

# Does Chaos Exist in Social Network Components? Role of Evolutionary Dynamics and Tool

Hameed Al-Qaheri  
Department of Quantitative  
Methods and Information Systems,  
Kuwait University, Safat, Kuwait  
alqaheri@cba.edu.kw

Soumya Banerjee  
Department of Computer Science,  
Birla Institute of Technology,  
Mesra, India  
dr.soumya@ieee.org

Goldina Ghosh  
Department of Computer Science,  
Birla Institute of Technology,  
Mesra, India  
goldinag@gmail.com

**Abstract**-Chaos and chaotic optimization is a global trend of optimization, customized for several engineering applications. This paper explores the behavior of chaos under social networking sites. Recalling chaotic dynamic characteristics emerging evolutionary network, the *News Feed* behavior of *Facebook* has been modeled for optimization. The research challenge is to address the emphasized presence of chaotic behavior under the social network paradigm. Subsequently, modeling could be collaborated with a class of *evolutionary network*, e.g., *vaccination network*. The outcome of investigating such non linear attribute could be of emerging relevance to demonstrate how far this will affect social network participants to be influenced over a specific social inference. In order to address the results obtained through evolutionary dynamics with clarity, the same problem has been tested through *Genetic Algorithm* components for different visualization and open access to this new research frontier using contemporary evolutionary algorithms.

**Keywords** - *Social Networking; Evolutionary Dynamics; Genetic Algorithms; Chaos.*

## I. INTRODUCTION

*Chaos* [1] is a general nonlinear phenomenon that sustains in the linear system. Chaos exhibits certain main characteristics such as *quasi-randomness*, *ergodicity* and *sensitive dependence* on initial conditions. Furthermore, ergodicity could be considered as an effective mechanism to avoid trapping into local minima in the searching process. As such, chaos could be considered as a novel and potential optimization tool of interest [12]. Since the inception of Web 2.0, different forms of social network have been envisaged and raised the relevance of chaos and chaotic optimization. A social network is basically represented as a graph, with individual persons represented as vertices, the relationships between pairs of individuals as edges, and the strengths of the relationships represented as the weights on edges (for the purpose of finding the shortest weighted distance, we treat lower-weight edges as stronger relationships). It has been suggested that there is amalgamation of social chaos model and evolutionary network noticed in its vaccination strategic behavior [10]. It is general practice that in any system of the evolutionary design, the intelligence emerges from a chaotic balance between individuality and sociality. The chaotic balances are the characteristic features of the complex system. For a given energy or cost function, by following

chaotic *ergodic* orbits, a chaotic dynamic system may converge towards a targeted global optimum.

Considering the broad hybrid evolutionary and chaotic behavior of social media services, (which are the direct or indirect function of user's participation via feeding of content and tagging) this paper explores the possibility of the existence of chaos in the functionality of *Facebook* social components, such as posts and news feed, which has the possibility of demonstrating non linear scales and being sensitive to initial condition. Subsequently, the paper also discusses the relevance of optimized function to distinguish the chaos on these components. This will help to achieve an optimized social container both from the users and social network service provider's space and time complexity perspective.

The remaining part of the paper is organized as follows. Section II introduces elementary background material, the problem statement and the relevance of evolutionary approach of visualizing chaos within social network components. In Section III, we present a news feed modeling technique coined from *Facebook*. Section IV presents the proposed algorithm of chaos modeling. In Section V, the implementation and results are presented, followed by the conclusion in Section VI.

## II. BACKGROUND, PROBLEM STATEMENT AND RELEVANCE OF EVOLUTIONARY DYNAMICS

*Facebook* introduced the *News Feed* feature on September 5, 2006. This feature gradually becomes clearer to the user, as he/she posts items to personal profile or wall. These items may be visible to friends or acquaintances connected to the *Facebook* connection. Both culturally, and technically, privacy of such news feed propagation has been questioned and the corresponding impact of the user after those news feed has also been scrutinized with a significant survey [3]. Such investigations also reveal the factors that are related to news, used on *Facebook*, and also demonstrate the other contemporary factors such as gender participation, emotional stability and so on [4]. News formation and distribution for various social networking sites could be analyzed by Exponential Random Graph Models (ERGMs), which show the importance of link and their non linear value of centrality [5]. In particular, it has also been observed that *Facebook* has several self-organized features like status

updates, wall posts, pokes, tagging and commenting, subscription services, games and applications (developed by external agencies). Among these user-oriented features, the majority of them are news feed or post related activities. Thus, *Facebook* becomes a well defined dynamic brand, referred to and maintained by its users [6]. The content feed subscription in *Facebook* follows dynamic distribution of content to all the subscribers, who are having common membership of the same service provider or of same interest. The most current content of two users can be shared when they meet to extend and optimize network coverage with lower cost of service. This could be another reason to be concerned with in social networking sites, especially when accessed through mobile devices [7]. Envisaging different news feed and content generation options, *Facebook* keeps the option open by forming similar opinion of clusters, which can be derived from the user's activities. Recent research emphasizes the model of opinion formation from social network [8], but the research challenge is to keep the contents and feed free from final chaotic behavior. Present *Facebook* structure has the option to subscribe or non-subscribe to certain content page or posts from a particular subscriber, but their trend of linearity and broadcast will still remain ambiguous.

