Low Complexity Long PN Code Acquisition Scheme for Spread Spectrum Systems

Jeehyeon Baek[1], Jonghun Park[2], Youngpo Lee[1], Sun Yong Kim[3], Gyu-In Jee[3], Jin-Mo Yang[4], and Seokho Yoon[1,†]
[1]School of Information and Communication Engineering, Sungkyunkwan University, Suwon, Korea
[2]Samsung Thales Co. LTD., Seongnam, Korea
[3]Department of Electronics Engineering, Konkuk University, Seoul, Korea
[4]Agency for Defense Development (ADD), Daejeon, Korea
[†]Corresponding author
(Email: bjh1987@skku.edu, johnjh.park@samsung.com, leeyp204@skku.edu,
{kimsy, gijee}@konkuk.ac.kr, jmy1965@dreamwiz.com, and [†]syoon@skku.edu)

***Abstract*—In this paper, a low complexity long pseudo noise (PN) code acquisition scheme is proposed for spread spectrum systems including global positioning system (GPS) and code division multiple access (CDMA) system. By using a phase-shift-network, the proposed scheme has less complexity than the conventional dual correlating sequential estimation scheme. From the analytic and numerical results, it is confirmed that the proposed scheme has the lower hardware complexity and the same mean acquisition time performance compared with the conventional scheme.**

***Index Terms*—acquisition, sequential estimation, phase-shift-network, PN code**

I. Introduction

In spread spectrum (SS) receivers, the pseudo noise (PN) code synchronization is one of the most important tasks, which is generally carried out in two stages: code acquisition and tracking [1]. In the code acquisition stage, the coarse alignment between the received and locally generated PN codes is performed, and subsequently, in the code tracking stage, the fine alignment between the two codes is performed.

In recent SS-based mobile communication systems including global positioning system (GPS) and code division multiple access (CDMA) system, a long PN code is essential to provide reliable positioning service or user distinction [2], [3]. However, acquisition of a long PN code leads to excessive acquisition time and hardware complexity for serial and parallel acquisition methods, respectively, which are typical acquisition schemes in SS-based systems [4].

To deal with acquisition of a long PN code, several schemes [5]-[7] have been proposed. Ward proposed an interesting code acquisition scheme based on sequential estimation of the received PN code [5], which is referred to as the traditional sequential estimation (TSE) scheme in this paper. The TSE scheme has a shorter mean acquisition time (MAT) and a lower hardware complexity than the serial and parallel acquisition schemes, respectively; however, the TSE scheme cannot achieve code acquisition when the PN code is inverted due to data modulation. To overcome this drawback, Chiu and Lee proposed an improved sequential estimation (ISE) scheme, which can achieve code acquisition regardless of whether the PN code is inverted or not by using an expanded primitive polynomial [6]. By incorporating the TSE and ISE schemes, Koller and Belkerdid proposed a dual correlating sequential estimation (DCSE) scheme, which can not only achieve acquisition for both inverted and non-inverted PN codes, but also demodulate the SS signal [7], whose hardware complexity is, however, inevitably high.

In this paper, thus, we propose a novel sequential estimation scheme for acquisition of a long PN code, which can achieve acquisition for both inverted and non-inverted PN codes and can also demodulate data as in the DCSE scheme, yet with only half the complexity of the DCSE scheme. We refer to the proposed scheme as phase-shift-network-based differential sequential estimation (PDSE) scheme since it employs a differential operator and a phase-shift-network. The numerical results demonstrate that the proposed scheme has a lower complexity regardless of the length of PN code while maintaining the same level of code acquisition performance compared to that of the conventional DCSE scheme.

This paper is organized as follows. Details of the system model and proposed PDSE scheme are described in Section II. The performance analysis and complexity comparison of conventional and proposed schemes are delineated in detail in Section III. Section IV concludes this paper with a brief summary.

