Creating an Intelligent, Integrated Mobility Assistance: The Elastic Trip-Chain Construction Problem

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***Abstract*—The number of transport modes and mobility services has increased significantly over the last decades. To support travelers, trip planning tools are required, which can take into account not only the broad spectrum of available transport modes but also personalized user requirements and their complete daily schedule. We present the architecture of a personal mobility-assistance service taking into account daily schedules to compute multimodal journeys and we identify the *Elastic Trip-Chain Construction Problem* as an important optimization problem for modern trip planning. We describe and analyze the optimization problem and model it as a Mixed Integer Quadratically Constrained Program.**

Keywords–Elastic Trip-Chain Construction Problem; Mobility Assistance; Travel Planning Service; Trip-Chain Composition

I. Introduction

Mobility behavior has always been subject to change. Today's changes seem to be mainly governed by increasing mobility demand, exhaustion of transport capacities, and an increasing consciousness of the environmental changes caused by traffic. At present, and especially in metropolitan areas, user's willingness and ability to employ a variety of transportation means apart from private owned cars can be observed [1]. On the one hand, this tendency is strongly supported by the ubiquitous availability of real-time services for nearly all kinds of transportation. Some transportation modes, e.g., ride sharing, even depend on such real-time services. On the other hand, real-time services offer the opportunity to influence and shape mobility behavior.

To efficiently assist (and influence) users traveling with this new variety of transport modes, new personal mobility assistance tools need to be employed. In the context of this work, we investigate a mobility assistant that plans trips along multiple activities, minimizing travel efforts on a per-user basis. Those trips usually start and end at a user's home location and interlink daily activities, such as going to work, (grocery-) shopping, or leisure activities. We call a trip including multiple activities a *trip chain*. Also, in the course of one single trip chain, the use of multiple different transportation modes to move from one activity to another is allowed.

The main two contributions of this paper are a) to sketch the function of a travel planner service as part of an integrated mobility assistance, the latter to be investigated with respect to its effect on mobility within the publicly funded joint project BiE (evaluation of integrated electric mobility) [2], and b) to describe and analyze the underlying optimization problem of computing valid and effective trip chains.

The rest of this paper is structured as follows: In Section II a short overview about trip-planning problems is given. Section III introduces our mobility assistance service, along with a detailed description of the integrated travel planning sub-service. The Elastic Trip-Chain Construction Problem is formulated and analyzed in Section IV. Moreover, a mathematical formulation of the problem in form of a Mixed Integer Quadratically Constrained Program is given. Section V concludes this paper.

II. Related Work

Existing commercial mobility assistance is usually confined to more or less elaborate choice of the opportunities to get from location a to location b at a given time. Recently, a few efforts have been made to develop context related (i.e., to the purpose of the trip) support for multiple activities on one journey, such as eCOMPASS [3].

Various trip planning related computation problems can be found in the literature; the closest ones to the problem at hand are the so-called *Tourist Trip Planning Problems* (for a survey, see [4]) and the *Traveling Purchaser Problem* [5]. These problems are $\mathcal{NP}$-hard, hence very hard to solve. The trip planning problem studied in this work differs from the already known problems mainly in the fact that activities (that are to be executed along the trip) do not have a single, fixed location where they should be executed, but a set of alternative locations to choose from. Timing constraints inferred by the activities are partly covered by some of the Tourist Trip Planning Problems but not to the same extent as in this work. Furthermore, in our case different transportation modes are taken into consideration, which increases the complexity of the problem considerably.

III. Architecture Mobility Assistance Service

A. Mobility Assistance Service

The architecture of the integrated mobility assistance service is shown in Fig. 1. This work focuses on a travel planning service as part of such an integrated system. Hence, Fig. 1 only depicts the essential parts from the travel planning point–of–view; the rest of the components are left out. The mobility assistant is designed as a distributed system aiming on efficient horizontal scaling. To this end, we use Vert.X [6]: an event-driven Java framework to create distributed systems. Vert.X provides an *Event-Bus* which is used as the communication medium by the system's components. The most important components for travel planing are *Organizer*, *UserPreferenceService*, *CalendarService*, *AssistanceApp*, and *TravelPlanningService*.

