

License Plates Recognition of Mexican Private Vehicles

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Abstract— In most of the investigations about the recognition of license plates from different countries it is assumed that they have a white background, without texture patterns and with black characters. Vehicle registration plates in Mexico are different because they have different texture patterns and colors in the background depending on the State of the Republic; that is why the algorithms of recognition of these plates are not always successful. This article proposes an algorithm for the recognition of vehicle registration plates of Mexico considering three phases: A) Normalization and Binarization of the plate, which is achieved by using a threshold factor, which separates dark colors that form letters from the clearings that are at the bottom. B) Characterization C) Modeling of symbols by techniques such as Hu's moment, Fourier descriptors and correlation cross factors and D) Classification, where we have used comparisons between different techniques of template matching, Bayesian classifier and Artificial Neural Networks to process images of plates from different states. The results obtained are discussed at the end of the present paper.

Keywords-license plate recognition; bayesian classifier; artificial neural network; correlation factor; principal component analysis; Hu's moments.

I. INTRODUCTION

Recognition of license plates of vehicles has been investigated throughout the world. We can find some examples of recognition of license plates in countries like Argentina [1], Bangladesh [2], China [3], Egypt [4], India [5], Japan [6], Malaysia [7], among others. Normally, these works consider four phases: 1) get the image of the vehicle 2) plates location inside the image, 3) characters extraction and 4) classification or recognition of characters.

In most algorithms found in literature, it is assumed that the plate has no textured patterns, the background is usually white and the characters black allowing better character recognition. However, in the case of Mexican plates it is not the case. On one hand, they have patterns of texture in the background and on the other hand, each State Government can design their own pattern of background texture; this generates more than 32 different plates, and this number increase with changes in the government administration.

These structures in the vehicle registration plates of Mexico make the traditional algorithms not working properly, mainly in the phase of extraction of characters.

Another important thing to mention is that although each State can design the background of its plates, the dimensions

of the plates and letters, as well as their style, must meet with the features that are designated in the official Mexican standard NOM-001-SCT-2-2000; these characteristics are used to recognize the registration.

This work proposes an algorithm that segments each character and recognizes them depending on their characteristics of color and form. To properly segment the characters, most of the background texture patterns were eliminated by using a threshold factor, which separated the dark colors that make up the letters from the light colors that make up the background. Once filtered the background texture, the image of the plate was binarized and both vertical and horizontal histograms were obtained using the technique of projection of profiles, just to obtain the coordinates of the position used to segment characters. When the images of the characters were obtained, we proceeded to model them and characterize them using the techniques of Hu's moment, Fourier Descriptors, and Cross-correlation factor. Data obtained at this stage was used as complementary in the classification phase. Finally, in the stage of classification, techniques of templates, Bayesian classifier and artificial networks neural were used. The results obtained are discussed at the end of the present work.

The article is organized in the following form: in Section 2 is presented the proposal of recognition of license plates. Experiments conducted in the section are presented in Section 3. A discussion of the results obtained is made in Section 4 and the article ends with conclusions in Section 5.

II. PROPOSAL OF VEHICLE LICENSE PLATE RECOGNITION

Automatic Number Plate Recognition (ANPR) presented in Fig. 1, consists essentially of four stages: 1) To get the image of the vehicle using a camera, 2) Extract the image plate, 3) Segment and remove the plate characters and 4) Recognize the characters extracted.

In the present work is only discussed the segmentation and recognition of characters, stages (3) and (4). Images employed contain exclusively the region that conforms to the plate, and has the following characteristics:

- Images of the plates must be obtained between 1 and 1.5 meters of distance between the camera and the registration.
- They must not have lighting variations.
- Images are frontal or near frontal, meaning images must have very small rotation angles.
- No occlusions or considerable physical damages.

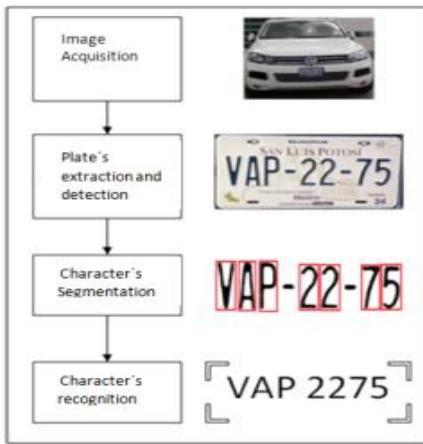


Figure 1. Stages of an ANPR system.