Hence, the concept of optimization has been proposed in social networking sites, such as *Facebook*, from different perspectives. Developers are always thriving to ensure that their sites and apps are publishing stories that make the lead profitable and popular, which has pioneered the concept of "News Feed Optimization" (NFO) in *Facebook*.

#### A. Background on Evolutionary Dynamics

Evolutionary dynamics is an emerging concept, evolved from the realm of game theory. Evolutionary game dynamics is conceived from a set of deterministic differential equations capable of addressing infinitely large and well-mixed populations [2]. In a well-mixed population, the chance of two individuals interacts is equally alike. Some recent approaches consider stochastic evolutionary dynamics in populations of finite size. Evolutionary game dynamics are also influenced by population structure [14]. Analogically, a well-mixed population typically opposes evolution of cooperation, but a structured population can promote it. The whole idea is to determine the strategy of two present strategies deployed by several sets of common or different players, then based on their interaction, the session calculates certain payoff. The payoff actually gives the effectiveness of any such propagated and linked approaches among those players. The trend of the evolutionary dynamics has been characterized through diversified mathematical treatments, and the utility of such treatments are of typical importance on social networking structure. The individuals finally try to converge and optimize certain deterministic measures at the end of a social session. We explore *Fermi's equation* [10] under this paradigm. Interested readers are advised to go

through the references mentioned to get a broader understanding on this emerging concept. Figure 1 shows a diagrammatic view of a *Facebook* screen shot, where the users of the particular community share or refer to a particular new item. The view will enable the implantation followed in the model.



Figure 1. Passive Mode of Information Sharing on *Facebook* Screen shot

#### B. General Mathematical Model

There is a generic mathematical model of the spread of messages and it follows a complex socio-psychological process. An adequate modeling of this process requires both a correct description of the underlying social networks, along which messages propagate, and a quantitative formulation of various behavioral mechanisms that motivate individuals to participate in the spread of any messages. The formal model comprises of a set of plausible rules. Generally, irrespective of any social network, we consider a population consisting of  $N$  individuals with respect to the propagation of messages, that is subdivided into different classes of participants. Inspired by Maki and Thompson [15], we can also assume that any messages propagate by direct contact of the participants with others in the population. However, these contacts can only take place along the links of an undirected social interaction network  $G = (V, E)$  where  $V$  and  $E$  denote the vertices and the edges of the network, respectively. The negotiation of contacts between the message originator and the rest of the population are governed by the different set of rules to define the originator, follower, and neutral. Assuming that a node  $j$  has  $k$  links,  $g$  can be considered as a stochastic variable which has the following binomial distribution-

$$\prod(g, t) = \binom{k}{g} \theta(k, t)^g (1 - \theta(k, t))^{k-g} \quad (1)$$

where,  $\theta(k, t)$  is the probability at time  $t$  that an edge transmitting from a neutral node with  $k$  links points to the other effected nodes.

### III. NEWS FEED MODELING PERSPECTIVE FROM FACEBOOK

During Facebook's developer conference, f8, on April 22, 2010, *EdgeRank*, a formula which determines the likelihood of an object appears in a *News Feed*, was introduced [11]. The formula is discussed below. However, two important *Facebook* concepts need to be defined prior to introducing the formula, and these are as follows-

- (1) An Object is any item, such as a post, a status update or a change to a profile picture that appears in *News Feed*.
- (2) An Edge is an interaction with an object, such as a comment or a tag, by another user.

Having defined these concepts, the *New Feed Optimization (NFO)* or the *Edge Rank* formula can be expressed mathematically as follows [11]:

$$\text{New Feed Optimization (EdgeRank)} = \sum_{\text{edges } e} \mathbf{u}_e \mathbf{w}_e \mathbf{d}_e \quad (1A)$$

where,  $\mathbf{u}_e$  is the affinity score between the viewing user and edge creator (a user gets higher affinity score if he/she sends more messages to friends or check their profiles often),  $\mathbf{w}_e$  is the weight for this edge type (create, comment, like, tag, etc.) and  $\mathbf{d}_e$  is time decay factor indicating how long ago the edge was created (the older an *Edge* is, the less important it becomes). The score returned by the formula indicates *Edge rank* of the object, the higher the value of the *Edge rank score* the more likely the desired object is to appear in the user's feed. It is also worth pointing out that the act of creating an object is considered an *Edge*, which is what allows objects to show up in user's friends' feeds before anyone has interacted with them.

