

# FBMC/COQAM: An Enabler for Cognitive Radio

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**Abstract**—To solve the issue of spectrum scarcity, new paradigms of spectrum access must be investigated. The Cognitive Radio (CR) is an efficient solution to response to the requirement. But it needs the support from physical layer design, especially the signal modulation. In the literature, Filter-Bank-based Multi-Carrier with Offset QAM (FBMC/OQAM) and Generalized Frequency Division Multiplexing (GFDM) were proposed as suitable enablers for CR. However, both of them have some drawbacks that eventually prevent them from a practical usage. In this paper, we present an improved modulation scheme, which successfully combines these two schemes into one named FBMC/Circular OQAM (FBMC/COQAM). With this scheme, we are able to overcome all the drawbacks of its predecessors, while still keeping their advantages.

**Keywords**-Cognitive Radio; FBMC; GFDM; OQAM.

## I. INTRODUCTION

In the vision towards future radio systems, the spectrum scarcity tends to be an inevitable issue, which urgently calls for some new spectrum access paradigms. The CR as an emerging application effectively addresses the requirement for efficient spectrum usage. Nevertheless, the implementation of CR solutions needs the support from a suitable modulation. Indeed, one important requirement is that the modulation scheme must provide a good spectrum localization. Orthogonal Frequency Division Multiplexing (OFDM) has been widely used in today's radio communications. However, the main drawback of the OFDM systems is the rectangular pulse shaping, which leads to an unsatisfactory energy localization in frequency domain. This drawback can cause several consequences in CR applications because the high out-of-band radiation may severely pollute the neighbors in the adjacent bands. To overcome this problem or, otherwise said, to enable the CR concept, more advanced modulation scheme is necessarily required. During the past years, two main alternatives have been proposed in the CR field, i.e., FBMC/OQAM and GFDM.

The FBMC/OQAM scheme shifts the conventional OFDM paradigm [1]. The key idea is that the modulated data at each subcarrier is shaped by a well-designed prototype filter, which is different from the rectangular pulse in OFDM, so that it indeed provides a large number of degrees of freedom to use different waveform shapes that can be intentionally optimized towards the localized frequency content, straightforwardly enabling the CR applications [2]. Since then, many research works have been conducted towards the CR applications, in particular for FBMC/OQAM [3]–[7]. However, since the FBMC/OQAM is a continuous transform based scheme, it

is not easy to employ a Cyclic Prefix (CP), which makes it less robust against the frequency selective fading. Another alternative, which is the GFDM, extends the traditional OFDM paradigm to a general framework by introducing a circular filtering at each subcarrier with an improved pulse shape. By this way, it trades the orthogonality with the possibility of using a non-rectangular pulse. This is the reason why the GFDM is recognized as a non-orthogonal system [8]. The GFDM was first presented for the communication over TV white space. Later, due to its good out-of-band energy attenuation, it rapidly captured a lot of attention in the CR field [9]–[12]. Nevertheless, its non-orthogonality could be a vital drawback in a practical usage. The in-band interference severely causes a performance degradation, which further was proved to be pulse shape dependent [13]. To mitigate the performance degradation, one possible solution is to use an iterative interference cancellation [11]. Thus, the receiver complexity gets largely increased and might even get exploded when considering it together with Multiple-Input-Multiple-Output (MIMO) transmission. Although, it is absolutely necessary to seek for an advanced Multi-Carrier Modulation (MCM) scheme at a price of increased complexity, when it exceeds an affordable limit, the practical implementation of such system cannot be envisaged in a near future.

In this paper, we propose a novel MCM concept that successfully combines FBMC/OQAM with GFDM. To be specific, we adopt the circular filtering for the FBMC/OQAM, which makes it a block transform scheme. Such that the CP can be easily inserted. We call this scheme FBMC/COQAM. Moreover, the core part of this scheme remains the FBMC/OQAM structure, which indeed allows to guarantee a true orthogonality system. Thus, the non-orthogonality issue of GFDM can be solved and in the meantime it enhances the robustness against the fading channels. Ultimately, it shows a good suitability for CR applications. The rest of the paper is organized as follows: in Section II, we give a brief recall of FBMC/OQAM and GFDM schemes. In Section III, we present the concept of the FBMC/COQAM and its motivation behind. In Section IV, we detail the FBMC/COQAM transmitter design in radio transmission. In Section V, we evaluate the FBMC/COQAM efficiency. Some conclusions are drawn in Section VI. For simplicity, in the following, we ignore the term FBMC for FBMC/OQAM and FBMC/COQAM, respectively.