The developed algorithm in this work is presented and consists of: 1) Standardization and binarization of the plate, 2) Segmentation of the characters, 3) Character modeling and 4) Recognition of characters.

A. Normalization and Binariization of the plate

Normalization of the images is based on the standard NOM-001-SCT-2-2000, where it is established that dimensions of the plates must be of 2:1, so, all images are resized at 700×350 pixels of being binarized. The same standard also establishes the position in which the characters must be collocated inside the plate, as well as the size and the distance between them, as it is shown in Fig. 2.

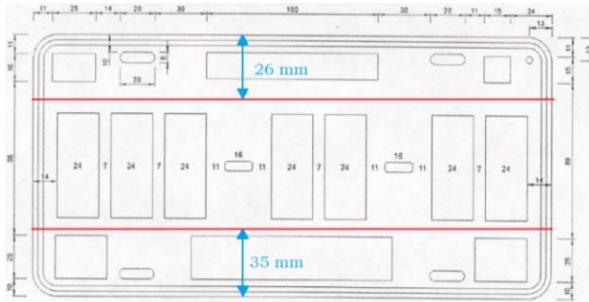


Figure 2. Position of the characters inside the registration.

With this information is eliminated the upper and lower sections of the original image, with the objective to get a section that contains exclusively characters. The result is shown in Fig. 3.



Figure 3. Example of sections elimination upper and lower: (a) original image, (b) image obtained after sections elimination.

Due to the great variety of background colors in plates, before being binarized, it was necessary to separate the

texture of the background from the numbers and letters of plates.

It is important to mention that, in the present work, the image colors are represented in the space RGB. In this space, the colors are represented as a linear combination of the vectors base in red, green and blue; the color of a pixel is represented as $\phi = [r, g, b]$. Fig. 4 shows the form of the RGB space.

To develop the separation process, we use an experimental threshold by graythresh function defined in Matlab, which chooses the threshold to minimize the interclass variance of the black and white pixels; the threshold (δ) obtained is of 0.73. To separate what is in the magnitude of a vector RGB, we start by performing a comparison to classify a pixel in a region or another, shown in Fig. 4.

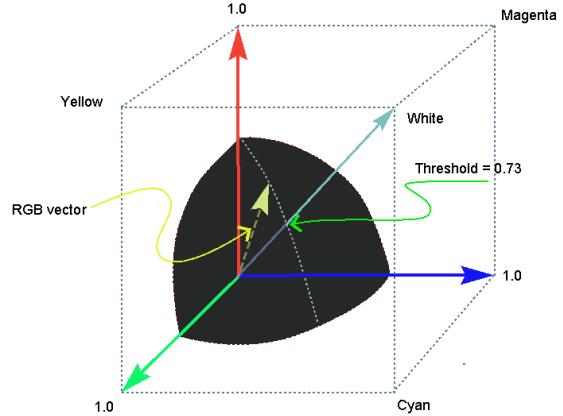


Figure 4. RGB space and threshold of intensity.

In Fig. 4 it is showed the part of the space where the colors from the characters are found. Separation was made using the equation (1).

$$\theta^* = \begin{cases} \vec{1}, & \|\phi\| \leq \delta \\ \vec{0}, & \|\phi\| > \delta \end{cases} \quad (1)$$

where $\eta^* = 0.73$, $\vec{1} = [1,1,1]$ and $\vec{0} = [0,0,0]$. With the proposed method, it is managed to differentiate the characters; the background was obtained and therefore the possibility of binarizing the images. In Fig. 5 we show two examples of binarized images using the Otsu method [8], which is normally used for gray scales; therefore, when using black and white is also a case for Otsu method.



Figure 5. Example of two images of plates binarized using the Otsu method.

B. Characters Segmentation

The horizontal and vertical projection method is used to segment characters [9]; this method has been used for similar investigations in [6][10][11].

