A Numerical Approach for Performance Evaluation of Cellular Mobile Networks with Channels Breakdowns

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***Abstract*—In this paper, we propose a numerical approach to study performance and reliability of cellular mobile networks, taking into account the repeated attempts of users whose call was refused due to the lack of available resources and random breakdowns of the base station channels, using the Generalized Stochastic Petri Nets (GSPNs) model as a support. In fact, one of the major drawbacks of this high-level formalism in performance evaluation of large networks is the state space explosion problem. Hence, the novelty of this investigation is the presentation of an approach which allows a direct computing of the infinitesimal generator describing the users behavior and channels allocation, without the generation of the reachability graph nor its reduction. In addition, we develop the formulas of the main stationary reliability and performance indices as a function of the network parameters, the stationary probabilities and independently of the reachability set markings.**

***Keywords*-Cellular Mobile Networks; Repeated calls; Channels breakdowns; Infinitesimal generator; Performance Evaluation.**

I. Introduction

Modeling and performance evaluation are essential for design of cellular mobile networks, where, the number of users and the need for higher data rates and multimedia services increase more and more. Hence, the study of users behavior and in particular, the repeated attempts (called repeated calls or retrials) of users whose call was refused due to the lack of available resources and the consideration of random breakdowns of the base station channels, are crucial to determine the network performance because they can have quite a negative impact on the quality of service offered to users and should therefore not be neglected in network design and planning.

The modeling of repeated attempts has been a subject of numerous investigations dealing with the performance analysis of switching systems, communication networks, cellular mobile networks [1], [2], [3] and wireless sensor networks [4].

In modern cellular networks, micro cells are under consideration. These small cells operate in licensed and unlicensed spectrum that have a range of 10 to 200 meters, compared to macrocells which might have a range of a few kilometers. Hence, the cell size gets smaller, and thus the number of users served in a cell will be relatively smaller, such that traffic models with a finite source of users should be considered.

This paper aims at presenting a numerical approach for performance and reliability evaluation of cellular mobile networks, where the supported area is divided into micro cells, each of them contains a finite number of users and is served by a base station having a limited number of channels which could be subject to random breakdowns. We focus specially on the effect of repeated calls of blocked users and channels breakdowns on the network performances.

Although the reliability study is of great importance, there are only few works that take into consideration repeated calls involving the unreliability of the servers and the finite source of users, as it can be seen in the recent classified bibliography of Artalejo [5]. Moreover, most studies deal with single unreliable server retrial queues or an infinite customers source [6], [7], [8], [9], [10]. However, papers treating finite-source retrial systems with multiple unreliable servers are fewer. The unreliable heterogeneous servers case was considered by Sztrik [11] using a retrial queueing model. On the other hand, retrial mobile networks with several homogeneous servers subject to breakdowns were modeled and analyzed by means of Generalized Stochastic Petri nets (GSPNs) in the recent paper of Gharbi [3].

From a modeling point of view, and compared to retrial queues, the generalized stochastic Petri nets (GSPNs) high level formalism allows an easier description of the behavior of complex systems. This is particularly true for mobile networks. However, the model analysis requires the generation of the reachability graph and then its reduction to obtain the corresponding Markov chain. These two steps require a large storage space and a long execution time. Moreover, the state space increases exponentially as function of the users source size and the base station channels number. So, for practical cellular networks, the models may have a huge state space. Hence, the novelty of this investigation is the presentation of an approach to deal with this problem. To this end, we develop an algorithm for automatically calculating the Markov chain infinitesimal generator, without the generation of the reachability graph nor its reduction. In that way, the storing of the entire state space is avoided. In

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addition, we develop the formulas of the main stationary performance and reliability indices, as a function of the base station channels number, the users source size, the stationary probabilities and independently of the reachability set markings.

The paper is organized as follows: First, we give the syntax and semantics of GSPNs formalism. Next, the GSPN model describing a network cell with repeated calls and base station channels breakdowns is developed in Section 3. Then, the proposed analysis approach is detailed. In Section 5, the computational formulas for evaluating exact performance indices of these networks are derived. Next, based on some experimental examples, we validate our approach in the reliable case and we illustrate the effect of retrial rate and base station channels number on the mean response time. Finally, we give a conclusion.