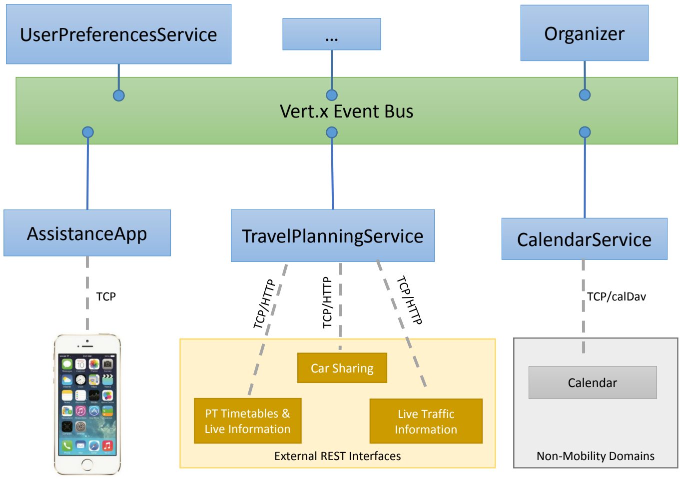

Figure 1. Architectural Overview of the Mobility Assistance Service

Both *AssistanceApp* and the *CalendarService* are the system's interfaces to external domains. The Assistance-App controls communication with the (human) users of the system. Among others, it proposes trips to the user, asks for preferences, and keeps track of the user's position. The CalendarService, on the other hand, interfaces directly with user's cloud-based digital calendars. Whenever there is a new entry in a calendar, this entry is compiled into an *activity* (the mobility assistance service's equivalent of a calendar entry) which is further analyzed and possibly annotated with additional information by the *Organizer*. The *UserPreferenceService* is the system's central component for storing and retrieving any sort of user preferences. If a user's profile is not completely filled out, it uses reasoning techniques to imply reasonable preferences. Those preferences are subsequently used by the *TravelPlanningService* to compute trip chains for all activities retrieved from external domains. The TravelPlanningService computes routes based on live traffic data. In addition, it uses REST interfaces to query for public transport information (such as time tables) and live information from car-sharing partners, for example availability of cars or the position of car-sharing stations.

B. Travel Planning Service

The travel planning service itself consists of multiple components, the most important ones for the scope of this work are *Trip-Chain Store*, *Trip-Chain Monitor*, *Trip-Chain Composer*, and *Routing Engine*.

The *trip-chain store* is the component responsible for efficient storage and retrieval of computed trip chains. It is used as a hub for other services of the integrated mobility assistance, as well as the central internal storage used by other components of the travel planning service.

The *trip-chain monitor* is used to keep trip chains up–to–date. The traffic situation undergoes constant change (e.g., due to unpredicted traffic jams). Hence, all trip chains have to be checked for feasibility under the projected future traffic situation. In the cases where they have been rendered infeasible by recent events, the trip-chain monitor triggers recomputation of the corresponding trip chains.

A *routing engine* is used to compute journeys. For any journey, all available modes of transportation are considered. Necessary real-time traffic information (public and private) is gathered via REST interfaces. These journeys are concatenated to trip chains by the *trip-chain composer*. To this end, the composer keeps a list of activities that need to be included into a trip chain and then creates journeys to connect the sites at which those activities are to be executed, making sure that no constraints, imposed by either the activities themselves or the user's preferences, are violated.

The trip-chain composer emerges as the crucial component with respect to the behavior of the travel planning service and thus of the integrated mobility assistance as a whole. As the trip-chain composer is also the component presenting the greatest challenges w.r.t. solving algorithms, it will be discussed and analyzed in more detail in the remainder of the paper.

