

# Study on the Improved D-S Evidence Theory and its Application on Gas Outburst Prediction

Y. Z. Miao<sup>1,2</sup>, Y. Chen<sup>1</sup>, H. X. Zhang<sup>2</sup>, J. W. Zhang<sup>2</sup>,

1. School of Information and Electrical Engineering,  
China University of Mining and Technology  
Jiefang South Road, 221008, Xuzhou, China  
miaoyanzi81@163.com

2. Institute of Technical Aspects of Multimodal Systems,  
Department of Informatics, University of Hamburg  
Vogt-Koelln-Strasse 30, 22527, Hamburg, Germany  
miao@informatik.uni-hamburg.de

**Abstract**-This paper introduces improved combination rules for the D-S Evidence Theory for dealing with the evidence conflicts which considers the coherence evidence and the conflicts evidences together and allocate the conflicts to various focal elements according to the credibility of the coherence evidence. Also after introduce the similarity degree to denote the similarity between two fuzzy focal elements, and extended the *Bel* and *Pl* functions for processing fuzzy data, the improved extension combination rules of the D-S evidence theory to fuzzy sets are described. The coal and gas outburst prediction experiments show that the fusion result with the improved combination rule of the D-S Evidence Theory is more reasonable and could give a more certain decision than each independent method.

**Keywords**-D-S Evidence Theory; Gas Outburst Prediction; Fuzzy Sets.

## I. INTRODUCTION

According to the information supplied by every sensor in a multi-sensor system, information fusion is a reasoning technology for object recognition and property judgment, but information from different sensors is usually abridged, imprecise and even inconsistent. As an uncertain reasoning method and as the generalization of the Bayesian reasoning method, the Dempster-Shafer Evidence Theory (D-S Evidence Theory) is put forward by Dempster in 1967 and expanded by Shafer by systemizing and elaborating the theory. Due to the advantages of the uncertain denotation, measurement and combination, the D-S Evidence Theory was applied widely [1-4]. As it can be combined with other methods, the Evidence Theory is more widely usable and can be extended very well in the future[5-7].

The Evidence Theory has many advantages, but it is not perfect in practical application, and even produces the opposite result to what our intuition tells us. The main reason for these deficits is the variance or conflict between the evidence[8-10]. And the evidence conflict is a problem that cannot be neglected during practical information fusion. The D-S Evidence Theory based on classical sets sometimes seems restrained and helpless concerning the fuzzy concept, so an advanced generalization of extending the evidence

theory to fuzzy sets is proposed to overcome the existing insufficiencies.

Since the gas outburst under the coal mine is a very complicated phenomenon, and there are many factors associated with the outburst [11, 12]. Also the warning signs before it happens are also unexpected and changing. So the D-S Evidence Theory which is an intelligent way of uncertain reasoning is the only potent approach to consider multiple associated factors and make a precise prediction.

In this paper, an improved D-S evidence theory is presented, and it is validated in the coal and gas outburst prediction experiments. In Section II, an improved D-S evidence theory is described, which can resolve the evidence conflict problem effectively. In Section III, we propose the generalized combination rule on fuzzy sets by extending the belief function *Bel* and the plausibility function *Pl* in the D-S evidence theory to fuzzy sets. And the Section IV introduces the application on outburst prediction with our improved combination rules of the D-S evidence theory on fuzzy sets. In the end, the related experimental results and conclusion would be given in Section V.

## II. IMPROVED EVIDENCE COMBINATION RULES

### A. Introduction of Correlative Conceptions

As proposed in [13] the concept of evidence distance, supposing  $\Theta$  is the frame of discernment (FOD) including  $N$  different propositions and  $M$  evidence sources:  $S_1, S_2, \dots, S_M$ , the corresponding BPAFs are:  $m_1, m_2, \dots, m_M$ . We consider each evidence source  $S_i$  as row vector  $S_i$  in  $2^N$  dimension; every part of the vector is the distributed probability  $m_i$  of every element in exponential sets  $2^\Theta$  of  $\Theta$ . So, the distance between two evidence sources  $S_i, S_j$  is defined as:

$$d(S_i, S_j) = \sqrt{\frac{1}{2}(S_i - S_j)\underline{D}(S_i - S_j)^T} \quad (1)$$