## II. OQAM AND GFDM BACKGROUND

In this section, we give a brief introduction of the OQAM and GFDM schemes which clearly shows the difference be-

tween these two schemes, paving the way for an introduction of our proposal.

### A. OQAM modulation

The remarkable contribution of the OQAM concept is that it introduced a staggered transmission structure, which allows it to escape from the Balian-Low Theorem (BLT) [14]. So that the OQAM scheme can simultaneously employ an improved pulse shape; keep full orthogonality; and transmit at the Nyquist rate. Contrary to the OFDM scheme that transmits complex-valued at subcarriers, in the staggered structure, the real and imaginary parts of the complex-valued symbols are transmitted separately with a delay of half OFDM symbol duration. More details for the OQAM concept can be found in [1] and the references therein. The baseband OQAM modulated signal writes as [15]

$$s_{\text{OQAM}}[k] = \sum_{m=0}^{M-1} \sum_{n \in \mathcal{Z}} a_{m,n} \underbrace{g[k - nN] e^{j \frac{2\pi}{M} m(k - \frac{D}{2})}}_{g_{m,n}[k]} e^{j\phi_{m,n}}, \quad (1)$$

where  $M$  is the number of carriers;  $g$  is the prototype filter with a length of  $L_g$  and  $D = L_g - 1$  (here,  $g$  is assumed to be real-valued and symmetrical);  $N = M/2$  is the discrete-time offset;  $\phi_{m,n}$  is an additional phase term at subcarrier  $m$  and symbol index  $n$  which can be expressed as  $\frac{\pi}{2}(n + m)$ . The transmitted symbols  $a_{m,n}$  are real-valued. They are obtained from a QAM constellation, taking the real and imaginary parts of these complex-valued symbols. To address a perfect reconstruction of real symbols, the prototype filter must satisfy the orthogonality condition:

$$\Re \left\{ \sum_{k \in \mathcal{Z}} g_{m,n}[k] g_{p,q}^*[k] \right\} = \delta_{m,p} \delta_{n,q}, \quad (2)$$

where  $*$  denotes the complex conjugation,  $\delta_{m,p} = 1$  if  $m = p$  and  $\delta_{m,p} = 0$  if  $m \neq p$ .

### B. GFDM modulation

The idea of GFDM is to group a set of complex-valued symbols from time-frequency lattice into one block. Then, for each block, a subcarrier-wise processing is carried out, which includes the up-sampling, pulse shaping, tail biting and finally is followed by a modulated operation to a set of subcarrier frequencies (cf. [8, Fig. 1]). The baseband GFDM modulated signal of one block, i.e., for  $k \in [0, MK - 1]$ , is expressed as

$$s_{\text{GFDM}}[k] = \sum_{m=0}^{M-1} \sum_{n=0}^{K-1} c_m[n] \tilde{h}[k - nM] e^{j \frac{2\pi k m}{N}}, \quad (3)$$

with  $M$  the subcarrier number;  $K$  the number of symbol slots considered in one block;  $c_m[n]$  the complex-valued symbols at  $m$ -th carrier and  $n$ -th symbol slot. The pulse shape  $\tilde{h}[k]$  indicates a periodic repetition of the prototype filter  $h[k]$  with a period of  $MK$ , i.e.,

$$\tilde{h}[k] = h[\text{mod}(k, MK)]. \quad (4)$$

The periodic filter is used to realize the circular convolution at the transmitter, which is equivalent to the tail biting process [8]. Note that there does not exist any orthogonality condition for the filter design because the GFDM itself is a non-orthogonal system.