Suppose we have an image $I(x, y)$ with $\text{wide} = N$ and $\text{height} = M$ (considering that $1 \leq x \leq N$ and $1 \leq y \leq M$), the horizontal and vertical projections are defined as:

$$P_{hor}(y_0) = \sum_{x=1}^N I(x, y_0), \forall y = 1, \dots, M \quad (2)$$

$$P_{ver}(x_0) = \sum_{y=1}^M I(x_0, y), \forall x = 1, \dots, N \quad (3)$$

The histograms obtained by equations (2) and (3) allow determining the coordinates of the area of each letter and number in the plates. Fig. 6 shows an example of horizontal and vertical projection of a binarized image.

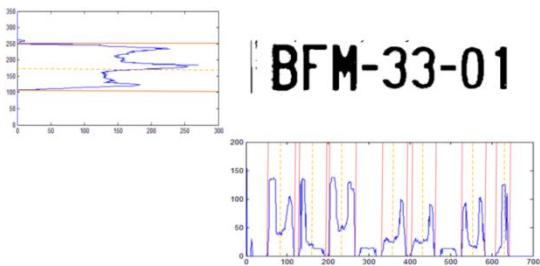


Figure 6. Horizontal and vertical projection of a binarized image.

Fig. 7 shows some examples of segmented characters used to do horizontal and vertical projections.



Figure 7. Examples of characters segmented using horizontal and vertical projections.

C. Modelling of the characters

The techniques of Hu moments, correlation factor and Fourier descriptors are compared because they are comparison techniques in which objective values are obtained, which identify classifications of letters and numbers that will be used in techniques such as Bayesian classifier and neural network.

Segmented characters are modeled using the Correlation factor, Hu's moment and Fourier descriptors.

1) *Correlation factor*: Correlation is a statistical technique that quantifies the strength of the linear relationship between two variables.

The quantification was performed using the coefficient of Pearson's correlation linear [12], whose value ranges between {-1 and 1}. Suppose we have the variables x and y, and on the other hand, $O = \{(x_1, y_1), \dots, (x_m, y_m)\}$ is the set of pixels coordinates to have a character extracted, the correlation coefficient between both variables is calculated as:

$$r = \frac{\sum_{(x,y) \in O} (x - \bar{x})(y - \bar{y})}{\sqrt{\sum_{(x,y) \in O} (x - \bar{x})^2 \sum_{(x,y) \in O} (y - \bar{y})^2}} \quad (4)$$

2) *Hu's moment*: Hu's moments have been employed to recognize characters in [13] and [14], to measure geometric features as ellipticities [15] or circularities [16]. In the Hu's moment, the most representative is the centralized and standardized moment, η_{pq} , obtained with:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{c+1}} \quad (5)$$

In equation (5), c and μ_{pq} is calculated as:

$$c = \frac{p + q}{2}, \quad p + q = 2, 3, \dots \quad (6)$$

$$\mu_{pq}(O) = \sum_{(x,y) \in O} (x - \bar{x})^p (y - \bar{y})^q \quad (7)$$

where (\bar{x}, \bar{y}) is the coordinate of the centroid of the objects, calculated as:

$$\bar{x} = \frac{1}{m} \sum_{(x,y) \in O} x \quad (8)$$

$$\bar{y} = \frac{1}{m} \sum_{(x,y) \in O} y \quad (9)$$

It is important to mention that Hu's moments are invariant to the position, rotation, and scaling.

3) *Fourier Descriptors*: Given $\{(x_1, y_1), \dots, (x_n, y_n)\} \subset \mathbb{Z}^2$ the set of points that form the outline of a letter, each point is represented as a complex number.

$$z_n = x_n + jy_n, \quad \text{where } j = \sqrt{-1}. \quad (10)$$

Discrete Fourier transform of the set of points that make up the outline of the characters can be calculated as follows:

$$F(u) = \frac{1}{u\pi} \sum_{n=1}^m \varphi_n \left[\cos\left(\frac{2\pi u s_n}{s_m}\right) - j \sin\left(\frac{2\pi u s_n}{s_m}\right) \right] \quad (11)$$

It can be written as:

$$F(u) = \frac{1}{u\pi} \sum_{n=1}^m \varphi_n \exp\left(\frac{-2j\pi u s_n}{s_m}\right) \quad (12)$$

Finally, the Fourier descriptors are obtained when calculating the absolute values of the complex numbers:

$$f(u) = |F(u)| \quad (13)$$

where $u = 1, 2, \dots, N$ and N is the total number of descriptors to obtain.