II. Syntax and semantics of Generalized Stochastic Petri nets

Generalized stochastic Petri nets (GSPNs) [12], [13] are a powerful mathematical and graphical formalism, well suited for modeling and evaluating the performances of stochastic systems involving concurrency, nondeterminism and synchronization. In the past decade, GSPNs have received much attention from researchers, and have been extensively used for analytical modeling of performance and performability of computer, communication, manufacturing and aerospace systems.

Formally, a GSPN can be defined as an eight-tuple $(P, T, \alpha, I, O, H, W, M_0)$ where P is the set of places, T is the set of transitions such that $T \cap P = \emptyset$, it consists of timed and immediate transitions, $\alpha : T \rightarrow \{0, 1\}$ is the priority function which associates the priority $\alpha(t) = 1$ to immediate transitions and $\alpha(t) = 0$ to timed transitions, I, O, $H : T \rightarrow Bag(P)$ are the input, output and inhibition functions, respectively, where $Bag(P)$ is the multiset on P, $W : T \rightarrow R^+$ is a function that assigns a rate of negative exponential distribution to each timed transition and a weight to each immediate transition, $M_0 : P \rightarrow IN$ is the initial marking, a function that assigns a nonnegative integer value to each place, and describes the initial state of the system.

In the graphical representation, places are represented by circles, timed transitions by boxes (or rectangles) and immediate transitions by thin bars. Arcs, leading from places to transitions (from transitions to places resp.) describe the input (the output resp.) function and the arcs, denoting the inhibition function are circle-headed. Arcs are labeled with an integer $d \geq 1$ called the multiplicity of the arc, a value of 1 is usually omitted for readability.

The system state is described by means of markings. The marking of a place is the number of tokens which the place contains. A marking of a Petri net is a mapping $M : P \rightarrow IN$, which specifies the number of tokens in each place of the net. The dynamic behavior of a GSPN results from the firing of transitions yielding other markings than M_0.

A transition t is enabled in a marking M iff each of its ordinary input places contains at least as many tokens as the multiplicity of the input arc, and each of its inhibitor input places contains fewer tokens than the multiplicity of the corresponding inhibitor arc. One more condition for timed transitions is that no immediate transition is enabled simultaneously in M because immediate transitions have priority over timed transitions. Moreover, an enabled timed transition t fires after a delay which is exponentially distributed with rate $W(t)$ while an enabled immediate transition t fires in zero time. In case of conflicts between immediate transitions in a marking M, a given transition t fires with probability $W(t)/\sum_{t':M[t'>} W(t')$. On the other hand, a timed transition has a single-server, n-servers or ∞-servers semantics. For the single-server semantics, the firing rate of a transition t equals its rate $W(t)$, however, for the infinite-servers semantics, the firing rate of transition t in marking M is marking dependent and so equals $W(t) \cdot ED(t, M)$, where $ED(t, M)$ is the enabling degree of t in the marking M. The condition of marking dependent firing is represented by the symbol $\#$ placed next to transition.

The firing of any enabled timed or immediate transition t from a marking M, produces a new marking $M' = M - I(t) + O(t)$. All markings created due to the firing of transitions are called reachable and the *reachability graph* is obtained by representing each marking by a vertex and placing a directed edge from vertex M_i to vertex M_j, if marking M_j can be obtained by the firing of some transition enabled in marking M_i. In the reachability graph, markings enabling no immediate transitions are called *tangible markings*. In this case, one of the enabled timed transitions can fire next (application of race policy commonly). Markings in which at least one immediate transition is enabled, are called *vanishing markings* and are passed through in zero time. Since the process spends zero time in the vanishing markings, they don't contribute to the dynamic behavior of the system, so, they are eliminated from the reachability graph by merging them with their successor tangible markings. This reduction process which corresponds to the elimination of vanishing markings results in a *tangible reachability graph*, which is isomorphic to a continuous time Markov chain (CTMC). The states of the CTMC are the markings in the tangible reachability graph, and the state transition rates are the exponential firing rates of timed transitions in the GSPN.

The solution of this CTMC at steady-state is the stationary probability vector π which can be expressed as the solution of the linear system of equilibrium equations $\pi.Q = 0$ with the normalization condition $\sum_i \pi_i = 1$, where π_i denotes the steady-state probability that the process is in state M_i and Q is the infinitesimal generator. Having the probabilities vector π, we can compute several stationary performance indices of the system.