where,  $\underline{D}$  is a matrix in  $2^N \times 2^N$

$$\underline{D}(A, B) = \frac{|A \cap B|}{|A \cup B|} \quad A, B \in 2^\Theta$$

The formula to get the evidence distance is also given in [13]:

$$d(S_i, S_j) = \sqrt{\frac{1}{2} (\|S_i\|^2 + \|S_j\|^2 - 2\langle S_i, S_j \rangle)} \quad (2)$$

in which,

$$\langle S_i, S_j \rangle = \sum_{s=1}^{2^N} \sum_{t=1}^{2^N} m_i(A_s) m_j(A_t) \frac{|A_s \cap A_t|}{|A_s \cup A_t|} \quad A_s, A_t \in 2^\Theta$$

So, the evidence distance represents the difference between two kinds of evidence, which is another reflection of the evidence conflict. On the other hand, the coherence degree of two kinds of evidence is also reflected. Obviously, the distance and coherence between two kinds of evidence are inversely proportional. When the distance between two kinds of evidence is equal to be 1, the two kinds of evidence are absolutely incoherent and the degree of coherence is 0; and when the distance becomes 0, the evidence is absolutely coherent and the degree of coherence is 1. Based on the fusion effect, we choose a simple formula to define the degree of coherence between two kinds of evidence. For example, the degree of coherence between  $S_i, S_j$  is described as:

$$coh(S_i, S_j) = 1 - d(S_i, S_j) \quad (3)$$

As can be seen from the definition, the coherence degree reflects the degree of mutual sustainment, so the degree to which evidence  $S_i$  is sustained by other instances of evidence is defined as:

$$sup(S_i) = \sum_{j=1, j \neq i}^M coh(S_i, S_j) \quad (4)$$

Hence, the greater the degree to which the evidence is sustained by other evidence, the higher its reliability. Therefore we can define the credibility of a single evidence source using the sustainment degree between two kinds of evidence. The credibility of evidence source  $S_i$  is defined as:

$$cre(S_i) = sup(S_i) / \sum_{i=1}^M sup(S_i) \quad (5)$$

The more number of highly coherent evidence sources, the more highly credible single evidence sources, and then the higher total credibility of M evidence sources; whereas it is lower. Furthermore, when all the evidence sources conflict absolutely, the coherence degree between two random kinds of evidence is 0 and the total credibility of the evidence sources should intuitively be 0. On the other hand, when all the evidence is absolutely coherent, the mutual coherence degree is 1 and the total credibility of the evidence sources should intuitively be 1. Therefore, we define the total

credibility hypersphere of the evidence sources and the total credibility hypersphere of absolute coherence (the perfect hypersphere), the dimensions of which are both  $C_M^2$ , and the respective radiuses  $r$  and  $R$  are defined as:

$$r = C_M^2 \sqrt{\sum_{i=1}^{M-1} \sum_{j=i+1}^M coh^{C_M^2}(S_i, S_j)} / 2 \quad (6)$$

$$R = C_M^2 \sqrt{C_M^2 \times 1^{C_M^2}} / 2 = C_M^2 \sqrt{C_M^2} / 2 \quad (7)$$

We consider the total credibility of the evidence sources as the degree closest to the credibility of perfect evidence sources in terms of absolute coherence, that is, the degree of the total credibility hypersphere is closest to the perfect hypersphere of evidence sources. Considering that the credit of evidence sources is one-dimensional, we define the total credit  $C$  of evidence sources as:

$$C = r/R \quad (8)$$

### B. Improved Combination Rules I

In this paper, we consider the conflict and coherence of the evidence combination synchronously, the conflict evidence combined with the credibility of the evidence source, and the coherence evidence combined with the AND-algorithm, which can reflect the degree of intersection fusion. Therefore, our improved rules are respectively based on the two different ideas mentioned above. To this end, we suppose  $A_i, B_j$ , and  $C_k$  to be the focal elements of M different evidence sources in the following section.