IV. The Elastic Trip-Chain Construction Problem

The Elastic Trip-Chain Construction Problem is the core problem solved by the Trip-Chain Composer. It asks for a series of journeys interlinking one or more activities of a user such that the user arrives at the site of the activity in time (i.e., *not too late*). The activities in question have, apart from their durations, *elastic* timing constraints defining a time *interval* in which they need to start. This section further defines the underlying activity model, formally states the Elastic Trip-Chain Construction Problem, discusses its complexity and ends with a Mixed Integer Quadratically Constrained Program (MIQCP) modeling the problem.

A. Model and Problem Statement

Definition 1 *An activity a is defined to be the 4-tuple $a = (d, s_e, s_l, l_{pos})$ where d is the duration of the activity, s_e is the* earliest *possible point in time where the activity can be started, s_l is the* latest *possible point in time where the activity can be started, and l_{pos} is a non-empty set of locations where it is possible to execute the activity.*

To travel from one location where an activity is executed to another location where the next activity is executed, we define a journey.

Definition 2 *A journey j is defined as the 5-tuple $j = (src, dest, t_{dep}, t_{arr}, m)$ where src is the location to start the journey , $dest$ is the destination of the journey, t_{dep} is the point in time when the journey starts, t_{arr} is the time when the journey ends (when it arrives at $dest$), and m is the transportation mode on the journey.*

In the remainder, we use $src, dest, t_{dep}, t_{arr}$, and m as functions on journeys and d, s_e, s_l, and l_{pos} as functions on activities to describe the corresponding property (e.g., $d(a_i)$ describes the duration of activity a_i).

Definition 3 *A trip chain t is an ordered tuple of n journeys $t = (j_1, j_2, ..., j_n)$ where $dest(j_i) = src(j_{i+1})\ \forall\ 0 < i < n$. Let $A = \{a_1, a_2, ..., a_{n-1}\}$ be a set of $n-1$ activities. We say a trip chain t is* valid *for A, if t interlinks each activity $a \in A$ such that one of a's locations from l_{pos} is visited* before or on *s_e and at least d time between s_e and $s_l + d$ is spent at the location. In addition, it must be possible to make all journeys without any overlap. Hence, for all journeys $j_l \in t$*

there must not exist another journey $j_k \in t$, $j_k \neq j_l$, such that $(t_{dep}(j_l), t_{arr}(j_l))$ overlaps with $(t_{dep}(j_k), t_{arr}(j_k))$.

The Elastic Trip-Chain Construction Problem (ETCCP) *Given a set of activities $A = \{a_1, a_2, ..., a_N\}$, a location l, an earliest time s to start and latest time o to arrive (each at l), and a set of available transport modes, the elastic trip-chain construction problem asks for a* valid *trip chain $t = (j_1, j_2, ..., j_{N+1})$ interconnecting all activities in A for which holds that $l = src(j_1) = dest(j_{N+1})$, $s \leq t_{dep}(j_1)$, and $t_{arr}(j_{N+1}) \leq o$.* It is worth noting that such a trip chain t does not always exist.

B. Notes on Complexity

The optimization variant of ETCCP not only asks for a *valid* trip chain that interlinks a set of activities but for the trip chain with *least* travel time or, in a general sense, *travel cost*.

The Traveling Salesperson Problem (TSP) can be reduced in polynomial time to the optimization variant of ETCCP. Reduction is done by assigning to each activity

- a time interval of unlimited size to be executed in,
- a duration of zero, and
- only one possible location to be executed at.

Due to space limitations, we omit the full construction here. However, it follows that TSP is a special case of the optimization variant of ETCCP meaning the optimization variant of our problem at hand is $\mathcal{NP}$-hard.

C. Mathematical Formulation

There is a variety of possible optimization criteria when constructing tripchains including shortest travel time, cheapest trip chain, or ecologically friendliest trip chain.

To keep the model description short, in the following we concentrate on computing trip chains with *shortest travel time* only. Note that the model can simply be adapted to meet the other optimization criteria by introducing additional model variables and using a different objective function.

1) Model variables: Our model features the following model variables as an input, describing a set of activities and their corresponding properties in detail.