II. Proposed Scheme

Fig. 1 shows the structure of the PDSE receiver proposed in this paper. At the receiver, as in the conventional DCSE scheme, PDSE scheme requires $n+1$ consecutive chips out of $L\ (=\ 2^n-1)$ chips of PN sequence period for PN code acquisition. The kth chip of the received signal can be expressed as

$$r_k = \sqrt{P}T_cD(2s_k-1)+w_k,\ \text{for}\ k=0,\ 1,\ \cdots,\ n, \quad (1)$$

where P, T_c, and D are the power of the PN signal, duration of a PN chip, and modulated data taking a value in $\{-1,1\}$ with equal probability, respectively, $s_k \in \{0,1\}$ is the kth chip

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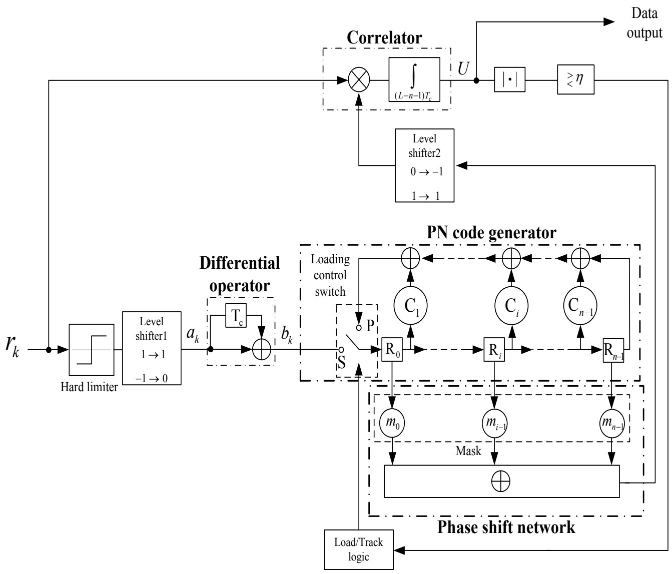

Fig. 1. A structure of the PDSE receiver.

of the PN code with period of L, and w_k is the kth additive white Gaussian noise (AWGN) sample with mean zero and variance of $\sigma_n^2 = N_0 T_c/2$.

First, r_k is mapped into 1 or -1 at the hard limiter with the threshold 0, and subsequently, the kth output a_k of level shifter1 is obtained by converting the value -1 (1) into 0 (1). To achieve code acquisition regardless of whether the PN code is inverted (i.e., $D = -1$) or not (i.e., $D = 1$), we use the differential operator whose output can be expressed as $b_k = a_k + a_{k+1}$ for $k = 0, \cdots, n-1$, which removes the effect of data modulation. Specifically, b_k equals to $s_k + s_{k+1}$ for both cases that $D = 1$ and $D = -1$ in the absence of noise. Moreover, b_k is equivalent to s_{k+l}, the phase-shifted version of s_k with arbitrary phase difference l $(0 \leq l \leq L)$ due to the shift-and-add property [1]. This is the key idea of the proposed PDSE scheme.

In the loading process (i.e., the loading control switch is set to 'S'), $\{b_k\}_{k=0}^{n-1}$ are loaded into registers $\{R_k\}_{k=0}^{n-1}$ in the PN code generator with coefficients of the primitive polynomial $\{c_j\}_{j=1}^{n-1}$. After the loading process, loading control switch is set to 'P' and we can construct a PN code $\{\hat{b}_k\}_{k=0}^{L-1}$ with length of L, the l-shifted version of the PN code with correct phase. Thus, in the absence of noise, we can write $\{\hat{b}_k\}_{k=0}^{L-1}$ as

$$\{\hat{b}_k\}_{k=0}^{L-1} = \{s_k + s_{k+1}\}_{k=0}^{L-1} = \{s_{k+l}\}_{k=0}^{L-1}. \tag{2}$$

To estimate the phase difference l between $\{\hat{b}_k\}_{k=0}^{L-1}$ and $\{s_k\}_{k=0}^{L-1}$, we express $\{\hat{b}_k\}_{k=0}^{L-1}$ in a polynomial forms as $\bmod\{(1+x), f(x)\}/f(x)$ and $\bmod\{x^l, f(x)\}/f(x)$ from equivalent expressions $\{s_k + s_{k+1}\}_{k=0}^{L-1}$ and $\{s_{k+l}\}_{k=0}^{L-1}$, respectively, where $f(x)$ is the primitive polynomial with coefficients $\{c_j\}_{j=1}^{n-1}$. From the polynomial forms of $\{\hat{b}_k\}_{k=0}^{L-1}$, we can obtain the phase difference l, and then, delaying the phase of the output for $L-l$ by using phase-shift-network, the phase of the output PN code comes to be the same as that of the PN