Based on the idea that conflict evidence can be used by parts, we think a part of the conflict information can be allocated to focal elements of the evidence, and the other parts to the unknown term  $m(\Theta)$ , moreover, the allocation depends on the credibility of the total evidence source and the single evidence. So the improved rule I is:

$$m(\Phi) = 0 \quad (9)$$

$$m(A) = \frac{1-K}{P} \cdot \sum_{A_i \cap B_j \cap C_k \cap \dots = A} \frac{|A|^M}{|A_i| |B_j| |C_k| \dots} \cdot m_1(A_i) m_2(B_j) m_3(C_k) \dots + KC \sum_{i=1}^M (m_i(A) \cdot cre(S_i)) \quad \forall A \neq \Phi, \Theta \quad (10)$$

$$m(\Theta) = \frac{1-K}{P} \cdot \sum_{A_i \cap B_j \cap C_k \cap \dots = \Theta} \frac{|\Theta|^M}{|A_i| |B_j| |C_k| \dots} \cdot m_1(A_i) m_2(B_j) m_3(C_k) \dots + KC \sum_{i=1}^M (m_i(\Theta) \cdot cre(S_i)) + K(1-C) \quad (11)$$

Where, the definitions of C and cre(Si) are the same as above. And in which,

$$K = \sum_{A_i \cap B_j \cap C_k \cap \dots = \Phi} m_1(A_i) \cdot m_2(B_j) \cdot m_3(C_k) \cdot \dots \quad (12)$$

This stands for the total conflict between all evidence.

$$P = \sum_{A_i \cap B_j \cap C_k \cap \dots \neq \Phi} Q \cdot m_1(A_i) \cdot m_2(B_j) \cdot m_3(C_k) \cdot \dots$$

$$= \sum_{A_i \cap B_j \cap C_k \cap \dots \neq \Phi} \frac{|A_i \cap B_j \cap C_k \cap \dots|^M}{|A_i| |B_j| |C_k| \dots} m_1(A_i) \cdot m_2(B_j) \cdot m_3(C_k) \cdot \dots \quad (13)$$

The formula above gives the total probability of coherence evidence after intercross fusion; the Q in the formula reflects the degree of intercross fusion of the evidence, including the AND-algorithm. Obviously, the more coherence there is between evidence sources, the higher the degree of intercross fusion; and (1-K)/P stands for the probability of unitary allocation of the coherence evidence after the intercross fusion. The second term in formula (10) and (11) stands for allocating the total conflict of the evidence source by weight logically, respectively considering the credit of the evidence source in total and individually.

Then we prove that function *m* from the rule above is a BPAF, that is, we only need to prove that:

$$\sum_{A \subset \Theta} m(A) = 1 \quad (14)$$

This detailed proof has been given in other paper which has been published [14].

### C. Improved Combination Rules II

Based on the idea that conflict evidence can be used completely (in the above rule) under the condition that the total credibility of the evidence sources  $C=I$ , and referring to the method in reference [15], we think the conflict information can be completely allocated to focal elements of evidence, and the allocation depends on the credibility of every single evidence source. Therefore, the improved rule II is as follows:

$$m(\Phi) = 0 \quad (15)$$

$$m(A) = \frac{1-K}{P} \sum_{A_i \cap B_j \cap C_k \cap \dots = A} \frac{|A|^M}{|A_i| |B_j| |C_k| \dots} m_1(A_i) m_2(B_j) m_3(C_k) \cdot \dots$$

$$+ K \sum_{i=1}^M (m_i(A) \cdot cre(s_i)) \quad \forall A \neq \Phi \quad (16)$$

Every term in the above formula is the same as that in rule I. When the frame of discernment  $\Theta$  is also the focal element, we can calculate the probability  $m(\Theta)$  using formula (16).

According to rule II, we also have the summary of the probability  $\sum m(A)$  is equal to be 1, and the reasoning is

similar to that in rule I, except that the total credibility of the evidence source used is  $C=I$ . So the *m* is also a BPAF function in rule II.

From the above proving process, we can conclude that rule II can be regarded as an example for rule I if we consider the total credit of the evidence source in rule I to be  $C=I$ , that means, the conflict information can be allocated to focal elements of evidence and unknown terms according to the total credibility of the evidence source in rule I. Therefore, rule I is more reliable and conservative than rule II, while rule II is a more changeable and less reliable decision-making method. However, in contrast to rule I, it is flexible and can adapt to special requirements.

