05$

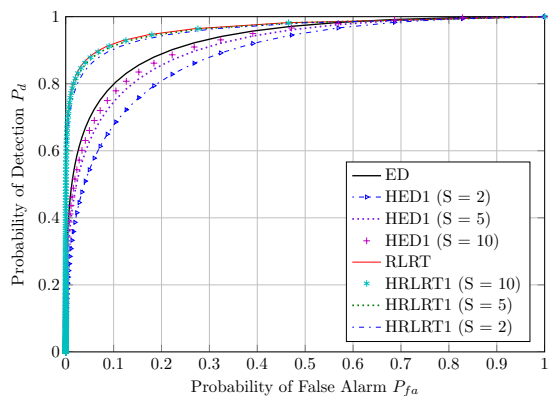


Fig. 7. ROC curves of ED/RLRT and their hybrid approach 1 (HED1/HRLRT1). Parameters: $N = 80, M = 80, K = 4$ and $SNR = -10dB$

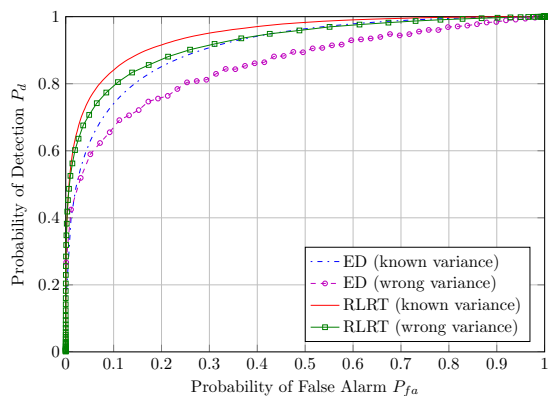


Fig. 8. Effect of Noise Variance fluctuation on ED and RLRT. Parameters: $N = 100, K = 4, var(\hat{\sigma}_v^2) = 0.0032(-25dB)$ given nominal noise variance $\sigma_v^2 = 1$.

The effect of the noise variance estimation uncertainty on ED and RLRT algorithms is considered in Fig. 8. Assuming the Gaussian distribution of the noise variance estimate with mean equal to nominal value, the ROC for ED and RLRT is plotted, setting $var(\hat{\sigma}_v^2) = 0.0032(-25dB)$. The result shows that, for the same uncertainty of the noise variance estimate,

the performance gap between the ideal curve and the curve with wrong variance is larger for ED as compared to RLRT. Thus, it can be easily noticed that RLRT is more robust to noise variance uncertainty as compared to ED algorithm.

VIII. CONCLUSION

In this paper, the analysis of two semi-blind spectrum sensing algorithms, ED and RLRT, is extended to hybrid approaches. Analytical expressions for the performance parameters, P_d and P_{fa} , are derived for each algorithm. Analytical results are verified by Monte Carlo simulation and by numerical methods. In addition, the impact of noise variance estimation on ED and RLRT was carried out based on ROC curves. The results showed that the fluctuation of noise variance estimate from nominal value is severe in case of small number of auxiliary slots used for the estimation of the noise variance. Moreover, for the same uncertainty on the noise variance estimate, the performance gap between the ideal curve and the curve with wrong variance is larger for ED as compared to RLRT.

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