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III. GSPN MODEL OF CELLULAR MOBILE NETWORKS WITH REPEATED CALLS AND CHANNELS BREAKDOWNS

We observe a cellular mobile network where a supported area is divided into small cells, with a finite source of users (mobiles) of size N in each cell and a base station that consists of c ($c \geq 1$) identical and parallel channels subject to breakdowns and repairs. Each user is either free, under service or in orbit at any time. Each channel can be in operational (up) or non-operational (down) state, and it can be idle or busy (on service). User requests are assigned to operational idle channels randomly and without any priority order. If one of the channels is *up and idle* at the moment of the arrival of a call, then the user starts being served immediately. Service times are independent identically-distributed random variables, whose distribution is exponential. After service completion, the channel becomes idle. Otherwise, if all channels are busy or down at the arrival of a request, the user joins the orbit.

In Fig. 1, we present the GSPN model describing the users behavior and the channels allocation. In this model, the place Cus_Free contains the free users, place $Choice$ represents the arrival of a primary or a repeated call for service and place Ser_Idle represents the operational idle channels. Initially, it contains c tokens because all channels are up and available. Place Cus_Serv contains the users in service. Place $Orbit$ represents the orbit and place Ser_Down contains the failed channels. Hence, the initial marking of the net is given by:

$$\begin{cases} M_0(Cus\_Free) = N \\ M_0(Ser\_Idle) = c \\ M_0(p) = 0 \quad \forall p \in P,\ p \notin \{Cus\_Free, Ser\_Idle\}. \end{cases}$$

The firing of transition $Arrival$ indicates the arrival of a primary request. The service semantics of this transition is ∞-servers (represented by symbol $\#$) because free users can independently generate primary calls. Hence, the firing rate depends on the marking of place Cus_Free and is equal to $\lambda.M(Cus\_Free)$.

At the arrival of a primary or repeated call to place $Choice$, if place Ser_Idle contains at least one token, i.e., if there is at least one idle operational channel, immediate transition $Begin\_Serv$ fires. Hence, the user starts being served and the channel moves into busy state. Otherwise, if place Ser_Idle is empty, immediate transition Go_Orbit fires and the user immediately joins place $Orbit$ and starts generating a flow of repeated calls with rate ν, until it finds an operational idle channel. In fact, users in orbit behave independently of each other and are persistent in the sense that they keep making retrials until they receive their requested service, after which they have no further effect on the network. The firing of transition $Retrial$ represents the arrival of a repeated call. As users independently generate repeated calls, this transition has an ∞-server semantics.

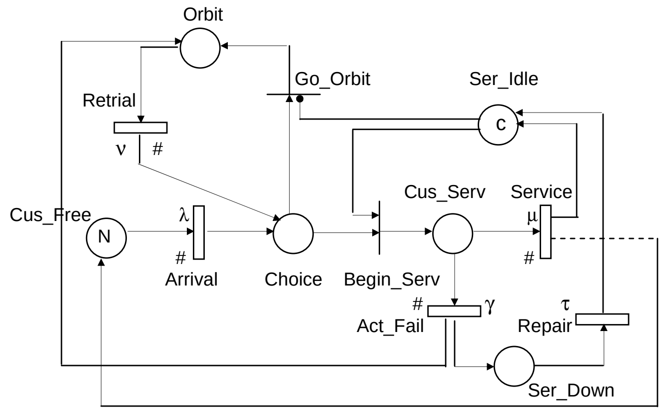

Figure 1. GSPN model of small cell networks with retrials and channels breakdowns

At the end of a service period, timed transition $Service$ fires. The users under service returns to free state (to place Cus_Free) and the channel becomes idle and ready to serve another user. As services take place in parallel, transition $Service$ has an ∞-servers semantics.

If a channel fails during a service period, which is represented by the firing of timed transition Act_Fail, the interrupted user joins the orbit and will restart service later, while the failed channel joins place Ser_Down, where it will be repaired. The firing of transition $Repair$ represents the end of the repair time which is exponentially distributed with rate τ, and the fact that the repaired channel returns to the operational idle state (to the place Ser_Idle). The repairman repairs one channel at a time. Thus, the service semantics of transition $Repair$ is single-server semantics. This means that the firing rate is constant.