### III. THE IMPROVED EXTENTION OF FUZZY D-S EVIDENCE THEORY

As we know, the evidence theory can express the “uncertainty” distinctly and correctly, and it also features the D-S combination rule which is based on sound mathematic rules. As the evidence theory has a good effect of combination, many researchers continually improved the D-S formulas. However, by extending it from classic sets to fuzzy sets, the denotation and algorithm of fuzzy sets and their intrinsic meanings underwent great changes compared to classic sets. Therefore, when we extend the evidence theory to fuzzy sets, we should change the way evidence is denoted as well as the corresponding combination rule.

Classical sets only need to consider the included elements which are certain, but fuzzy sets consider not only the included elements, but also the degree to which every element is subjected to the fuzzy sets. So taking the fuzzy sets into account, we cannot only heed the included elements like the classic sets, as there several degrees to which an element “belongs to” the set. These degrees are confirmed only by the subjection degree.

Example1: Three fuzzy sets:

$$\tilde{A} = \left\{ \frac{0.9}{1}, \frac{1}{2}, \frac{0.9}{3}, \frac{0.8}{4}, \frac{0.6}{5}, \frac{0.3}{6}, \frac{0.1}{7} \right\},$$

$$\tilde{B} = \left\{ \frac{0.1}{2}, \frac{0.1}{3}, \frac{0.1}{4}, \frac{0.2}{5}, \frac{0.5}{6}, \frac{0.8}{7}, \frac{1}{8}, \frac{0.9}{9} \right\},$$

$$\tilde{C} = \left\{ \frac{0.9}{2}, \frac{1}{3}, \frac{0.9}{4}, \frac{0.8}{5}, \frac{0.4}{6}, \frac{0.2}{7}, \frac{0.1}{8}, \frac{0.1}{9} \right\},$$

According to the definition of the intersection of fuzzy sets [16], we have:

$$\tilde{A} \cap \tilde{B} = \left\{ \frac{0.1}{2}, \frac{0.1}{3}, \frac{0.1}{4}, \frac{0.2}{5}, \frac{0.3}{6}, \frac{0.1}{7} \right\}$$

$$\tilde{A} \cap \tilde{C} = \left\{ \frac{0.9}{2}, \frac{0.9}{3}, \frac{0.8}{4}, \frac{0.6}{5}, \frac{0.3}{6}, \frac{0.1}{7} \right\}$$

From the formulas above we see that the intersections  $\tilde{A} \cap \tilde{B}$  and  $\tilde{A} \cap \tilde{C}$  seem to have the “same” fuzzy elements on the surface, but a different subjection degree. But according to the definition of fuzzy sets, the two fuzzy sets have a very

different nature. Therefore, when considering fuzzy sets, the subjection degree of every element to the fuzzy set is more important, and it is necessary to extend the combination rules of the evidence theory to fuzzy sets.

Example2: Supposing the frame of discernment is  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , and the fuzzy set is  $\tilde{A} = \{\frac{0.9}{\theta_1}, \frac{1}{\theta_2}, \frac{0.9}{\theta_3}, \frac{0.8}{\theta_4}, \frac{0.6}{\theta_5}\}$ , the degree of subjection of the

elements  $\theta_i (i = 6, 7, \dots, n)$  to the fuzzy set  $\tilde{A}$  is equal to 0. The subjection degree of every element in the frame of discernment  $\Theta$  to an empty set also seems to be 0, so the subjection degrees of elements  $\theta_i (i = 6, 7, \dots, n)$  to an empty set and a fuzzy set  $\tilde{A}$  are both equal to 0. This is the similarity between them, thus the degree of their similarity should be larger than 0. This means that the frame of discernment can be considered as a special fuzzy set with subjection degrees of equal to 1 for all included elements, therefore the empty set and the frame of discernment  $\Theta$  can be considered as absolute opposites, and the degree of their similarity should be 0. Furthermore, the similarity of any fuzzy set to itself should be equal to 1.