#### A. How Chaos exists in Facebook: Functional Validation

Inspired by the evolutionary dynamics of social network of new propagation, where we consider the analogy of process of *Page* diffusion via *Facebook's* *News Feed*, the diffusion followed leads to chaotic behavior while feeds towards optimization. *EdgeRank* is similar in semantics to a fan page structure. As such, analytically, diffusion of pages occurs when a user becomes fan of the page and finally their friends of friends become fans of the same page as well. Therefore, the question is raised as how to accomplish optimization in terms of feed option. We model diffusion approximation for large populations of size  $N$  under *Facebook*.

Let  $m/N$  be the fraction of targeted individuals, who are interacting in that propagation of news network of *Facebook*. *Facebook participants* follow each others broadcast based on the group presence, which preferentially copies others with higher influential impact friend or friends of friends. In each round, a randomly chosen individual  $i$  selects another random individual  $j$  as cascaded friend, and compares his/her

position of influence to that of the actual pivotal friend. Individual  $i$  adopts the strategy of individual  $j$  with the probability of feeding the particular post given by the Fermi function [10].

$$\phi(s_i \leftarrow s_j) = f(p_j - p_i) = \frac{1}{1 + \exp[-\beta(p_j - p_i)]} \quad (2)$$

where,  $\beta$  represents the intensity of selection of probability of most acclaimed post. The population can change, only if individuals  $i$  and  $j$  have different strategies. Hence, subsequently, the probability that the number of news fed individuals increases from  $m$  to  $m + 1$  (denoted  $T_m^+$ ) and the probability that the number decreases from  $m$  to  $m - 1$ ; this statistics will influence *news feed* and thus the linear behavior of page rank will not sustain any more. We model the affect of chaos, for some emergency and popular pages under these typical phenomena of vaccination network. It has been observed that, for some popular pages, more than 90% of the fans can be part of a single group of people who are all somehow connected to one another. We solicit our prepositions through an example taken from "*Facebook of August 21, 2008, 71,090 of 96,922 fans (73.3%) of the NastiaLiukin (an American Olympic gymnast) Page were in one connected cluster*" [9]. The chain data feed existing in this page and several parameters could also exist analogous to our validation of vaccination network.

Based on the cited instance, we prepare a shadow data set as detailed in Section V.

### IV. PROPOSED ALGORITHM

The paper explores an elementary, yet evolutionary, optimized approach to minimize the chaos associated with the social network site. The ambiguity of posts and other shareable activities is also partially addressed. The model contains certain variables in the algorithm. The algorithm describes the flow of message among different friends and even to the friends of friends. This flow of message leads to chaos, when propagated to friends of friends. The flow of messages is restricted among the friends, only if there is a decrease in chaos, but the rate of message flow remains the same. Different variables and their corresponding semantics as used in the algorithm are detailed below.

TABLE I. Facebook News and wall Variables with semantics post

Category	Semantics
Message	existing 'message received' record of the friends
u message	new messages with different point value sent by a person to his friends
Link	presence or absence of friend of a friend
Nm	message, but no representation of chaos
rec1	summation of all the values of the maximized function

rec 2	summation of all the values of the minimized function
Rank list	The summation of age, Facebook age and activity count denoting the active user's rate. A scale from 1 to 100
News feed	The different types of newsfeeds with a specific value ranging from 1 to N

Table I summarizes the different parameters associated with variables, pertain to the semantic post of the user and it also solicit the same to devise the proposed algorithm.

```

1 Begin
2 Initialization of values to the variables link
  present or not present
3 Link present =1
4 Link Not present=0
5 Values for variables like Age, FaceBook
  age, Activity count and message received
  are generated on the user's access report
6 Generating Rank List
7 Rank list = ∑ Age, FaceBook Age, Activity count
8 Generating different valued Newsfeed
9 Newsfeed =1, 2... N
10 Delivering Newsfeed without any restriction
11   for i=1 to a do
12     for j= 1 to c do
13       rec (i) = message (i) + umessage(j)
14     end
15   end
16 Delivering Newsfeeds with a restriction
17   for m=1 to c do
18     if link(m) = 0 then
19       for i = 1 to a do
20         for j= 1 to c do
21           ch(m)=message(i) +
22             umessage(j)
23         end
24       else
25         ch(m) = message(i)
26       end
27 Separating Chaos and Newsfeed
28   for i = 1 to a do
29     for m = 1 to e do
30       nm(i) = rec(i) – ch(m)
31     end
32   end
33 Generating Maximization and Minimization value of
  the Function
34 f(x)=x2 - 10 cos(2x) + 10
35 value of cos (x) lies between -1<= cos(x) <=1
36   while cos (x)<= -1 do
37     cos(2x) = -1 = cos 1800
38     2x = 1800
39     x = 0.5
40