## III. COQAM: MOTIVATION AND CONCEPT

As previously stated, the OQAM and GFDM schemes inherit some weak points. For the OQAM scheme, due to its continuous transform nature, it is not straightforward to use a CP. Thus, in the radio environment, the frequency selective fading usually ruins the orthogonality of the OQAM system, leading to a performance degradation. Therefore, more complex multi-tap equalizers are needed to warrant the quality of the transmission [16]–[18]. Similar to the OQAM, the GFDM itself is a non-orthogonal scheme. Thus, the performance must be guaranteed with a more powerful receiver design. The authors in [13] have shown that even in a distortion free channel the matched filter cannot address a satisfactory performance. Although the degradation can be relieved by an appropriate filter, with an example of a raised cosine filter given in [13], the filter must be driven close to its ideal limit, which increases the complexity. Alternatively, the GFDM can employ an iterative interference cancellation method [11], resulting, nevertheless, in a high complex receiver, in particular when it is in combination with MIMO transmission.

The drawback of OQAM is due to the fact that the OQAM is not a block processing scheme, while the drawback of GFDM is due to that its non-orthogonality is restricted by the BLT. Knowing the problematic for these schemes, an intuitive question is whether we can find an improved modulation that keeps all the benefits of OQAM and GFDM and at the same time gets rid of their drawbacks. With this motivation in mind, we investigated a new MCM scheme called COQAM, whose idea is to replace the linear convolution inherited in the OQAM with a circular convolution used in the GFDM. By this way we get a modulation scheme which, as CP-OFDM and GFDM, corresponds to a block transform. The baseband COQAM modulation structure is depicted in Fig. 1. For a discrete-time signal  $s[k]$  defined in a block interval such

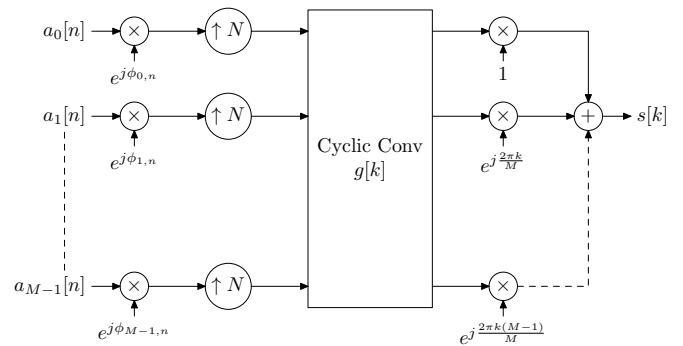


Figure 1. FBMC/COQAM baseband modulation.

that  $k \in [0, MK - 1]$ , the baseband COQAM modulation is expressed as

$$s_{\text{COQAM}}[k] = \sum_{m=0}^{M-1} \sum_{n=0}^{K-1} a_m[n] \tilde{g}[k - nN] e^{j \frac{2\pi}{M} m(k - \frac{D}{2})} e^{j\phi_{m,n}}, \quad (5)$$

with  $K$  the number of real symbol slots per each block. Like GFDM, to implement a circular convolution with a prototype filter  $g$  of length  $L = KM$ , we introduce a pulse shaping filter denoted  $\tilde{g}$ , obtained by the periodic repetition of duration  $KM$

of the prototype filter  $g$ , i.e.,

$$\tilde{g}[k] = g[\text{mod}(k, MK)]. \quad (6)$$

The orthogonality condition for the filter design is in line with (2). This can be readily proven using the symmetrical property of the filter.

#### IV. COQAM TRANSMITTER DESIGN IN A RADIO SYSTEM

In order to maintain a perfect orthogonality after a transmission through a multi-path channel, we introduce a CP to cancel the inter block interference. Moreover, as we want to prevent an alteration of the Power Spectral Density (PSD), resulting from a spectral leakage due to the block processing, a windowing is applied after CP insertion. The transmission system resulting from these two operations is named windowed CP-FBMC/COQAM (WCP-COQAM). Denoting the CP length by  $L_{CP}$ , we get  $L_{CP} = L_{GI} + L_{RI}$ , where  $L_{GI}$  is the CP part used to fight against the multi-path channel interference and  $L_{RI}$  is the portion devoted to windowing.