According to the definitions of the contribution factor, the belief function and the similarity function of fuzzy evidence reasoning were described as follows:

$$Bel(\tilde{B}) = \sum_i F_*(\tilde{B}, \tilde{A}_i)m(\tilde{A}_i) \tag{17}$$

$$Pl(\tilde{B}) = \sum_i F^*(\tilde{B}, \tilde{A}_i)m(\tilde{A}_i) \tag{18}$$

Taking the important degree of every element in the frame of discernment into account, the following formulas of the corresponding contribution factor result:

$$F_*(\tilde{B}, \tilde{A})_\omega = 1 - \frac{1}{|\tilde{A}|} \sum_i \omega_i |\mu_{\tilde{B}}(\theta_i) - \mu_{\tilde{A}}(\theta_i)| \tag{19}$$

$$F^*(\tilde{B}, \tilde{A})_\omega = 1 - \frac{1}{|\Theta|} \sum_i \omega_i |\mu_{\tilde{B}}(\theta_i) - \mu_{\tilde{A}}(\theta_i)| \tag{20}$$

Putting the formulas of  $F_*(\tilde{B}, \tilde{A})_\omega$  and  $F^*(\tilde{B}, \tilde{A})_\omega$  above into  $F_*(\tilde{B}, \tilde{A}_i)$  and  $F^*(\tilde{B}, \tilde{A}_i)$  in the formulas (17) and (18) separately, we get a measurement method for the weight of the belief function and the similarity function.

The combination rule of fuzzy evidence reasoning adopted the idea of Haenni[17], which is to modify the belief allocation model, not to change the form of Dempster's combination rule which has well characters. Before combining the evidence, the Basic Probability of Assignment Function (BPAF) of the fuzzy focal elements needs to be

amended. Based on the similarity between fuzzy sets, for amending the BPAF of fuzzy focal element  $\tilde{A}$ , the weight of the similarity between fuzzy focal elements  $\tilde{C}$  and  $\tilde{A}$  was confirmed as:

$$\omega(\tilde{C}, \tilde{A}) = 1 - \frac{1}{|\Theta|} \sum_i |\mu_{\tilde{C}}(\theta_i) - \mu_{\tilde{A}}(\theta_i)| \tag{21}$$

Supposing  $Bel_1$  and  $Bel_2$  are the belief functions of the same frame of discernment  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , it has the basic probability assignment function  $m_1$  and  $m_2$ , the fuzzy focal elements of which are  $\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_p\}$  and  $\{\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_q\}$ , so the BPAF  $m : 2^\Theta \rightarrow [0, 1]$  of a nonempty set  $\tilde{C}$  can be put forward as:

$$m(\tilde{C}) = m_1 \oplus m_2(\tilde{C}) = \frac{\sum_{\tilde{A}_i \cap \tilde{B}_j = \tilde{C}} \omega(\tilde{C}, \tilde{A}_i)m_1(\tilde{A}_i)\omega(\tilde{C}, \tilde{B}_j)m_2(\tilde{B}_j)}{1 - \sum_{\tilde{A}_i, \tilde{B}_j} (1 - \omega(\tilde{A}_i \cap \tilde{B}_j, \tilde{A}_i)\omega(\tilde{A}_i \cap \tilde{B}_j, \tilde{B}_j))m_1(\tilde{A}_i)m_2(\tilde{B}_j)} \tag{22}$$

in which,  $\omega(\tilde{C}, \tilde{A}_i)$  is the weight of the fuzzy focal element  $\tilde{A}_i (i = 1, 2, \dots, p)$ ,  $\omega(\tilde{C}, \tilde{B}_j)$  is the weight of the fuzzy focal element  $\tilde{B}_j (j = 1, 2, \dots, q)$ .

Accordingly, taking the important degree of the elements in the frame of discernment  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$  into account, the corresponding weight and combination rule could be described as:

$$\omega'(\tilde{C}, \tilde{A}) = 1 - \frac{1}{|\Theta|} \sum_i \alpha_i |\mu_{\tilde{C}}(\theta_i) - \mu_{\tilde{A}}(\theta_i)| \tag{23}$$

$$m'(\tilde{C}) = m_1 \oplus m_2'(\tilde{C}) = \frac{\sum_{\tilde{A}_i \cap \tilde{B}_j = \tilde{C}} \omega'(\tilde{C}, \tilde{A}_i)m_1(\tilde{A}_i)\omega'(\tilde{C}, \tilde{B}_j)m_2(\tilde{B}_j)}{1 - \sum_{\tilde{A}_i, \tilde{B}_j} (1 - \omega'(\tilde{A}_i \cap \tilde{B}_j, \tilde{A}_i)\omega'(\tilde{A}_i \cap \tilde{B}_j, \tilde{B}_j))m_1(\tilde{A}_i)m_2(\tilde{B}_j)} \tag{24}$$