IV. STOCHASTIC ANALYSIS

When modeling real cellular mobile networks, generating the GSPN reachability graph and then the reduced underlying Markov chain, may require a huge storage space and a long execution time, since the state space increases exponentially as a function of the users source size and channels number. To overcome this problem, this paper aims to avoid these two steps by designing an algorithm that compute directly the Markov chain infinitesimal generator as a function of system parameters and without generating neither the reachability graph nor the reduced Markov chain. In that way, the complete storing of the reachability set is avoided.

In the following, we describe in detail, the applied steps to derive the corresponding algorithm.

Whatever the values of N and c (with $c < N$), the conservation of users and channels gives the following equations:

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$$\left\{\begin{array}{c} M(Cus\_Free) + M(Cus\_Serv) + M(Orbit) = N \\ M(Ser\_Idle) + M(Cus\_Serv) + M(Ser\_Down) = c \end{array}\right. \tag{1}$$

Observing these two equations, we note that the system state at steady-state can be described by means of three variables (i, j, k), where:

- i represents the number of users in service (in place Cus_Serv);
- j is the number of users in orbit (in place $Orbit$);
- and k is the number of failed channels (in place Ser_Down).

Hence, having (i, j, k), the markings of all places can be obtained. On the other hand, applying (1), we can deduce:

$$\left\{\begin{array}{l} 0 \leq i + j \leq N \\ 0 \leq i + k \leq c \end{array}\right. \tag{2}$$

The behavior of the system can be described by a CTMC, whose infinitesimal generator is an $R \times R$ matrix Q. When there are i users in service, the remaining $N - i$ users must be dispatched between places Cus_Free and $Orbit$, and the remaining $c - i$ channels are idle or down. However, when active breakdowns are considered, state $(0, 0, c)$ where all users are free and all channels are down is not reachable, because channels can fail only in busy state. But the model with (in)dependent breakdowns includes this state. Hence, the number R of accessible tangible markings equals: $R = [\sum_{i=0}^{c} (N - i + 1).(c - i + 1)] - 1$, which can be rewritten as: $R = [\sum_{i=1}^{c+1} (N - c + i) * i] - 1$.

The infinitesimal generator Q is constructed as follows:

$$Q[(i,j,k),(x,y,z)] = \left\{\begin{array}{l} \theta[(i,j,k),(x,y,z)] \\ \text{if } (i,j,k) \neq (x,y,z), \\ -\sum_{(x,y,z)\neq(i,j,k)} \theta[(i,j,k),(x,y,z)] \\ \text{if } (i,j,k) = (x,y,z). \end{array}\right.$$

where $\theta[(i,j,k),(x,y,z)]$ is the transition rate from state (i,j,k) to state (x,y,z). By analyzing the firings of the GSPN transitions, we obtain the following rates:

- $[k > 0] : (i,j,k) \xrightarrow{\tau} (i,j,k-1)$
- $[i > 0] : (i,j,k) \xrightarrow{i\mu} (i-1,j,k)$ and $(i,j,k) \xrightarrow{i\gamma} (i-1,j+1,k+1)$
- $[j > 0$ and $i + k < c] : (i,j,k) \xrightarrow{j\nu} (i+1,j-1,k)$
- $[i + j < N$ and $i + k < c] : (i,j,k) \xrightarrow{(N-i-j).\lambda} (i+1,j,k)$
- $[i + j < N$ and $i + k = c] : (i,j,k) \xrightarrow{(N-i-j).\lambda} (i,j+1,k)$

As a consequence, the infinitesimal generator can be automatically calculated by means of Algorithm 1. In this case, when dealing with line 7, the case where $i + j = 0$ should not be considered, as the state where all users are free and all channels are down does not exist. The same holds for line 29 when $i = j = 0$ and $k = c$.