The  $l$ -th block of the WCP-COQAM signal  $s_{WCP-COQAM}[k]$ , for  $k = 0, \dots, KM + L_{CP} - 1$ , can be obtained from the  $l$ -th block of the COQAM signal, for  $k = 0, \dots, KM - 1$ , by

$$s_{WCP-COQAM}[k] = \sum_{r=l-1}^{l+1} s_{COQAM}[\text{mod}(k - L_{CP}, KM)] \times w[k - rQ], \quad (7)$$

where  $Q = KM + L_{GI}$  and  $w[k]$ , defined in the  $k = 0, \dots, KM + L_{CP} - 1$  interval, is the window function computed as follows

$$w[k] = \begin{cases} \text{window coeffs.} & k \in [0, L_{RI} - 1] \\ 1 & k \in [L_{RI}, KM + L_{GI} - 1] \\ w[KM + L_{CP} - 1 - k] & \text{otherwise.} \end{cases}$$

The WCP-COQAM transmitter structure is depicted in Fig. 2, where  $s_1[k]$  denotes  $s_{COQAM}[k]$ , which is obtained from Fig. 1 and  $s_2[k]$  stands for  $s_{WCP-COQAM}[k]$ .

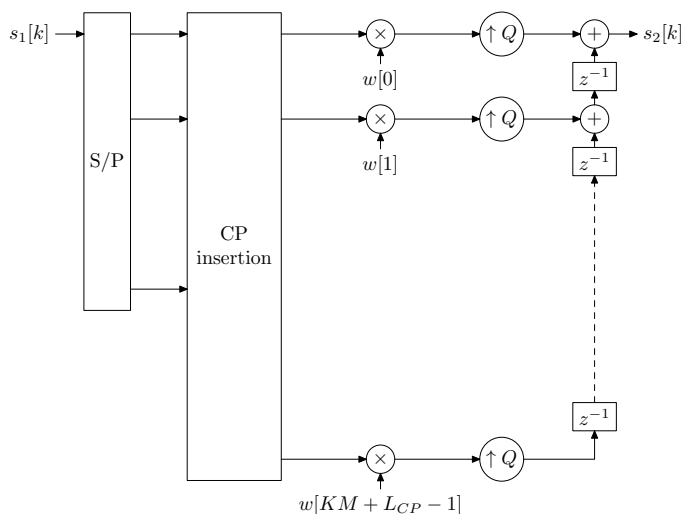


Figure 2. Transmitter of the windowed CP-FBMC/COQAM system.

There are extensive works on the window design [19]. In this paper, we simply use the Hamming window for this

windowing process. Particular investigation on the window design in the context of COQAM will be envisaged in the future step. Furthermore, it is worth noting that the additional part  $L_{RI}$  does not reduce the spectral efficiency as it falls only in the overlapped samples between two successive blocks.

#### V. SIMULATION EVALUATION

To illustrate the efficiency of the proposed WCP-COQAM scheme, we first evaluate the PSD of the afore-mentioned MCM candidates. In our simulation, the PSD is estimated using the Welch method [20]. The parameters used in our simulation are: number of subcarriers is fixed to  $M = 128$  for all MCM schemes. Regarding the prototype filter for OQAM, WCP-COQAM and GFDM, in our simulation a Square Root Raised Cosine (SRRC) filter is used with the roll-off factor 0.5 and length up to  $6M$ . The CP contains 32 samples for CP-OFDM, WCP-COQAM and GFDM. In particular, the windowing interval for WCP-COQAM is  $L_{RI} = 16$  samples. The result of the PSD comparison is given in Fig. 3. It clearly

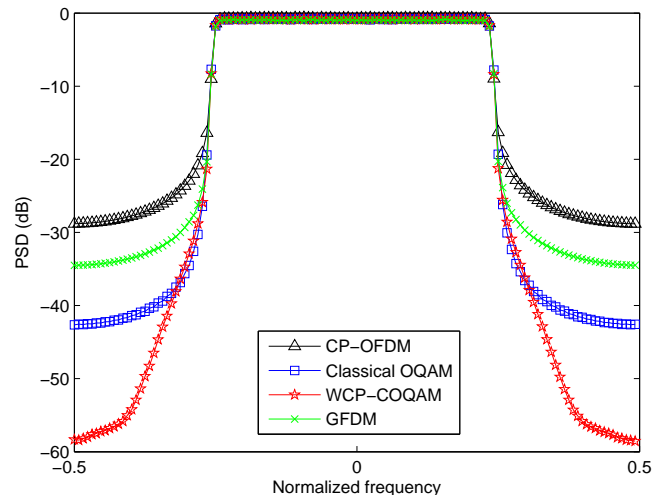


Figure 3. PSD simulation,  $M = 128$ , SRRC with roll-off 0.5.