```

```

41   end while
42   while cos (x)>= 1 do
43     cos(2x) = 1 = cos 0
44     2x = 0
45     x = 0
46   end while
47   if x == 0.5 then
48     f(x)= (-x2 + 10 Cos(2x) – 10)
49   end if
50   if x == 0 then
51     f(x)= x2 - 10 Cos(2x) + 10)
52   end if
53 Maximizing Newsfeed and Minimizing Chaos
54   mf= maxfun(x)
55   mif= minfun(x)
56   for i = 1 to a do
57     rec1(i)= rec(i) + mf
58   end for
59   for m = 1 to e do
60     rec2(i) = rec(i) + mif
61   end for
62 Plotting the graph
63 Transfer of newsfeeds to all
64 Tracking of chaotic behavior due to transfer of
  newsfeed
65 Maximization of message flow
66 Minimization of chaos
67 End Begin

```

V. IMPLEMENTATION AND RESULTS: PREPARATION OF DATA SET FOR THE PROPOSED MODEL

A list of 40 different friends of a specific person from Facebook is used as sample. These individual friends have their own identity, nature of behavior, and activities. Some of the friends in the list are also interrelated with a few other friends in the list. This signifies the community of that person's common friend circle. The messages transmitted by the person to all his friends at some point in time are also cited.

The attributes of Table II are operationally defined as follows:

**Friend List-** The friends are denoted by F1 to F40.

**Age-** It is the age of the individual friends. Based on the age the nature of activity of an individual is also reflected.

**Facebook Age-** It denotes that since how many years they are connected to this Social Network site.

**Activity count-** It includes the information like message sent, photo uploaded or Facebook's Wall post. Any such activity done is kept as a count in the record set.

**Link present/Not present-** This denotes the idea of friend of a friend. Link present=1 and Not present=0

**Received Messages-** This is a count, which is based on the number of messages, accepted by the friends from the

person, denoting the depth of the connectivity. Based on all these attributes we generate a *Rank list* to judge the liveliness of a member. This is the summation of age, *Facebook* age, and activity count denoting the active user’s rate. A scale from 1 to 100 is provided to plot the rank.

TABLE II. List of Friends and their Associated Attributes

FRIEND LIST	PARAMETERS				
	Age	Facebook Age (in years)	Activity count	Link Present=1 Not present=0	Received Messages
F1	20	5	10	1	2
F2	18	0	6	1	4
F3	30	5	12	0	4
F4	19	1	7	0	2
F5	28	5	15	1	3
F6	28	2	10	1	1
F7	33	5	25	1	1
F8	24	5	25	0	3
F9	40	4	15	0	4
F10	28	1	1	0	5
F11	36	4	1	1	1
F12	41	5	10	0	3
F13	15	1	0	0	3
F14	45	5	5	1	1
F15	20	2	10	1	2
F16	18	1	11	1	1
F17	22	3	7	0	4
F18	32	3	10	0	2
F19	30	4	9	1	3
F20	48	5	15	0	1
F21	50	4	15	1	4
F22	28	5	15	0	2
F23	22	1	7	0	4
F24	42	3	15	0	4
F25	50	6	9	0	4
F26	22	5	10	0	3
F27	20	4	6	1	1
F28	50	5	17	1	2
F29	48	5	17	1	5
F30	40	3	15	0	1
F31	29	6	15	1	5
F32	42	5	1	0	3
F33	50	4	6	1	5
F34	31	1	17	0	1
F35	20	4	17	1	3
F36	30	1	4	1	1
F37	18	1	4	1	5
F38	47	4	9	0	1
F39	33	5	7	1	4
F40	50	4	17	1	5

The data set shown in Table II has been considered, for the proposed validation using **MATLAB and C++**. In Figure 2, the plot represents the message transferred to all friends along with friend of friends and it leads to chaotic behavior. The different valued messages that are transferred via the *newsfeed* are plotted along the x-axis. Whenever a new message is received by the friend its “*received message*” record in the database is updated. The friend’s rate of activity is judged on the basis of the rank. This is plotted on the y-axis. The lines in graph denote that any valued message that is sent is received, in each range of ranks in between 10 to 80. The arrow marks denotes the flow of information in every direction, i.e. is, first to the friends and then to the ‘N’ number of links connected to each

friend.