D. Characters Recognition

For the characters recognition four techniques are used: 1) Template matching, 2) Bayesians classifier and 3) Artificial Neural Networks (ANN) and Principal Component Analysis, which will be explained in more detail below.

1) *The template matching*: The comparison of templates is a technique that consists of comparing the image of the character to be recognized with a series of known templates; its similarity can be measured to identify the character that contains the image to be classified. This technique has been used for the same purpose in [18] and [21]. When an image must be classified, it is compared with all the images of the templates and the maximum value of similarity is obtained:

$$\Xi_\sigma = \text{Max}(r_\sigma), \forall \sigma \in (0, 1, \dots, 9, A, B, \dots, Z) \quad (14)$$

With the value of Ξ_σ it is defined what symbol is the content in the image.

2) *Bayesian classifier*: The Bayesian classifier [19] is based on the Bayes theorem where it is assumed that the vector of characteristics is a multivariate Gaussian distribution. $C = \{k_1, \dots, k_n\}$ is the set of n class; the probability that an object A is a class k_i is denoted by $p(k_i|A)$. The Bayes theorem:

$$p(k_i|A) = \frac{p(A|k_i)p(k_i)}{p(A)} \quad (15)$$

For Bayesian classification, it is chosen the class k_i where $p(A|k_i)p(k_i)$ is larger. This way the observed object A is assigned to the class k_i . We assume that the probability of the feature vector of object A is of class k_j ($p(A|k_i)$), has a Gaussian Distribution with mean μ and with covariance matrix Ω defined as follows:

$$P(A|k_i) = \frac{1}{\Delta} \exp \left[-\frac{1}{2} (A - \mu_j)^T \Omega_j^{-1} (A - \mu_j) \right]$$

Where $\Delta = (2\pi)^{m/2} (\det \Omega_j)^{1/2}$ and m is the dimension of the feature vectors.

3) *Artificial Neural Network*: It is a mathematical model that is inspired by the way biological neurons work. The ANN [20] is applied in the learning of tasks where each instance is described by a set of values that represent its Special features and where the function objective $f(A)$ can take any value from a finite set V . In the present work, there are implemented two ANN, one dedicated to the classification of numbers and another for letters. The ANN deployed is of type Backpropagation, activated by the sigmoid function.

4) *Principal Component Analysis*: The Principal Components Analysis (PCA) is a technique in which a set of correlated variables are transformed to another set of not correlated variables that is a linear combination from the

original variables in which most of these variables can be removed with minimal loss of the original information; this characteristic allows the PCA to be employed to reduce the dimensionality of a large set of data losing the least of information.

III. EXPERIMENTS

To perform the experimentations phase it was employed a database of 70 images of plates from the 31 States of the Mexican Republic, except Mexico City because these entity license plates are different (contain 6 characters instead of 7). This database was chosen because it was considered the sampling by convenience. The images were captured at a distance of 1.5 to 3 meters between the camera and the plate, seeking a uniform lighting and subsequently normalized to 700×350 pixels.

Experimentation was made in two phases:

- Training to recognize letters and numbers
- License plate recognition.

A. Training to recognizer letters and numbers

To test the recognizers, 738 characters, 374 letters, and 364 numbers were used. In Table I we showed the number of characters used in each case. That number is not equal for all symbols since they do not appear with the same frequency in the license plate.

Three types of characters recognizers were implemented: 1) coefficient of correlation 2) Bayesian classifier and 3) Artificial Neural Network (ANN). These last 2 networks were implemented, one for numbers and another for letters.

TABLE I. NUMBER OF EXAMPLES USED DURING THE TRAINING.