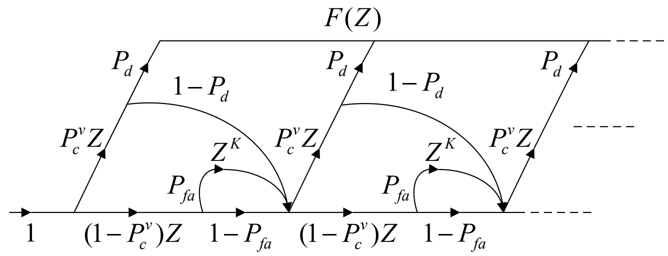

Fig. 2. A generation function flow graph of the acquisition schemes based on the sequential estimation.

code in the received signal, which can be generated by using mask polynomial and expressed as [8]

$$\begin{aligned} m(x) &= \bmod\{x^{(L-l)}, f(x)\} \\ &= m_0 + m_1 x + \cdots + m_i x^i + \cdots + m_{n-1} x^{n-1}, \end{aligned} \tag{3}$$

where $m_i \in \{0, 1\}$ denotes the ith coefficient of the mask polynomial.

To determine whether acquisition is achieved or not, we obtain the correlation value U between $\{r_k\}_{k=0}^{n-1}$ and the output of the phase-shift-network changed into bipolar PN code by level shifter2. Then, the absolute correlation value $|U|$ is compared with a given threshold value η. If $|U|$ is smaller than η, the acquisition is determined to be failed and the receiver repeats the acquisition process. If $|U|$ is larger than η, it is determined that the acquisition is achieved. Finally, the PDSE receiver demodulate the data D by using the correlation U comparing with the threshold of zero.

III. Performance Analysis

A. Mean acquisition time

The expression of MAT can be obtained by using a generation function flow graph [9], [10]. Fig. 2 shows a generation function flow graph of the acquisition schemes based on the sequential estimation including TSE, ISE, DCSE, and PDSE, where P_c is the correct chip probability defined as the probability that the chip is estimated correctly, P_c^v is the correct chip probability of consecutive v chips (in DCSE and PDSE schemes, $v = n+1$), P_d is the detection probability defined as the probability to achieve the acquisition when all v chips are estimated correctly, and P_{fa} is the false alarm probability defined as the probability that $|U|$ is larger than η when there exist at least one error in the process of the sequential estimation. K is the penalty factor associated with the false alarm. From Fig. 2, the generation function $F(Z)$ can be derived as

$$F(Z) = \frac{P_c^v P_d Z}{1 - P_c^v(1-P_d)Z - (1-P_c^v)\{P_{fa} Z^{K+1} + (1-P_{fa})Z\}}. \tag{4}$$

From (4), the MAT can be achieved as

$$E[T_{acq}] = \left.\frac{dF(Z)}{dZ} T_e\right|_{Z=1} = \frac{1 + (1-P_c^v) K P_{fa}}{P_c^v P_d} T_e, \tag{5}$$

where T_e is the estimation time spent for loading and estimation process. As we can see in (5), the MAT of the DCSE and PDSE schemes depends on the correct chip, detection, and false alarm probabilities.

Since both the hard limiters of the DCSE and PDSE schemes perform the same operation, those schemes have same correct chip probability P_c. The correct chip probability of the kth chip of the received signal is represented as

$$\begin{aligned} P_c &= 1 - \frac{1}{2}\Pr[r_k < 0 | s_k = 1] - \frac{1}{2}\Pr[r_k > 0 | s_k = 0] \\ &= 1 - \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{ \frac{(x - \sqrt{P}T_c)^2}{2\sigma_n^2} \right\} dx \\ &\quad - \frac{1}{2}\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{ \frac{(x + \sqrt{P}T_c)^2}{2\sigma_n^2} \right\} dx \\ &= 1 - \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{ \frac{(x - \sqrt{P}T_c)^2}{2\sigma_n^2} \right\} dx. \end{aligned} \tag{6}$$