where  $\alpha_i$  is the weight of  $\theta_i \in \Theta$ ,  $\omega'(\tilde{C}, \tilde{A}_i)$  is the weight of fuzzy focal element  $\tilde{A}_i (i = 1, 2, \dots, p)$ ,  $\omega'(\tilde{C}, \tilde{B}_j)$  is the weight of fuzzy focal element  $\tilde{B}_j (j = 1, 2, \dots, q)$ .

This above idea of the combination rule put forward by Lin Zhigui in [18] seems more reasonable, and using numeral experiments, he validated the advantage of the defined concept of similarity between fuzzy sets and the combination rule, which is more sensitive to the changing of fuzzy focal elements. However, this concept also has deficiencies. Firstly, he only defined the similarity between fuzzy focal elements, and according to the combination formula, the similarity degree is equal to 0 between an empty set and any nonempty

fuzzy set. Secondly, the combination formula he put forward has a deficiency, that is, the function  $m$  after combination is no longer a basic probability assignment function, because now  $\sum m(\tilde{C}) < 1$ , and it cannot satisfy the basic condition  $\sum m(\tilde{C}) = 1$ .

From the above chain of reasoning, we can define the similarity between random fuzzy sets as follows:

**Definition 1** Supposing the frame of discernment to be  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , and the fuzzy sets  $\tilde{A}$  and  $\tilde{C}$  are the two random fuzzy subsets, then the similarity between fuzzy sets  $\tilde{A}$  and  $\tilde{C}$  is defined as:

$$\omega(\tilde{C}, \tilde{A}) = 1 - \frac{1}{|\Theta|} \sum_i |\mu_{\tilde{C}}(\theta_i) - \mu_{\tilde{A}}(\theta_i)| \quad (25)$$

As this definition of real function  $\omega$  satisfies the four basic

$$m(\emptyset) = 0 \quad (26)$$

$$m(\tilde{C}) = m_1 \oplus m_2(\tilde{C}) \quad (27)$$

$$\frac{\sum_{\tilde{A} \cap \tilde{B}_j = \tilde{C}} \alpha(\tilde{C}, \tilde{A}) m_1(\tilde{A}) \alpha(\tilde{C}, \tilde{B}_j) m_2(\tilde{B}_j)}{\sum_{\tilde{A} \cap \tilde{B}_j} \alpha(\tilde{A}, \tilde{B}_j) \alpha(\tilde{A}, \tilde{A}) m_1(\tilde{A}) m_2(\tilde{B}_j) - \sum_{\tilde{A} \cap \tilde{B}_j = \emptyset} \alpha(\emptyset, \tilde{A}) \alpha(\emptyset, \tilde{B}_j) m_1(\tilde{A}) m_2(\tilde{B}_j)}$$

Then we also prove that the function from the formulas (26) and (27) above is a BPAF, that is, we only need to prove that  $\sum_{\tilde{C}} m(\tilde{C}) = 1$  in other reference [19].

#### IV. APPLICATION ON GAS OUT BURST PREDICTION

In order to validate the improved combination rule of the D-S evidence theory on fuzzy sets, we did our experiments of gas outburst prediction with this method. The intelligent prediction way based on multi-sensor information presented in this paper is choosing the preliminary prediction results from different main methods as the evidences of the D-S evidence theory, and then making an extended reasoning fusion decision with the improved fuzzy combination rule.

The data in Table I show the indexes from five different mines and the actual results are divided into two kinds, The “A” stands for occurrence of outburst, “B” is no occurrence.

