Neural Network-Based Reasoning for Solving the Tower of Hanoi

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***Abstract*—In this paper, we propose a reasoning solution, which can infer the rules of game, such as Tower of Hanoi. Neural networks require large amounts of data to improve performance. However, human intelligence requires only a simple exposition. The goal of this paper is to learn to solve a Tower of Hanoi without much learning. We collect sequential data from participants' experiments that are supposed to solve the Tower of Hanoi. And then, we observe relations between different objects and their actions to establish our reasoning model. We train our reasoning model to solve the Tower of Hanoi. Finally, we show that our reasoning model can infer the rule of game through observations from objects, their relationships and actions.**

***Keywords*–Reasoning;Inference;Neural Networks; Machine Learning;Tower of Hanoi.**

I. Introduction

From a psychological point of view, the reasoning of human being can be considered as the process of drawing conclusions. Human beings' endeavors to solve problems or to make decisions are goal-oriented [1]. Reasoning has long been regarded as exclusive domain of the human being. It was one of the most difficult areas to implement mechanically. Because we need to understand contextual meanings, such as texts and images, and we have to consider contextual relationships that change depending on the situation, even with the same information. Therefore, reasoning has received a lot of attentions in the last few decades [2][3]. In recent years, advances in deep learning, especially the evolution of learning algorithms, are leading to rapid research and development of artificial intelligence in the field of reasoning [4]. In this paper, we pay our attention to a matter of reasoning about objects, relations and interactions. This task is a matter of reasoning about things and relationships in order to solve problems. The French mathematician Edouard Lucas introduced the Tower of Hanoi (TOH) puzzle in 1883. Figure 1 shows a standard example of Tower of Hanoi. There is a pile of different size disks on the right peg. The largest disk is at the bottom of the peg and the smallest at the top of the pile. The goal of TOH is to move the whole stack of disks from the right peg to the left peg. There are three rules as constraints: One disk at a time should be moved, in a location, the smallest disk is the one to take and a large disk cannot be placed on top of a smaller one. In this paper, our hypothesis is that we can infer the rule of game through observing the movements of the participants action. In order to test this hypothesis, we carry out an experiment for which participants were given two successive tasks: to solve the three-disk Tower of Hanoi task, then to solve this problem with four disks. Based on the acquired data, we observed relations between different objects and their actions to establish our reasoning model. As we know, neural networks require amounts of data to improve performance. But, in this paper we tried to infer the solution of Tower of Hanoi without much learning. We trained with 3 disks and predicted out model with 4 disks.

II. Reasoning model

As mentioned at the introduction, only one disk at a time can be moved, a disk can only be moved if it is the top disk on a pile, and a larger disk can never be placed on a smaller one (See Figure 1). From the rule of game, we can infer this participant's action. How can we make such inference possible without any information about the rule of game ? We have received great inspiration from Interaction Networks(Ins) [5] and Relation networks(RNs) [6]. These models, which have a neural network architecture, are very helpful for inference learning by observing from objects, their relationships and actions.

Figure 1. The smallest disk in the four is at the top, and participant grabs it and moves it.

As with the Relation Networks [6], we assume reasoning of rule that depends on the relations $\mathcal{R}$ between objects. According to our sequential data, relation ($\mathcal{R}$, with elements r) is consist of location, and inclusion relation. For example, the smallest disk is the relation with other disks (Ex:The smallest disk is in contact with the second smallest disk.). In the case of the Tower of Hanoi with 3 disks, we have six objects (3 disks, 3 pegs) with a vector of 3 features encoding properties, such as the object's position (beneath and on) and inclusion relation. We can express the smallest disk like this, $o_1 = (o_1^{beneath}, o_1^{on}, o_1^{inclusion})$. We are interested in models generally defined by the composite form $f \cdot g$, where f is a function that returns a prediction y. The function g_ψ is defined to operate on a particular factorization of D

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(ex: $g_\psi(D) \equiv g_\psi(o_1^1, o_1^2, ..., o_i^1, ..., o_m^n)$). As shown in Figure 2, there is the smallest disk on the second small disk. In this case, there is a relation with the disk below. It means $g_\psi(o_1, o_2) = beneath(o_1, o_2)$. But, there is not the disk on the smallest disk. For inclusion, There are four disks in peg1. But the peg2 and peg3 do not have a disk.