When all consecutive $n + 1$ chips are estimated correctly (i.e., correct estimation), in the PDSE scheme, the phase of the generated PN code after loading process and that of the PN code from the transmitter are same (the received PN code is non-inverted) or opposite (the received PN code is inverted). Thus, the probability density function (PDF) of the correlation value U can be written in two cases. First, when the received PN code is non-inverted, the PDF of the correlation value U of the PDSE scheme can be written as

$$f_{U|c}(x) = \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left\{ -\frac{(x - \sqrt{P}MT_c)^2}{2\sigma_N^2} \right\}, \tag{7}$$

where $M\ (= L - n - 1)$, $\sqrt{P}MT_c$, and $\sigma_N^2 = N_0 M T_c/2$ are the length of the correlation, the mean, and variance of U, respectively. Next, when the received PN code is inverted, the PDF of the correlation value U of the PDSE scheme can be written as

$$f_{U|c}^{\text{inv}}(x) = \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left\{ -\frac{(x + \sqrt{P}MT_c)^2}{2\sigma_N^2} \right\}. \tag{8}$$

(8) has an opposite mean and the same variance compared with (7). Since the receiving probabilities of the inverted and non-inverted PN codes are the same, the detection probability of the PDSE scheme can be written as

$$\begin{aligned} P_d &= \frac{1}{2} P\{|U| \geq \eta | \text{correct estimation, non-inverted}\} \\ &\quad + \frac{1}{2} P\{|U| \geq \eta | \text{correct estimation, inverted}\} \\ &= \frac{1}{2}\left\{ \int_{\eta}^{\infty} f_{U|c}(x)dx + \int_{-\infty}^{-\eta} f_{U|c}(x)dx \right\} \\ &\quad + \frac{1}{2}\left\{ \int_{\eta}^{\infty} f_{U|c}^{\text{inv}}(x)dx + \int_{-\infty}^{-\eta} f_{U|c}^{\text{inv}}(x)dx \right\}. \end{aligned} \tag{9}$$

In (9), $f_{U|c}$ and $f_{U|c}^{\text{inv}}$ are symmetric. Thus, we can rewrite (9) as

$$\begin{aligned} P_d &= \frac{1}{2}\left\{ \int_{\eta}^{\infty} f_{U|c}(x)dx + \int_{-\infty}^{-\eta} f_{U|c}(x)dx \right\} \\ &\quad + \frac{1}{2}\left\{ \int_{-\infty}^{-\eta} f_{U|c}(x)dx + \int_{\eta}^{\infty} f_{U|c}(x)dx \right\} \\ &= \int_{\eta}^{\infty} f_{U|c}(x)dx + \int_{-\infty}^{-\eta} f_{U|c}(x)dx \\ &= 1 - \Phi\left(\frac{\eta - \sqrt{P}MT_c}{\sigma_N}\right) + \Phi\left(\frac{-\eta - \sqrt{P}MT_c}{\sigma_N}\right), \end{aligned} \tag{10}$$

where $\Phi(x)$ means the cumulative distribution function (CDF) of Gaussian distribution with zero mean and unit variance.

Since the DCSE scheme exploits the absolute correlation value and threshold same as in the PDSE scheme for acquisition, the detection probability of the DCSE scheme is the same as that of the PDSE scheme.

In the other case, if there is at least one chip which is not estimated correctly (i.e., wrong estimation), the phase of the PN code generated after loading process is different with that of the PN code from the transmitter. Let us assume that we use long PN code thus the period of the PN code L is long enough. If the PN codes are miss matched, then the correlation value during M chips can be approximated as $-1/L$. Since $L \gg 1$, the correlation value can be approximated as 0, and thus, the PDF of the correlation value of the PDSE scheme (same as that of the DCSE scheme) in the case of wrong estimation can be written as

$$f_{U|w}(x) = \frac{1}{\sqrt{2\pi}\sigma_N} \exp\left\{ -\frac{x^2}{2\sigma_N^2} \right\}. \tag{11}$$