**TABLE I** The Indexes from Different Areas

Number	Preliminary Velocity	Solid Coefficient $f$	Gas Pressure $P$ (Mpa)	$S_{max}$ (L/m)	Composite Index K	Composite Index D	Result
19	10.5	0.3297	0.97	11.2	32.02	3.43	A
20	12	0.2252	0.40	5.3	53.29	-7.89	A
21	3	0.4762	0.21	4.8	6.30	-8.29	B
22	9.1	0.4167	0.63	5.5	14.4	-2.00	A
23	4	0.6610	0.47	3.9	6.05	-2.40	B

conditions in reference [17], the definition of the similarity above is reasonable. And based on the property of the basic probability assignment function, we define the combination rule of the evidence theory on fuzzy sets as definition 2.

**Definition 2** Supposing  $Bel_1$  and  $Bel_2$  to be the belief functions of the same frame of discernment  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , the basic probability assignment function of which to be  $m_1$  and  $m_2$ , and the fuzzy focal elements to be  $\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_p\}$  and  $\{\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_q\}$ , then the basic probability assignment function is defined to be  $m: 2^\Theta \rightarrow [0,1]$  :

In this experiment, we chose neural network, single index method--Preliminary Velocity, D-K composite index, and cutting desorption index  $S_{max}$  way as four evidence source. The prediction results from these four ways are gotten respectively not only for comparison, but also for evidence source of extended decision of the D-S Evidence Theory.

To compare the fusion result with each of the single methods above, we achieved the following prediction results listed in Table II as follows:

**TABLE II** Fusion Results of Each Method

No.		19	20	21	22	23
Neural Network	$m_1(A)$	0.9133	1	0.0133	0.9998	0.1181
	$m_1(B)$	0.0867	0	0.9867	0.0002	0.8819
Single Index	$m_2(A)$	1	1	0.0926	0.8606	0.1220
	$m_2(B)$	0	0	0.9074	0.1394	0.8780
Composite Index	$m_3(A)$	1	1	0.0620	0.9328	0.0588
	$m_3(B)$	0	0	0.9380	0.0672	0.9412
Cutting Desorption	$m_4(A)$	1	0.9524	0.3571	1	0.0816
	$m_4(B)$	0	0.0476	0.6429	0	0.9184
D-S Fusion	$m(A)$	0.9982	0.9994	0.0554	0.99	0.0316
	$m(B)$	0.0018	0.0006	0.9446	0.01	0.9684

From this fusion results, we can make a determinate decision, areas in 19, 20, 22 have the risk of outburst, and areas in 21, 23 have no risk of outburst, which accord with the factual results.

From the table above, we can conclude that the result after fusion with D-S evidence theory is more reasonable than the result from each single present prediction method. Also it is much easier to make a determinate decision from the fusion result. The results proved that the evidence theory fusion could “compensate” the deficiencies in each single prediction, thus adding to other evidence sources. For example, according to the data from area in 21, if we only use cutting desorption index method, the prediction result is uncertain, the probability of no risk is only 64.29%, and we could not give an assured conclusion, but after “compensating” of other methods, the result is improved, the probability of no risk is up to 94.46%, and we can give the conclusion exactly. Another example, the prediction in area 23, the credibility of no risk after evidence fusion is up to 96.84%, which is higher than the credibility from each other method.

## V. CONCLUSION

This paper improves the combination rules of the D-S Evidence Theory to deal with coherent or incoherent evidence obtained from multiple sources. According to the credibility of coherence evidence, the improved rules allocate the conflicts to various focal elements, so the new rules can process both highly conflicting and coherent evidence effectively, and the rules can provide reasonable results with better convergence efficiency than other rules in the case of highly conflicting evidence sources.

We introduced the similarity degree to denote the similarity between two fuzzy focal elements, and extended the *Bel* and *Pl* functions for processing fuzzy data. Then the improved extension combination rule of the D-S evidence theory to fuzzy sets is considered.

The coal and gas outburst prediction experiments show that the fusion result with the improved combination rule of the D-S Evidence Theory is more reasonable and could give a more certain decision than each independent method.

In the future work, the experience of the experts could be added as a new evidence source, combing external measurement data with human subjective experience, the fusion result must be better and more reasonable.