shows that the WCP-COQAM can remain a satisfactory low level of the out-of-band spectral leakage. In fact, for some CR applications, non-contiguous bands are sometimes considered for addressing dynamic spectrum sharing. In this sense, the in-band notch spectrum shape becomes extremely important, as it directly reflects the spectrum management granularity, which can be used to drive its dynamics to the ultimate limit. A good MCM scheme is expected to provide a low spectral radiation in the notch band, in order not to create severe interference to the neighboring bands. As shown in Fig. 4, the WCP-COQAM is able to keep a low power leakage in the notch band. On the other hand, we note that although the PSD of GFDM is improved compared with CP-OFDM, it cannot provide a similar PSD as for the OQAM and WCP-COQAM schemes, because the block processing of GFDM still generates the spectral leakage due to the discontinuity on the edge of each block [19]. Hence, the windowing process can alternatively be employed to the GFDM for further spectral leakage mitigation. However, this cannot relieve its non-orthogonality issue.

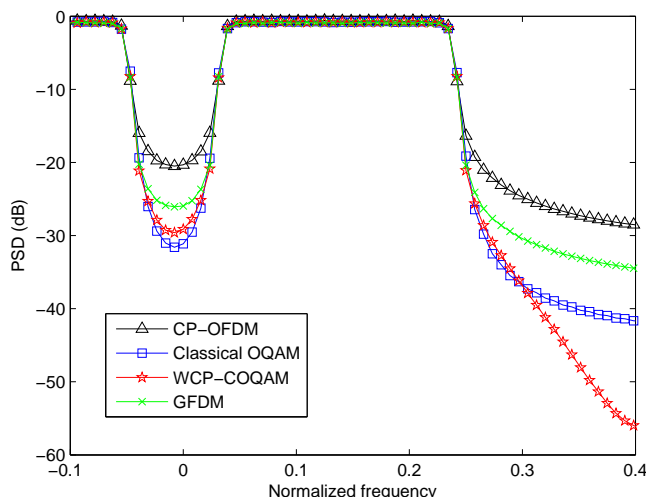


Figure 4. PSD simulation with notch,  $M = 128$ , SRRC with roll-off 0.5..

Next, we provide an orthogonality evaluation to show that the WCP-COQAM perfectly retains the orthogonality. Moreover, it is quite robust against the frequency selective fading. Our evaluation follows two steps. In the first step, which is similar to [13], we evaluate the Bit Error Rate (BER) performance in an Additive White Gaussian Noise (AWGN) channel. We keep the same parameter setting as in the PSD evaluation and the symbol constellation is QPSK. The receiver does not employ any equalizer but a matched filter. The simulation result is reported in Fig. 5, where we use the CP-OFDM curve as a reference because it reflects the QPSK Matched-Filter (MF) bound in AWGN. It is clearly shown that

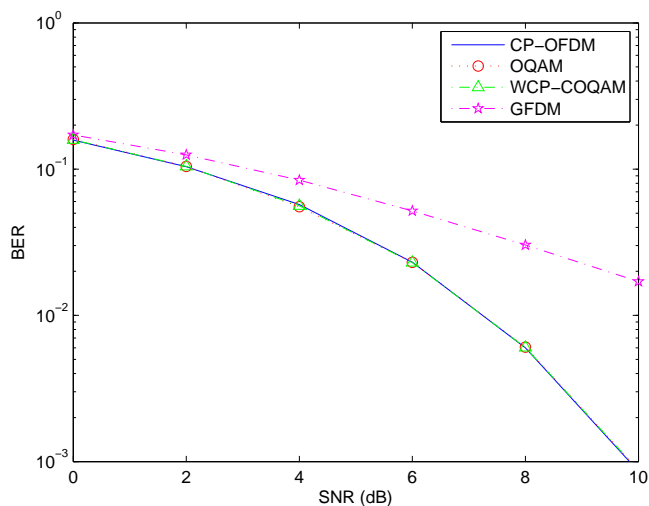


Figure 5. Matched filter BER for uncoded QPSK in AWGN channel.