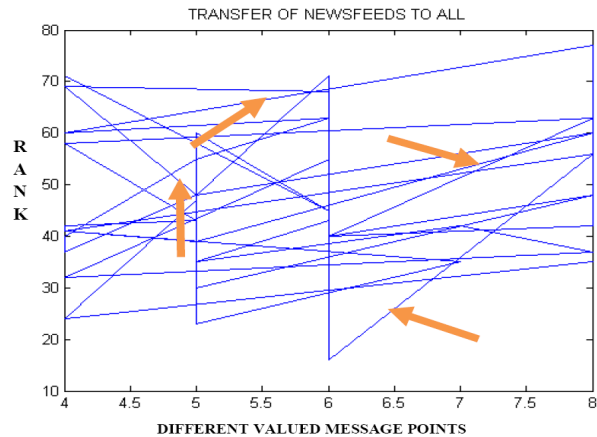


Figure 2. Random Message passing lead to chaos

As it becomes difficult to identify the message sent and the chaos that is generated, a filter is used to distinguish only the chaos, where conditions for  $link = 1$ , i.e. presence of immediate link is checked to identify friends of friends. Based on this condition, the graph is plotted in Figure 3. This figure demonstrates the different message values along x-axis after updating it in the “*received message*” section to only those who are linked. The y-axis is the rank. The multiple parallel straight lines in the graph denote that any valued messages that are received each time by all the linked friends create a meshed structure.

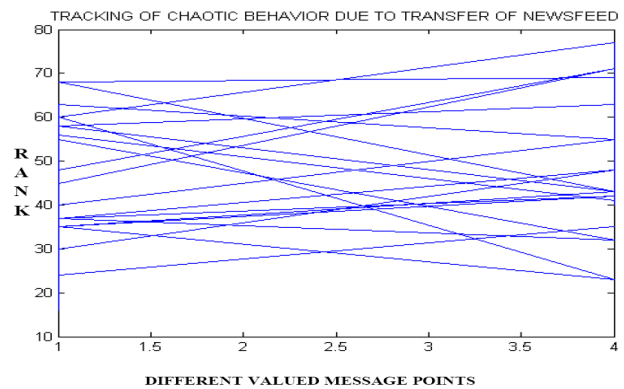


Figure 3. Chaos Present in messages

Hence, the concept of minimizing the chaos arises and maximizing the message transfer takes place. Without affecting the message transfer, we try to reduce the chaos and maximize the message flow, (number of news feed points) which implies no transferring to the friends of friends. By maximizing, we mean transferring all different valued messages to all different ranked users, who do not have any link. This is achieved by maximizing the message chaos function:

$$f_1(x) = \sum_{i=1}^n [x_i^2 - 10\cos(2\pi x_i) + 10] \quad (3)$$

where,  $0 < (x) < 0.5$  and  $0 < f(x) < 19.75$ . The graphical representation is given in Figure 4, where the x-axis gives the representation for all the different messages uploaded for only the unlinked friends with respect to the y-axis that represents the rank. In this graph it is observed that the multiple straight lines are much distinct. The directions are indicated by the arrows.

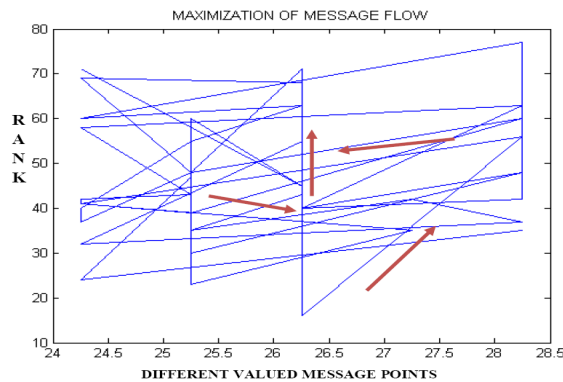


Figure 4. Maximizing the message flow out of chaos

Similarly, after minimizing the function application of the result for the chaotic control is shown in Figure 5. Here the graph denotes the presence of chaos in much lesser number of locations.