In the description, with $A = \{a_1, a_2, ..., a_N\}$, we denote the set of activities to include in the trip chain. $origin$ denotes the origin (and final destination) of the trip chain. Time is modeled as a discrete series of equally spaced numbers on a time line analogous to UNIX timestamps where the spacing (i.e., the granularity) can be chosen arbitrarily.

$L \in \mathcal{N}^+$	Number of distinct possible locations from l_{pos} over all activities and the origin of the trip chain $\left(\mid origin \cup \bigcup_{a_i \in A} l_{pos}(a_i) \mid\right)$
$N \in \mathcal{N}^+$	Total number of activities on the trip chain $(\mid A \mid)$
$f^D_{k,l,t,m} \in \mathcal{N}^+$	Travel time between locations k and l with *departure* time t and using modality m. The origin of the trip chain goes by index 1 (i.e, $f^D_{\mathbf{1},l,t,m}$ denotes the travel time when departing at $origin$)
$f^A_{k,l,t,m} \in \mathcal{N}^+$	Travel time between locations k and l with *arrival* time t and using modality m.
$potloc_{i,k} \in \{0,1\}$	Potential locations for an activity. 1, if location k corresponds to one of $l_{pos}(a_i)$, 0 otherwise
S	The earliest possible point in time to start the trip chain
O	The latest possible point in time the trip chain needs to arrive at it's origin
M	Total number of available transport modes

2) Decision variables: The following *decision variables* tell when to execute which activity at what location and which mode to use to travel between activities.

$loc_{i,k} \in \{0,1\}$	1, if activity i is executed at location k, 0 otherwise
$t_i \in \mathcal{N}^+$	The point in time when activity i is to be started
$start_{i,k} \in \{0,1\}$	1, if execution of activity i starts at time k (in the granularity chosen), 0 otherwise
$term_{i,k} \in \{0,1\}$	1, if execution of activity i ends at time k (in the granularity chosen), 0 otherwise
$ord_{i,j} \in \{0,1\}$	The ordering of activities: 1, if activity i is the j-th activity on the trip chain, 0 otherwise.
$tt_j \in \mathcal{N}^+$	The time it takes to travel from the j-th activity to the (j+1)-th activity on the trip chain
$ttf \in \mathcal{N}^+$	The time it takes to travel from origin to the first activity
$ttl \in \mathcal{N}^+$	The time it takes to travel from the last activity back to origin
$mode_{i,m}$	1, if transport mode m is used for the i-th journey on the trip chain, 0 otherwise

3) Constraints: The decision variables have to be selected with respect to the following constraints. The notation $[a, b]$ denotes the set of natural numbers starting from a to b inclusive.

Starting activities in their „time frame" only

$$t_i >= s_e(a_i) \qquad \forall i \in [1, N] \tag{1}$$

$$t_i <= s_l(a_i) \qquad \forall i \in [1, N] \tag{2}$$

order of activities on the trip chain

$$\sum_{j=1}^{N} ord_{i,j} = 1 \qquad \forall i \in [1, N] \tag{3}$$

$$\sum_{i=1}^{N} ord_{i,j} = 1 \qquad \forall j \in [1, N] \tag{4}$$

$$\sum_{i=1}^{N} ord_{i,j} \cdot t_i < \sum_{i=1}^{N} ord_{i,j+1} \cdot t_i \qquad \forall j \in [1, N-1] \tag{5}$$

no overlap of activities

$$term_i = t_i + d(a_i) \qquad \forall i \in [1, N] \tag{6}$$

$$\sum_{i=1}^{N} ord_{i,j} \cdot term_i + \sum_{i=1}^{N} tt_i \cdot ord_{i,j+1} \qquad \forall i \in [1, N-1] \tag{7}$$