the conventional OQAM and WCP-COQAM systems are fully orthogonal because their BER curves are overlapped with the CP-OFDM curve. On the other hand, the GFDM curve cannot merge with the other ones due to its non-orthogonality. A performance degradation can be seen, confirming that the matched filter cannot cancel the interference brought by the neighboring

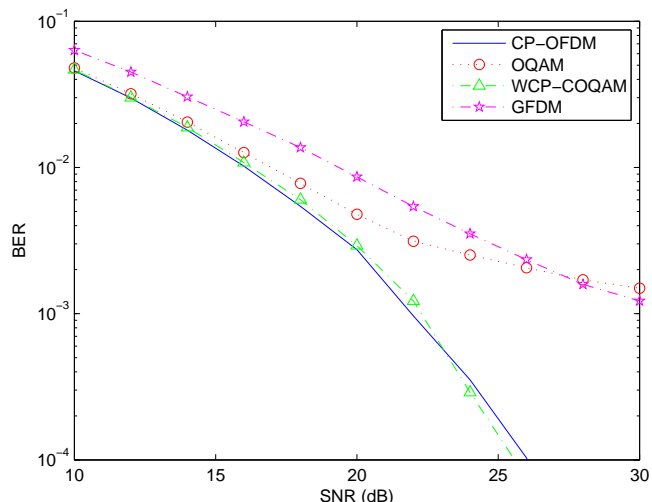


Figure 6. BER for uncoded QPSK in dispersive channel with ZF equalizer.

symbols. Thus, it needs a more advanced equalizer. In the second step, we show that the WCP-COQAM is still quite robust against the frequency selective fading. In our simulation, we consider a wireless dispersive fading channel, similar to [21], with impulse response  $h_{ch}[k] = \sum_p \alpha_p \delta(k - p)$ , where  $\alpha_p$  are complex i.i.d. Gaussian variables with the power profile following  $\mathbb{E}[|\alpha_p|^2] = e^{-p/r}$  and,  $r$ , the normalized delay spread. In our simulation, we choose  $r = 4$ , reflecting a severe frequency selective fading. The channel is normalized and is truncated at  $-20$  dB. The receiver equalization technique uses a one-tap Zero-Forcing (ZF) equalization for all the MCM schemes. The simulation result, presented in Fig. 6, shows that even under a severe fading environment, the WCP-COQAM curve is still merged with the CP-OFDM. While for the OQAM and GFDM, a degradation by several decibels is displayed. Hence, the result proves that the WCP-COQAM is very robust against the frequency selective fading. With our simulations, we can conclude that the WCP-COQAM can keep all the benefits of the proposed MCM schemes and get rid of their drawbacks. Thus, this scheme can be seen as a good physical layer enabler for the CR applications.

## VI. CONCLUSION AND FUTURE WORK

In this paper, we presented a novel multi-carrier modulation scheme, which combines the conventional OQAM and GFDM concepts and successfully keeps all their benefits and at the same time overcomes their drawbacks. The simulation confirmed the achievements of this new scheme and further proved its efficiency as a suitable physical layer modulation for cognitive radio applications. In the next step, we will investigate a dedicated window design, as well as an analysis on the Peak-to-Average-Power-Ratio (PAPR).

## REFERENCES

- [1] B. Le Floch, M. Alard, and C. Berrou, "Coded Orthogonal Frequency Division Multiplex," Proceedings of the IEEE, vol. 83, Jun. 1995, pp. 982-996.
- [2] M. Bellanger, T. Ihalainen, and M. Renfors, "Filter bank based cognitive radio physical layer," in Proc. of ICT-Mobile Summit, Santander, Jun. 2009, ISBN: 978-1-905824-12-0.