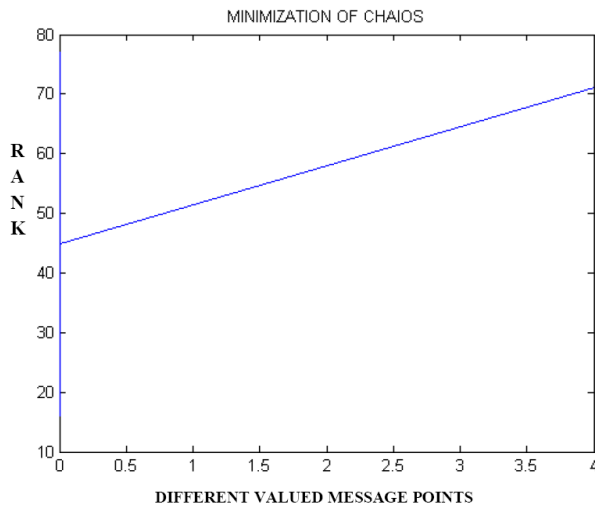


Figure 5. Minimizing the Number of recipients and chaos

A. Observation of news feed using Genetic Algorithm (GA) for similar objective function

To compare the efficacy and implication of the present proposed module for the same optimization problem concerned with Facebook news feed and post, the idea of the natural selection process that drives the biological evolution

is applied through Genetic Algorithm (GA). MATLAB 7.0.4, toolbox has been utilized on the current population set. Genetic Algorithm considers potentially huge search spaces and navigates them looking for optimal solutions. If we are intending to search for a solution to a problem, we look for the best among possible solutions. The space for all possible solutions is called “Search Space”. “Looking for a solution” signifies looking for extremes (either maximum or minimum) in the search space. Fitness function quantifies the optimality of a solution. Fitness function is derived from objective function and then used in the successive genetic operations. After obtaining the suitable values from the fitness function, we go to the next step of Genetic Algorithm which is selection procedure. In this procedure, an individual called ‘parent’ that combines the population at the next generation. Cross Rule helps in forming children in the next generation and hence obtaining only the optimized solutions.

Here, by applying equation (3), the maximization function is used to represent the maximum amount of message transfer only to the friends without link. After maximizing the function, a graph is plotted as shown in Figure 6. X-axis in the graph represents the range of ranks of the friends who receive the message and y-axis represents the maximum number of the messages with different values that are transferred (measured by the fitness value).

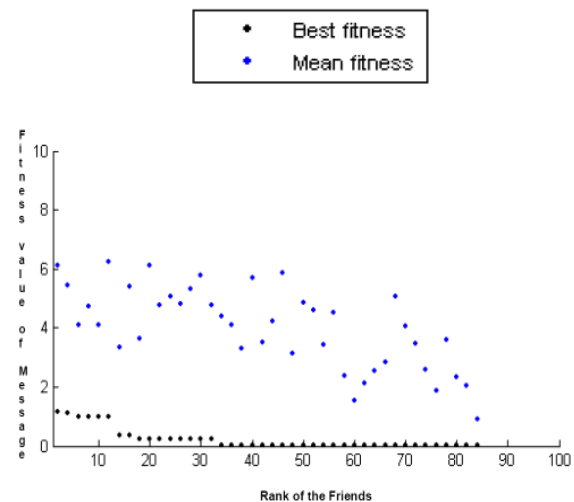


Figure 6. Rank of the Friends with fitness

The blue dots in the scattered graph represent the best fitness value and the black dots represent the mean fitness value. In Figure 7, the average distance between the messages transferred among each individual is shown, where x-axis is the range of ranks of the friends, who receive the message and y-axis is the average points calculated based on the fitness value of the different valued messages that are sent.



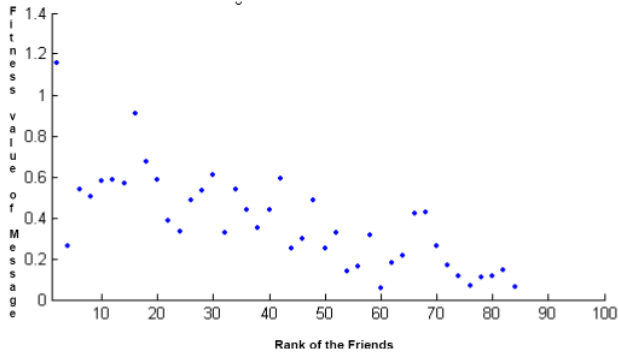


Figure 7. Distance between the messages with rank of friends

In Figure 8, the graph plotted represents the range, *Best*, *Worst* and *Mean*, of the fitness function value of each group of rank in each generation due to the transfer of different valued messages.