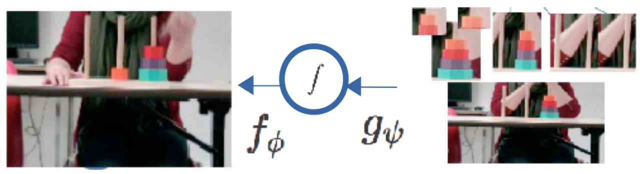

Figure 2. Model type

The model we want to predict is given by $y = f_\phi(\sum_{i,j} g_\psi(o_i, o_j))$ where f_ϕ and g_ψ are Multi-Layer Perceptron. We optimize this model by means of training. Finally, our model gives us probabilities about whether we can move the disk in each different situation or not.

III. Experiments

A. Experiment: Tower of Hanoi

We recruited 14 participants (Average age 41, Standard Deviation=8.51). The blind group consisted of 6 women and 1 man (Average age 39, Standard Deviation=6.65). The sighted group consisted of 6 women and 1 man (Average age 43, Standard Deviation=10.30). Sitting down at the table, the participants were then given four disks of the Tower of Hanoi that they had to solve. The instructions were given to the participants. The participants were requested to solve the four disks TOH as we collected their solution. Through these research experiments, we have obtained the sequential data concerning about the solution of Tower of Hanoi.

B. Experiment: Reasoning model

The model has 5 input units, one or two hidden layers and one output unit. There are different neurons (5,10) in each layers. Rectified linear activation functions are used in each hidden layer and a sigmoid activation function is used in the output layer, for binary classification. Reasoning model with different architectural variations was trained to find the rule of Tower of Hanoi. The model with different number of neurons reaches a cross entropy loss below 0.6. However, the fourth model (MLP:5 inputs, two hidden layers with 10 neurons, 1 output) performed well as compared to other models.

As we can see in Figure 3 and Table I, the test accuracy with 3 disks for the first and fourth model are 95.24% and 100%, respectively. When we try to infer the rule of game for The tower of Hanoi with 4 disks, we find our reasoning model achieves 75% prediction accuracy for the first model and 96.67% for the fourth model.

IV. Conclusion

Human being's reasoning and thinking do not require much learning data. Human intelligence requires only a simple exposition [7]. Based on this idea, without much learning data, we propose a reasoning model that can infer the rule of the game through observing the characteristics, relationships, and actions of objects. In particular, the objective of Tower of Hanoi is to find the solution in a way that is the shortest possible movement. For future work, we will apply our model to find the shortest path of Tower of Hanoi. Furthermore, after proving the effectiveness of this model, we will apply it to explore the more complicated tasks.

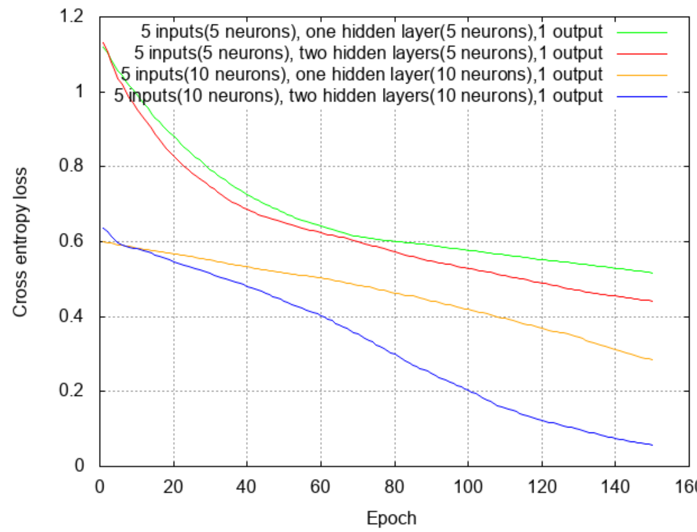

Figure 3. Reasoning model with different number of neurons was trained to find the rule of TOH.

TABLE I. MODEL PERFORMANCE FOR TOWER OF HANOI WITH 3 DISKS (TRAINING) AND 4 DISKS (PREDICTION)

	Accuracy	
Model	training(%)	prediction(%)
Five inputs (5 neurons), one hidden layer (5 neurons)	95.24	75
Five inputs (5 neurons), two hidden layers (5 neurons)	90.48	76.67
Five inputs (10 neurons), one hidden layer (10 neurons)	90.48	71.67
Five inputs (10 neurons), two hidden layers (10 neurons)	100	96.67

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