$$< \sum_{i=1}^{N} ord_{i,j+1} \cdot t_i$$

transportation mode of journeys

$$\sum_{m=1}^{M} mode_{i,m} = 1 \qquad \forall i \in [1, N+1] \tag{8}$$

fixing a location for each activity

$$\sum_{k=1}^{L} loc_{i,k} = 1 \qquad \forall i \in [1, N] \tag{9}$$

$$potloc_{i,k} >= loc_{i,k} \qquad \forall i \in [1, N] \; \forall k \in [1, L] \tag{10}$$

do not start trip chain before S

$$\sum_{t=S}^{O}\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{k=1}^{L} loc_{i,k} \cdot f^{A}_{1,k,t,m} \cdot mode_{i,m} \cdot ord_{i,1} \cdot start_{i,t} < \sum_{i=1}^{N} t_i \cdot ord_{i,1} - S \tag{11}$$

do not arrive after O

$$\sum_{t=S}^{O}\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{k=1}^{L} loc_{i,k} \cdot f^{D}_{k,1,t,m} \cdot mode_{i+1,m} \cdot ord_{i,N} \cdot term_{i,t} < O - \sum_{t=S}^{O} term_{N,t} \cdot t \tag{12}$$

travel times between activities

$$tt_j = \sum_{m=1}^{M}\sum_{u=1}^{N}\sum_{v=1}^{N}\sum_{k=1}^{L}\sum_{l=1}^{L}\sum_{t=S}^{O} ord_{u,j} \cdot ord_{v,j+1} \cdot loc_{u,k} \cdot loc_{v,l} \cdot f^{D}_{k,l,t,m} \cdot term_{j,t} \cdot mode_{u,m} \qquad \forall j \in [1, N-1] \tag{13}$$

travel time from origin to first activity

$$\sum_{t=S}^{O} start_{i,t} = 1 \qquad \forall i \in [1, N] \tag{14}$$

$$\sum_{t=S}^{O} start_{i,t} \cdot t = t_i \qquad \forall i \in [1, N] \tag{15}$$

$$ttf = \sum_{t=S}^{O}\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{k=1}^{L} ord_{i,1} \cdot loc_{i,k} \cdot mode_{i,m} \cdot f^{A}_{1,k,t,m} \cdot start_{i,t} \tag{16}$$

travel time from first activity back to origin

$$ttl = \sum_{t=S}^{O}\sum_{m=1}^{M}\sum_{i=1}^{N}\sum_{k=1}^{L} ord_{i,N} \cdot loc_{i,k} \cdot mode_{i+1,m} \cdot f^{D}_{k,1,t,m} \cdot term_{i,t} \tag{17}$$

The optimization objective to minimize travel time is:

$$\min \left(\sum_{j=0}^{N} tt_j + ttf + ttl \right)$$

As this mathematical formulation contains products of decision variables in both its constraints and its objective function, it is a Mixed Integer Quadratically Constrained Program.

V. Conclusion

We described the architecture of a travel planning service as part of an integrated mobility assistance system, emphasizing that the crucial component of such a service is the trip-chain composer. This component computes the concatenation of journeys and determines when to execute which activity at what location such that travel time (or *travel cost* in general) is minimized. We call the underlying optimization problem the Elastic Trip-Chain Construction Problem and argued that it is $\mathcal{NP}$-hard. To solve the problem for small instances and to act as a benchmark for future approximation algorithms and meta heuristics, we formulated a MIQCP computing trip chains with least travel time.

Solving the MIQCP is—even for small instances—computational expensive. This not only holds for the MIQCP itself, which has a non-linear objective function but also for pre-computing the model variables. Prior to solving an instance of the problem using the MIQCP, we need to compute $f^{D}_{k,l,t,m}$ and $f^{A}_{k,l,t,m}$ which denote travel times between all locations for all departure and arrival times. As this is not practical for larger instances, future work will focus on finding fast and accurate heuristic algorithms. Further work of practical interest will address an extension of the Elastic Trip-Chain Construction Problem, which will yield multiple trip chains for a set of given activities. This will address those cases where activities are spaced so far from each other (on the time line) that one single trip chain would lead to unacceptably long waiting times between arriving at a site and starting the activity.

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