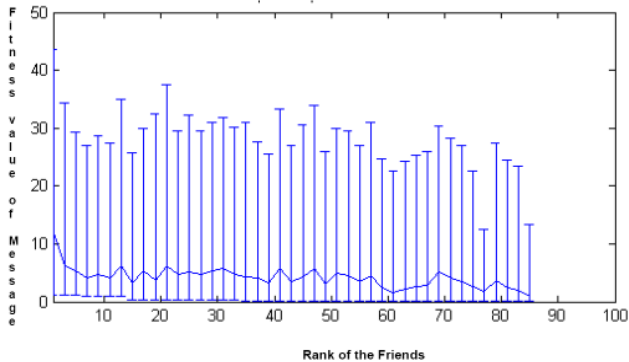


Figure 8. Diversified Fitness Function

Similar to the previous proposed module, the minimization function is used for reducing the chaos, but the message transfer is kept as is with regards to the friends without links. Hence, after minimizing the function, a graph is plotted as shown in Figure 9. X-axis in this graph represents the range of ranks of the friends who receive the message and y-axis as the minimized chaos and maximum number of the messages with different values that are transferred (measured by the fitness value).

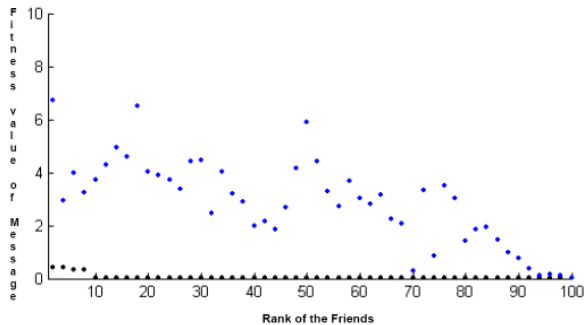


Figure 9. Minimization of chaos

In Figures 10 and 11, the average distance between the messages transferred among each individuals and the minimized fitness value generated are plotted in the graphs

that represent Best, Worst and Mean of the fitness function value of each group of rank in each generation due to the transfer of different valued messages respectively.

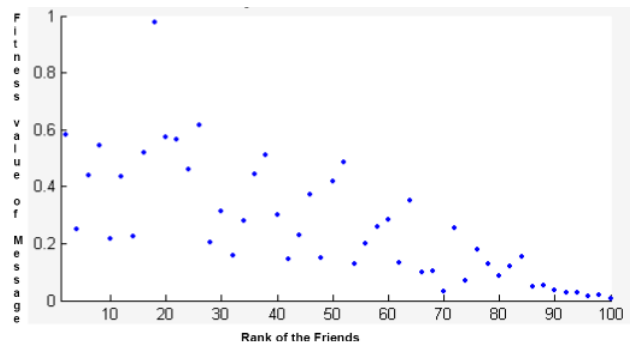


Figure 10. Average Distances of Messages

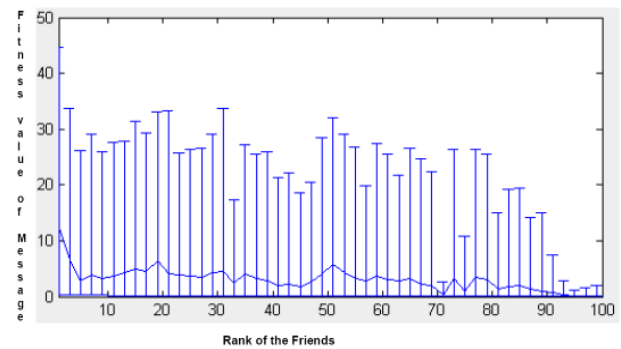


Figure 11. Diversified Fitness values for different Propagated Messages

### B. Implication on the Results

Careful observation of two sets of graphs generated for observing chaos, rank of friends and link of message propagation using different evolutionary methodologies reveal certain interesting observations. Firstly, set of results generated on the basis of Fermi's equation model and its associated evolutionary dynamics concept (equation 2, section 3.1), demonstrates more clarity on the chaos optimization as shown in Figure 4. The counterpart evolutionary tool through GA produces more scattered and distributed optimization index. Moreover, the choice of fitness also requires certain heuristics across all diversified message values and rank of friends. Hence, the first set of evolutionary dynamics paradigm could be considered a better choice for social networking sites to develop software plug-ins in the form of optimization function.

### C. Major Observation on Figure.6-Figure.11

Figures 6 to 11 represent the alteration in tendency of message propagation of friends depending on the value of message and distribution strategy. Figure 10 specifically, gives an estimation of distance of message transferred to the individuals. The work discusses about rank list of the shared group messages among the members, reflected in Figure 8.

From Figure 9 to Figure 11, the status of the plots has been revisited and significant minimization of chaos, while incorporating Genetic Algorithm has been observed. This is only possible, if distributions of objective function of messages have been optimally configured. The optimization could assist the social network message distribution and privacy scheme in a more validated processes and more such optimal function could enhance design space flexibility of social network.