## REFERENCES

- [1] X. Hu and C. Xie, “Security Operation Center Design Based on D-S Evidence Theory”, *Mechatronics and Automation, Proceedings of the 2006 IEEE International Conference*, Luoyang, China, pp.2302-2306, June, 2006.
- [2] H. Su and G. Zheng, “A Non-Intrusive Drowsiness Related Accident Prediction Model Based on D-S Evidence Theory”, *Proceedings of the 1st International Conference on Bioinformatics and Biomedical Engineering, Wuhan, China*, pp.570-573, July, 2007.
- [3] M. Bauer. “Approximations for decision making in the Dempster-Shafer Theory of evidence”, *Proceedings of the Twelfth annual conference on Uncertainty in Artificial Intelligence (UAI-96)*, San Francisco, California. Morgan Kaufmann, pp. 73-80, 1996.
- [4] V. Kaftandjian, Y. Zhu, O. Dupuis, and D. Babot, “The combined use of the evidence theory and fuzzy logic for improving multimodal nondestructive testing systems”, *IEEE Transactions on Instrumentation and Measurement*, vol.54, no.5, pp.1968-1977, 2005.
- [5] F. Rottensteiner, J. Trinder, and S. Clode, “Using the Dempster-Shafer method for the fusion of LIDAR data and multi-spectral images for building detection. Information fusion”, *Proceedings of the 7th International Conference on Information Fusion*, pp. 1396-1403, 2005.
- [6] S. Regis, J. Desachy, and A. Doncescu, “Evaluation of biochemical sources pertinence in classification of cell’s physiological states by evidence theory”, *Proceedings of 2004 IEEE International Conference on Fuzzy Systems*, pp. 867-872, 2004.
- [7] D. Tan, X. Yan, S. Gao, and Z. Liu, “Fault diagnosis for spark ignition engine based on multi-sensor data fusion”, *Proceedings of IEEE International Conference on Vehicular Electronics and Safety*, pp.311-314, 2005.
- [8] Y. Xu, X. Wu, J. Shen, and X. San, “Feature fusion algorithm based on DT-CWT texture”. *7th International Conference on Signal Processing, (ICSP '04)*, vol.2, pp.1492-1495, 2004.
- [9] W. Lu, Q. Wu, and Q. Shao, “The D-S Evidence Theory and Application to Feasibility Evaluating of New Product Developing”, *Journal of Operational Research and Management*, vol.13, no.5, pp.111-115, 2004.
- [10] L. Xu, Z. Lin, and S. Yang, “Vague Sets Based Evidence Combinational Rule With Generalized Belief Function”, *Proceedings of IEEE International Conference on Information Acquisition*, pp.764-769, 2006.
- [11] R. Zhang. “Application of advanced information technology on coal and gas outburst prediction” *Ph.D thesis, Chongqing University*, 2004.
- [12] C. Li. “Research on gas classification and prediction for coal and gas outburst quicksand”, *Ph.D thesis, China University of Mining and Technology*, 2005.
- [13] A. Jousselme, D. Grenier, and E. Bosse, “A new distance between two bodies of evidence”, *Journal of Information Fusion*, vol. 3, pp. 237-239, 2002.
- [14] Y. Miao, J. Zhang, H. Zhang, and X. Ma. Improvement of the Combination Rules of the D-S Evidence Theory Based on Dealing with the Evidence Conflict. *Proceedings of the 2008 IEEE International Conference on Information and Automation*, pp.331-336, 2008.
- [15] L. Hagarat, S. Mascle, D. Richard, and C. Ottle “Multi-scale data fusion using Dempster-Shafer evidence theory”, *Proceedings of IEEE International Conference on Geoscience and Remote Sensing Symposium*, vol.2, pp. 911-913, 2002.
- [16] S. Chen, J. Li, and X. Wang, “Fuzzy set theory and its application”, Beijing, Science publish company, 2005.
- [17] R. Haenni, “Are alternatives to Dempster’s rule of combination real alternatives? Comments on “About the belief function combination and the conflict management problem””. *Information Fusion*, Vol.3(4), pp.237-239, 2002.
- [18] Z. Lin, “Study on Information Fusion Based on Evidence Theory and its Application in Water Quality Monitoring”. *Ph.D. thesis, Nanjing Hohai University*, 2005.
- [19] Y. Miao, X. Ma, H. Zhang, and J. Zhang “An Improved Extension of the D-S Evidence Theory to Fuzzy sets”. *Proceedings of the 3th International Multi-Conference on Computing in the Global Information Technology, Athens, Greece*, pp.148-153, 2008.