- [3] M. Shaat and F. Bader, "Computationally Efficient Power Allocation Algorithm in Multicarrier Based Cognitive Radio Networks: OFDM and FBMC systems," *EURASIP Journal on Advanced Signal Processing (ASP)*, vol. 2010, 2010:528378, 13 pages.
- [4] H. Zhang, D. Le Ruyet, D. Roviras, Y. Medjahdi and H. Sun, "Spectral Efficiency Comparison of OFDM/FBMC for Uplink Cognitive Radio Networks," *EURASIP Journal on Advanced Signal Processing (ASP)*, vol. 2010, 2010:621808, 14 pages.
- [5] H. Zhang, D. Le Ruyet, and M. Terré, "Spectral Correlation of Multicarrier Modulated Signals and its Application for Signal Detection," *EURASIP Journal on Advanced Signal Processing (ASP)*, vol. 2010, 2010:794246, 13 pages.
- [6] M. Renfors, "Spectrum monitoring schemes for FBMC cognitive radios," in *Proc. Future Network and MobileSummit (FNMS 2010)*, Florence, Italy, Jun. 2010, pp. 1–9.
- [7] T. Ihalainen, A. Viholainen, T.H. Stitz and M. Renfors, "Reappearing primary user detection in FBMC/OQAM cognitive radios," in *CROWNCOM conference*, Cannes, France, Jun. 2010, pp. 1–5.
- [8] G. Fettweis, M. Krondorf, and S. Bittner, "GFDM-Generalized Frequency Division Multiplexing," in *IEEE Vehicular Technology Conference (VTC Spring'09)*, Barcelona, Spain, Apr. 2009, pp. 1–4.
- [9] N. Michailow, M. Lentmaier, P. Rost, and G. Fettweis, "Integration of a GFDM Secondary System in an OFDM Primary System," in *Future Network and Mobile Summit (FNMS'11)*, Warsaw, Jan. 2011, pp. 1–8.
- [10] R. Datta, N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Generalized Frequency Division Multiplexing in Cognitive Radio," in *European Signal Processing Conference (EUSIPCO'12)*, Bucharest, Romania, Aug. 2012, pp. 2679 – 2683.
- [11] R. Datta, N. Michailow, M. Lentmaier, and G. Fettweis, "GFDM Interference Cancellation for Flexible Cognitive Radio PHY Design," in *IEEE Vehicular Technology Conference (VTC Fall'12)*, Quebec city, Canada, Sept. 2012, pp. 1–5.
- [12] R. Datta, D. Panaitopol, and G. Fettweis, "Analysis of Cyclostationary GFDM Signal Properties in Flexible Cognitive Radio," in *International Symposium on Communication and Information Technologies (ISCIT'12)*, Gold Coast, Australia, Oct. 2012, pp. 663 – 667.
- [13] N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Bit Error Rate Performance of Generalized Frequency Division Multiplexing," in *IEEE Vehicular Technology Conference (VTC Fall'12)*, Quebec city, Canada, Sept. 2012, pp. 1–5.
- [14] H.G. Feichtinger and T. Strohmer, *Gabor Analysis and Algorithm - Theory and Applications*, Birkhäuser, Boston-Basel-Berlin, 1998.
- [15] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank theory," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, May 2002, pp. 1170–1183.
- [16] T. Ihalainen, T. H. Stitz, M. Rinne, and M. Renfors, "Channel Equalization in Filter Bank Based Multicarrier Modulation for Wireless Communications," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, no. ID 49389, Aug. 2007, pp. 1–17.
- [17] P. Tanguy H. Lin, P. Siohan and J-P. Javaudin, "An analysis of EIC for OFDM/OQAM systems," *Journal of Communications (JCM)*, vol. 4, no. 1, Feb. 2008, pp. 52–60.
- [18] G. Ndo, H. Lin, and P. Siohan, "FBMC/OQAM equalization: Exploiting the imaginary interference," in *PIMRC'12*, Sydney, Australia, Sept. 2012, pp. 2359 – 2364.
- [19] F.J. Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," *Proceedings of the IEEE*, vol. 66, no. 1, Jan. 1978, pp. 51 – 83.
- [20] P. D. Welch, "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms," *IEEE Tans. Audio and Electroacoust.*, vol. AU-15, Jun. 1967, pp. 17–20.
- [21] T. Fusco, A. Petrella, and M. Tanda, "Data-Aided Symbol Timing and CFO Synchronization for Filter Bank Multicarrier Systems," *IEEE Tans. on Wireless Communications*, vol. 8, May 2009, pp. 2705–2715.