Algorithm 1 Computation of the infinitesimal generator

▷ Primary arrivals : $i + j < N$
1: **for** $i \leftarrow 0, c-1$ **do**
2:   **for** $j \leftarrow 0, N-i-1$ **do**
3:     **for** $k \leftarrow 0, c-i-1$ **do**
4:       ▷ admission in service
5:       $\theta[(i,j,k),(i+1,j,k)] \leftarrow (N-i-j).\lambda$
6:     **end for**
7:     ▷ admission in orbit
8:     $\theta[(i,j,c-i),(i,j+1,c-i)] \leftarrow (N-i-j)\lambda$
9:   **end for**
10: **end for**

▷ Successful retrials : $j > 0$ and $i + k < c$
11: **for** $i \leftarrow 0, c-1$ **do**
12:   **for** $j \leftarrow 1, N-i$ **do**
13:     $\theta[(i,j,k),(i+1,j-1,k)] \leftarrow j.\nu$
14:   **end for**
15: **end for**

▷ End of service and channel breakdown : $i > 0$
16: **for** $i \leftarrow 1, c$ **do**
17:   **for** $j \leftarrow 0, N-i$ **do**
18:     **for** $k \leftarrow 0, c-i$ **do**
19:       ▷ end of service
20:       $\theta[(i,j,k),(i-1,j,k)] \leftarrow i.\mu$
21:       ▷ channel breakdown
22:       $\theta[(i,j,k),(i-1,j+1,k+1)] \leftarrow i.\gamma$
23:     **end for**
24:   **end for**
25: **end for**

▷ Repairs : $k > 0$
26: **for** $i \leftarrow 0, c-1$ **do**
27:   **for** $j \leftarrow 0, N-i$ **do**
28:     **for** $k \leftarrow 1, c-i$ **do**
29:       $\theta[(i,j,k),(i,j,k-1)] \leftarrow \tau$
30:     **end for**
31:   **end for**
32: **end for**

V. PERFORMANCE AND RELIABILITY INDICES

The aim of this section is to derive the formulas of the most important stationary performance and reliability indices. As, the proposed models are bounded and the initial marking is a home state, the underlying process is ergodic. Hence, the steady-state solution exists and is unique.

The infinitesimal generator Q can be obtained automatically by applying the above algorithms. Then, the steady-state probability vector π can be computed by solving the linear equation system:

$$\begin{cases} \pi.Q = 0 \\ \sum_{(i,j,k)} \pi_{i,j,k} = 1, \\ \text{where } (i,j,k) \text{ satisfy the conditions given in (2).} \end{cases}$$

Having the probability distribution π, we can derive several exact performance and reliability measures. Although state $(0, 0, c)$ is not reachable, we consider it in order to have an homogeneous presentation of formulas. In this case, we assign it a null probability.

- `Mean number of busy channels` (n_s)`:` This corresponds to the mean number of tokens in place Cus_Serv which is also the mean number of customers under service.

$$n_s = \sum_{i=0}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} i.\pi_{i,j,k} \qquad (3)$$

- `Mean number of users in orbit` (n_o)`:` This corresponds to the mean number of tokens in place $Orbit$.

$$n_o = \sum_{i=0}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} j.\pi_{i,j,k} \qquad (4)$$

- `Mean number of users in the system` (n)`:`

$$n = n_s + n_o = \sum_{i=0}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} (i + j).\pi_{i,j,k} \qquad (5)$$

- `Mean number of failed channels` (n_f)`:` This represents the mean number of tokens in place Ser_Down.

$$n_f = \sum_{i=0}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} k.\pi_{i,j,k} \qquad (6)$$

- `Mean number of operational idle channels` (n_i)`:` This represents the average number of tokens in place Ser_Idle.

$$n_i = c-(n_s+n_f) = c-\sum_{i=0}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} (i + k).\pi_{i,j,k} \qquad (7)$$

- `Mean rate of generation of primary calls` ($\overline{\lambda}$)`:` This represents the throughput of transition $Arrival$, which equals the throughput of transition $Service$.

$$\overline{\lambda} = (N - n).\lambda = \sum_{i=0}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} (N - i - j).\lambda.\pi_{i,j,k} \qquad (8)$$

- `Mean rate of service` ($\overline{\mu}$)`:` This represents the throughput of transition $Service$.

$$\overline{\mu} = \mu.n_s = \overline{\lambda}$$

- `Mean rate of generation of repeated calls` ($\overline{\nu}$)`:` This represents the retrial frequency of customers in orbit. It corresponds to the throughput of transition $Retrial$.

$$\overline{\nu} = \sum_{i=0}^{c} \sum_{j=1}^{N-i} \sum_{k=0}^{c-i} j.\nu.\pi_{i,j,k} = \nu.n_o$$

- `Failure frequency of busy channels` ($\overline{\gamma}$)`:` This represents the throughput of transition Act_Fail.