From (11), the false alarm probability can be expressed as

$$\begin{aligned} P_{fa} &= P\{|U| \geq \eta | \text{wrong estimation}\} \\ &= \int_{\eta}^{\infty} f_{U|w}(x)dx + \int_{-\infty}^{-\eta} f_{U|w}(x)dx \\ &= 1 - 2\Phi\left(\frac{\eta}{\sigma_N}\right). \end{aligned} \tag{12}$$

So far, we have analyzed the correct chip, detection, and false alarm probabilities of the PDSE and DCSE schemes. Finally, by substituting (6), (10), and (12) in (5), we can obtain the MATs of the PDSE and DCSE schemes.

We prove the analytic results by using Monte Carlo simulations. Fig. 3 shows the MAT performances of the PDSE and DCSE schemes. We use the primitive polynomials as $1 + x^2 + x^3 + x^4 + x^8$ and $1 + x^3 + x^{10}$ and assume $T_e = LT_c$; $K = 10L$. To calculate the threshold, the false alarm probability is fixed to 0.01. As the analytic results, the simulation results demonstrate that the MAT performances of both DCSE and PDSE schemes are same.

B. Complexity Comparison

In this section, we compare the hardware complexity of the PDSE scheme with that of the DCSE scheme. To compare the hardware complexity, we count the number of materials which compose each scheme as in [11]. Table I shows the number

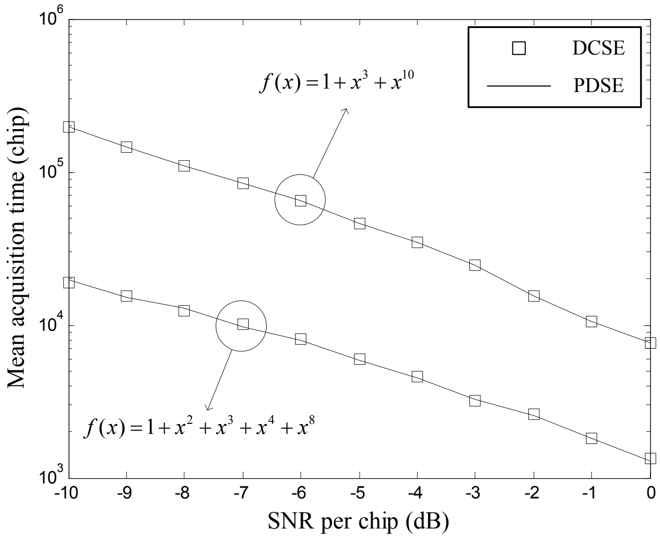

Fig. 3. The MAT performance of the proposed and conventional schemes with primitive polynomials $1+x^2+x^3+x^4+x^8$ and $1+x^3+x^{10}$.

TABLE I
THE NUMBER OF HARDWARES FOR THE PDSE AND DCSE SCHEMES.

Hardwares	PDSE scheme	DCSE scheme
Registers	$n+1$	$2n+1$
Correlators	1	2
Absolute operators	1	2
Threshold comparators	1	2
Differential operators	1	0
Inverting controllers	0	1
Combination logics	0	1

of hardwares for the PDSE and DCSE schemes. As shown in Table I, the PDSE scheme needs a half of hardware complexity of the DCSE scheme in terms of the required registers, correlators, absolute operators, and threshold comparators (generally, $n \gg 1$). Moreover, the PDSE scheme only needs a differential operator unlike DCSE scheme which requires logic blocks and an inverting control switch in order to incorporate the TSE and ISE schemes [7].

The proposed PDSE scheme can be applied to the synchronization process of the SS-based communication systems such as GPS and CDMA system, where the fast acquisition of long PN sequence with low hardware complexity is necessary for practical implementation.

IV. CONCLUSION

In this paper, we have proposed a novel sequential estimation scheme for acquisition of a long PN code based on differential estimation and the phase-shift-network. The proposed scheme has a lower complexity compared with the conventional scheme regardless of the length of PN code. Analytic and numerical results confirm that the proposed scheme, with only half complexity, has the same level of the MAT performance to that of the conventional scheme.

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