A	B	C	D	E	F	G	H	J	K	L
14	9	11	17	16	18	26	21	20	13	14
M	N	R	P	S	T	U	V	W	X	Y
13	14	20	17	12	18	18	21	13	15	18
Z	0	1	2	3	4	5	6	7	8	9
16	27	27	40	32	41	50	43	34	33	37

In the case of the Bayesian classifier and ANN, a vector of 8-dimension characteristics was used at first, formed by 7 Hu's moment and correlation Factor; the results to classify the numbers and letters are shown in Table II. In the present experiment, the networks are two layers with the following structure: {8,10} for numbers and {8,23} for letters.

TABLE II. RESULTS OF CLASSIFICATION WITH 7 HU'S MOMENT AND THE CORRELATION FACTOR

Character	Bayesian classifier			
	Hits	%	Hits	%
Numbers	246/364	67.58	333/364	91.48
Letters	294/374	78.60	273/374	72.99

Subsequently, vectors were added with characteristics 30 and 60 Fourier descriptors, forming vectors of dimension 38: 7 Hu's moment, 30 Fourier descriptors and correlation factor, and dimension 68: correlation factor, 7 Hu's moment and 60 Fourier descriptors. Results obtained in the tests with the Bayesian classifier can be observed in Table III.

In this experiment, neural networks were also implemented for the 38 and 68 features, establishing several elements per layer experimentally. For example, the structure for network with 38 features is: letters = {38, 76, 23} and numbers = {38, 38, 10}. Both networks correctly classified 100% of the examples.

TABLE III. BAYESIAN CLASSIFIER RESULTS WITH 38 AND 68 FEATURES.

Character	Bayesian classifier			
	38 hits	%	68 hits	%
Numbers	325/364	89.40	331/364	91.50
Letters	360/374	96.4	369/374	98.7

Finally, the Principal Components Analysis was observed to reduce the dimension of the vector of characteristics in the following way:

- 1) 38 features vector: The number of vectors decreased to 9 components, while the letters vectors went from 38 to 8 items. Neural networks implemented in this case have the following composition: numbers: {9, 27, 36, 10}, Letters: {8, 16, 32, 23}.
- 2) 68 features vector: The number of vectors decreased to 13 components, while the letters vectors went to only 11. Neural networks implemented in this case have the following composition: Numbers: {13, 26, 13, 10}, Letters: {11, 22, 46, 23}.

B. Recognition of Vehicle's registration.

Table IV shows the number of images of plates used by State.

TABLE IV. EXAMPLES NUMBER OF REGISTRATIONS BY STATE.

Aguascalientes	Baja California Norte	Baja California Sur	Campeche	Chihuahua
2	1	0	2	2
Colima	Coahuila	Chiapas	Durango	Estado de México
1	2	2	1	3
Guerrero	Guanajuato	Hidalgo	Jalisco	Michoacán
3	4	3	4	4
Morelos	Nuevo León	Nayarit	Oaxaca	Puebla
2	2	3	2	2
Querétaro	Quintana Roo	San Luis Potosí	Sinaloa	Sonora
4	1	4	3	1
Tabasco	Tamaulipas	Tlaxcala	Veracruz	Yucatán
2	2	1	2	2
Zacatecas	Total			

The results obtained in the test what the recognizers developed are shown in Figure 8.

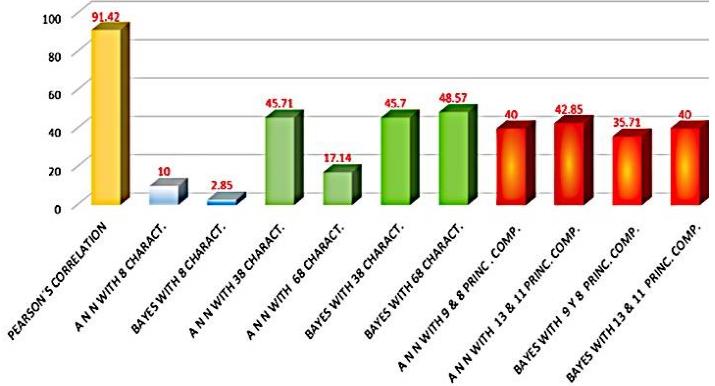


Figure 8. Results of classification plates to different three organizers implemented.

To consider recognition of the registration as a success we use the following criteria: A registration is successfully recognized if all their characters are classified correctly. Otherwise, those registrations are not recognized successfully.