$$\overline{\gamma} = \sum_{i=1}^{c} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i} i.\gamma.\pi_{i,j,k} = \gamma.n_s$$

- `Failure frequency of idle channels` ($\overline{\delta}$)`:` This represents the throughput of transition $Idle\_Fail$.

$$\overline{\delta} = \sum_{i=0}^{c-1} \sum_{j=0}^{N-i} \sum_{k=0}^{c-i-1} (c - i - k).\delta.\pi_{i,j,k} = \delta.n_i$$

- `Blocking probability of a primary call` (B_p)`:`

$$B_p = \frac{\sum_{i=0}^{c} \sum_{j=0}^{N-i-1} (N - i - j).\lambda.\pi_{i,j,c-i}}{\overline{\lambda}}$$

- `Blocking probability of a repeated call` (B_r)`:`

$$B_r = \frac{\sum_{i=0}^{c} \sum_{j=1}^{N-i} j.\nu.\pi_{i,j,c-i}}{\overline{\nu}}$$

- `Blocking probability` (B)`:`

$$B = \frac{\overline{\lambda}}{\overline{\lambda} + \overline{\nu}} \cdot B_p + \frac{\overline{\nu}}{\overline{\lambda} + \overline{\nu}} \cdot B_r$$

- `Mean rate of repair` ($\overline{\tau}$)`:` This represents the throughput of transition $Repair$.

$$\begin{aligned} \overline{\tau} &= \tau.\sum_{i=0}^{c-1} \sum_{j=0}^{N-i} \sum_{k=1}^{c-i} \pi_{i,j,k} \\ &= \begin{cases} \overline{\gamma}, & \text{in active breakdowns,} \\ \overline{\gamma} + \overline{\delta}, & \text{in dependent breakdowns.} \end{cases} \end{aligned}$$

- `Utilization of` s `channels` (U_s)`:` ($0 \le s \le c$) This corresponds to the probability that s channels are busy :

$$U_s = \sum_{j=0}^{N-s} \sum_{k=0}^{c-s} \pi_{s,j,k}$$

- `Availability of` s `channels` (A_s)`:` ($0 \le s \le c$) This corresponds to the probability that s channels are operational and idle.

$$A_s = \sum_{i=0}^{c-s} \sum_{j=0}^{N-i} \pi_{i,j,c-s-i}$$

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Table I
VALIDATION IN THE RELIABLE CASE

	Reliable [14]	Non-reliable
Number of channels	4	4
Number of users	20	20
Primary call generation rate	0.1	0.1
Service rate	1	1
Retrial rate	1.2	1.2
channel's failure rate	-	1e-25
channel's repair rate	-	1e+25
Mean number of busy channels	1.800748	1.800764
Mean number of sources of repeated calls	0.191771	0.191786
Mean rate of generation of primary calls	1.800748	1.800745
Mean waiting time	0.106495	0.1065036

- `Failure probability of` s `channels` (F_s): $(0 \leq s \leq c)$ This corresponds to the probability that s channels are failed:

$$F_s = \sum_{i=0}^{c-s} \sum_{j=0}^{N-i} \pi_{i,j,s}$$

- `Utilization of the repairman` (U_r): This corresponds to the probability that at least one channel is failed:

$$U_r = \overline{\tau}/\tau$$

- `Mean response time` $(\overline{R})$: The mean response time is defined as the mean time from the instant a customer generates a primary request until it is served. In the steady state, it can be obtained using Little's formula:

$$\overline{R} = \frac{n_o + n_s}{\overline{\lambda}}$$

- `Mean waiting time` $(\overline{W})$:

$$\overline{W} = \frac{n_o}{\overline{\lambda}} = \overline{R} - \frac{1}{\mu}$$

VI. EXPERIMENTAL EXAMPLES

In order to test the feasibility of our approach, we developed a tool to implement the above algorithm and the performance indices formulas. Hence, we tested it for a large number of examples. In particular, the results obtained in the reliable case were compared to those generated by the program given in the book of Falin and Templeton [14] for analysis of finite-source retrial queues with reliable servers, since if the failure rate in non-reliable models is very low and repair rate is very high, the performance parameters should approach the corresponding ones in reliable models. From table I, we can see that the results of the proposed approach are very close to those obtained in the reliable case.