## VI. CONCLUSION

This paper investigates the presence of chaotic behavior under Facebook's news and post events. The paper also suggests an optimized solution from the perspective of *evolutionary network*, e.g., *vaccination network*. Real life *Facebook* instances have been envisaged and different observational points of chaotic behavior were studied with an evolutionary optimization function. Several research directions have been evolved out of the present initiatives: particularly, the extraction of *Facebook* data and its associated crawling, or graph visualization which could be made simpler. We came across an unbiased sample of Facebook users by crawling its social graph using certain popular algorithms e.g., *Metropolis-Hasting random walk (MHRW)* and *a re-weighted random walk (RWRW)* [13]. Similarly, other evolutionary heuristics like *Particle Swarm Optimization (PSO)* or *Differential Evolution (DE)*, could be tested on the same data set of *Facebook* to quantify the chaotic behaviour. The present study can also facilitate *Facebook* application developers to intricate certain smarter and optimized mechanism for wall and message post events.

## REFERENCES

- [1] T. Y. Li and J. A. Yorke, Period three implies chaos, *Amer. Math. Monthly*, 82, 1975, pp. 985-992.
- [2] Martin A. Nowak, Corina E. Tarnita, Tibor Antal, Evolutionary dynamics in structured populations *Phil Trans R Soc B*, 365, 2010, pp.19-30.
- [3] Christopher M. Hoadley, HengXu, Joey J. Lee, Mary Beth Rosson, Privacy as information access and illusory control: The case of the *FaceBook* News Feed privacy outcry in *Electronic Commerce Research and Applications Elsevier*, Vol. 9, 2010, pp. 50-60.
- [4] Carroll J. Glynn, Michael E. Huges, Lindsay H. Hoffman All the news that's fit to post: A profile of news use on social networking sites, *Computers in Human Behavior*, 2011, Elsevier, doi:10.1016/j.chb.2011.08.017.
- [5] Sandra Gonzalez-Bailon Opening the black box of link formation: Social factors underlying the structure of the web, *Elsevier Social Networks Vol. 31* 2009, pp. 271-280.
- [6] Patterson A, Social-networkers of the world, unite and take over: A meta-introspective perspective on the *FaceBook* brand, *J Bus Res*, 2011, Elsevier, doi:10.1016/j.jbusres.2011.02.032.
- [7] Stratis Ioannidis, AugustinChaintreau and Laurent Massoulié, Optimal and Scalable Distribution of Content Updates over a Mobile Social Network, in proceedings of IEEE INFOCOM, 2009, pp.1422-1430.
- [8] Gerardo Iñiguez, Rafael A. Barrio, JánosKertész, Kimmo K. Kaski, Modeling opinion formation driven communities in social networks, *Computer Physics Communications*, Elsevier, Vol.182, 2011, pp.1866-1869.
- [9] Eric Sun, ItamarRosenn, Cameron A. Marlow, Thomas M. Lento, Gesundheit! Modelling Contagion through *FaceBook* News Feed, Association for the Advancement of Artificial Intelligence, 2009, [www.aaai.org](http://www.aaai.org).
- [10] Feng Fu, Daniel I. Rosenbloom, Long Wang and Martin A. Nowak, Imitation dynamics of vaccination behavior on social network, *Proc. R. Soc. B* 2011, 278, pp. 42-49 first published online, 28 July 2010.
- [11]. <http://www.facebook.com/f8> (Facebook Developers Conference September 22, 2011)
- [12] Wei Gong, Shoubin Wang. Chaos Ant Colony Optimization and Application.2009 Fourth International Conference on Internet Computing for Science and Engineering. DOI 10.1109/ICICSE.2009.38, pp. 301-303.
- [13] M. Gjoka, M. Kurant, C. Butts, and A. Markopoulou. Walking in facebook: a case study of unbiased sampling of OSNs. In Proceedings of the 29<sup>th</sup> conference on Information communications, pp.2498. 2506, IEEE Press, 2010.
- [14] Taylor C, Fudenberg D, Sasaki A, Nowak MA, Evolutionary game dynamics in finite populations. *B Math Biol* 66: 2004,pp. 1621-1644.
- [15] D.P. Maki, *Mathematical Models and Applications, with Emphasis on Social, Life, and Management Sciences*, Prentice-Hall, Englewood Cliffs, NJ, 1973.
- [16] S. Boccaletti, V. Latora, Y. Moreno, D.-U. Hwang, M. Chavez, Complex networks: structure and dynamics, *Phys. Rep.* 424, 2006.