IV. DISCUSSION

In the development of this proposal, there were implemented 3 types of recognizers: 1) correlation factor, 2) Bayesian classifier and 3) Artificial neuronal network. Out of these techniques, the correlation Factor had the best performance as it can be seen in Fig. 8. The main problem with this technique is comparing the examples of plates presented in Table IV with a properly chosen template. This is due, on one hand, to the segmented characters that may have an excess of noise, and on the other hand, to the size and shape of the character.

Using the Bayesian classifier with only 8 features we got a percentage of hits of 2.85%. However, when it increased the number of features with Fourier descriptors, the percentage increased to 45.7% with 38 features and 48.57% with 68 characteristics. When the Principal Components was incorporated, the percentage decreased by 8%. This shows that the cumulative variance cannot be greater than 95%.

For neural networks, with 38 features we obtained 47.50% of hits and 17.14% with 68 features. When using 8 and 9 principal components with 38 features, performance decreased only by 5%. On the other hand, when using 13 and 11 principal components with 68 features, the number of hits increased to 43.85%. This shows that increasing the number of elements in character modeling is not always the best alternative.

The classification mistakes in the Bayesian classifier recognizer is presented in the following form: (a) Number 9 is classified as 6 and 4, 1 is recognized as 7 and 8 is

classified as 0. (b) Letters: J and T are classified as L and H, respectively, and letters X and M are recognized as W.

The classification mistakes in artificial neural networks are: (a) Number 9 is recognized as 6 and vice versa, 5 as 3 and in some cases 4 as 9. (b) Letter R is classified as K; L is recognized as J; M is rated as W and vice versa; V is recognized as A.

In the two previous classifiers, an error that occurs in both is that they cannot recognize characters that have a very similar structure; some examples are 5 and 8 and H and K, or characters that look like their rotated version such as M and W, 6 and 9, and V and A.

V. CONCLUSIONS

In this paper, we proposed an algorithm for recognition of vehicle registration plates of Mexico, which consider different texture patterns and colors in the background of plates. This is an improvement with respect to other works, since most consider plates formed with black characters on white background.

In the case of the vehicle registration plates of Mexico, they have different texture patterns and colors in the background; so, when we applied the traditional algorithms, their performance was considerably affected. This was mainly due to the segmentation of characters; they are usually extracted with part of the texture of the background, getting incorrect characters.

The results showed that the proposed algorithm is robust because in tests recognition using the Pearson correlation coefficient we obtained a 91.42% of recognized characters. In this case, we used techniques of template matching, which showed that characters segmentation was adequate.

This proposed algorithm used the Mexican official standard NOM-001-SCT-2-2000, which specifies the dimensions of the plate, and the size and location of the characters; this information provided the phase of segmentation of characters.

By experimentation, it was obtained a threshold of pixel intensity for each character that allows us to separate them from the background characters with more precision.

Although the recognition was minimal, it should be considered that the misclassification of a single character implies that the plate is not acknowledged successfully, therefore, a finer threshold to decide when a plaque is or is not recognized must be established.

Using the characterization of the correlation coefficient, Hu's moments and Fourier descriptors allowed to recognize the characters successfully. So, even though results obtained in the phase of license plate recognition were low, good results were achieved in an individual way.

For some alphanumeric characters of the plates, such as the M and W, 6 and 9, A and V, based on the characteristic that the moments of Hu are invariant, the recognition was wrong. This was because, in an invariant process, the characters are described through a set of quantifiable features (very similar in the previous characters), that are insensitive to any type of deformation. For the rotation characters in the

Hu's moments is necessary to apply mechanisms to reduce the number of misrecognized characters, for example, Chain Codes. For these, we chose a starting point and travelled the border in a clockwise direction indicating the direction the border is following, thus having a qualitative and quantitative recognition.

This study is original with respect to Automatic Number Plate Recognition (ANPR) literature and addresses harder use-case than those commonly evaluated. We believe the performances of the proposed algorithm should be compared to open-source ANPR engines addressing textured immatriculation plates management.

The dataset we used in this study is small: 2 plates per Mexican state, resulting in 9 to 26 examples for each letter, which is rather low to train and evaluate robust systems. This is the reason why it is necessary to increase the size of the database to improve the results of the proposed algorithm.

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