Next, we illustrate the effect of retrial rate and base station channels number on the mean response time. The results are presented in Figure 2 and Figure 3, respectively, where the two curves correspond to the reliable case and non-reliable one. From these two figures, we see that the mean response time is a decreasing function of retrial rate and channels number. Moreover, the reliable model gives the best mean response times in the two cases.

Figure 2 also shows that the retrial rate has a significant influence on the mean response time for low retrial rate values. However, when more and more repeated requests arrive, the decrease is not considerable in the case of channels breakdowns.

Figure 3 shows that a small change in the number of base station channels, particularly from 1 to 3 channels, produces a big difference in the mean response time ($\approx -61\%$ for the unreliable channels). However, after a certain value ($c = 4$), the decrease is not considerable.

VII. CONCLUSION AND FUTURE WORK

This paper aims at presenting an approach that allows performance evaluation of cellular mobile networks, taking into account the repeated calls of blocked customers, the finite number of customers served in a cell and the breakdowns of base station channels. The flexibility of GSPNs modeling approach allowed us a simple construction of detailed and compact models for these systems. The models are used as a support to derive the balance equations of the networks, so that the infinitesimal generator can be obtained without building the reachability graph of the model nor reducing it. Exact stationary performance and reliability parameters can then be computed.

In conclusion, the GSPNs method holds promise for the solution of several systems with repeated attempts. Hence, it is worth noting that our approach can be further extended to more complex systems with different breakdowns disciplines.

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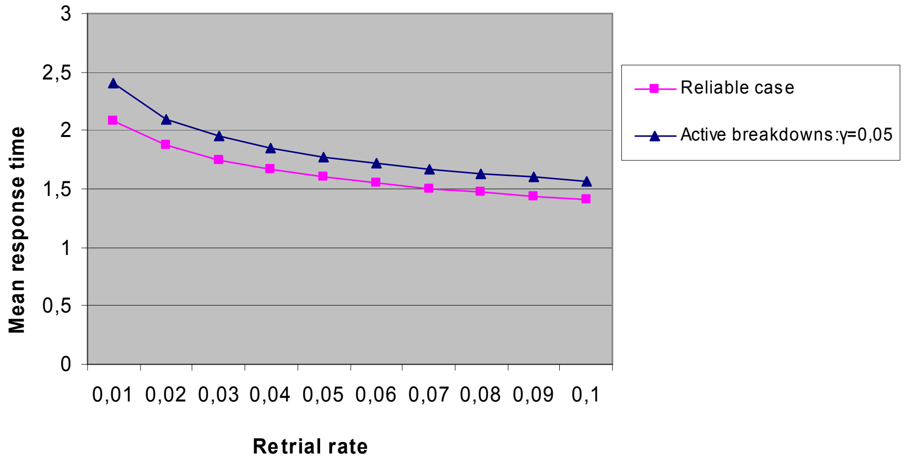

Figure 2. Mean response time versus retrial rate

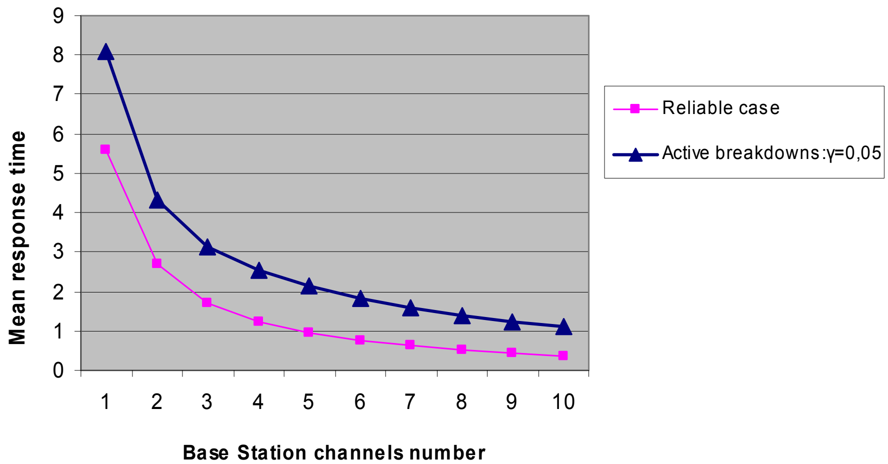

Figure 3. Mean response time versus base station channels number

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