Performance Evaluation of the Opportunistic Spectrum Access in a Cognitive Radio Network with Imperfect Sensing

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***Abstract*—Cognitive radio networks have emerged as a promising technology to shift the actual paradigm of spectrum scarcity by allowing the cognitive users (secondary users) to use the licensed spectrum on an opportunistic basis. A continuous-time Markov chain (CTMC) can be used to model the *opportunistic spectrum access* (OSA) in a cognitive radio network with imperfect sensing (false alarms and misdetections). In this paper, an analytic model is developed and the performance of the system is evaluated and the numerical results are presented in terms of capacity (average number of calls completion per time unit), blocking probability, force termination probability and spectrum utilization.**

Keywords—performance evaluation, cognitive radio networks, continuous-time Markov chain (CTMC) model, imperfect sensing.

I. INTRODUCTION

Cognitive radio technology offers an alternative spectrum access, which could result in an effective use of the spectrum. In cognitive radio networks, there are two types of users: primary users, who own (have license for the use of) a frequency band, and secondary users, who attempt to use the licensed frequency band in an opportunistic way. Primary users have priority over secondary users and secondary users (unlicensed) can access the licensed spectrum band if the owner of that band is absent. In order to access those frequency bands without interfering with primary users transmissions, secondary users must accurately detect the presence of primary users by sensing the spectrum. Different sensing techniques can be used to perform spectrum sensing [10], but sensing techniques cannot provide perfect sensing. So, false alarms and misdetection cannot be fully avoided. False alarms happen when a channel is free and the cognitive user classifies that channel as busy, and misdetections happen when a busy channel is classified as free. In [2], analysis of the access of cognitive radio networks was studied using continuous time Markov chain (CTMC), but the effect of imperfect sensing was not taken into account. A similar state diagram based approach was taken in [6]-[8] for modeling dynamic spectrum access with channel assembling, but, also perfect sensing accuracy was assumed. In [3], the effect of imperfect sensing is studied alongside with the opportunistic spectrum access in a cognitive radio network. Explicit expressions for state dependent transition rates were presented for the specific case of three channels. However, a generalization of the study for more channels was not done.

The prominent feature of the state diagram in [3] is that it has state-dependent transition rates at the nodes. These transitions are found by going through all possible transitions from a state taking into account the channel state and the sensing decision.

In this paper, we generalize the approach used in [3] and provide expressions to compute the state transition rates in a generic way, which can be used for any numbers of channels. Also, additional performance metrics are presented to analyze the performance of the system. The rest of this paper is organized as follows. Section II describes the system model and assumptions. Section III describes the CTMC analysis. Section IV goes into details with respect to the network performance analysis. Numerical results are presented in Section V. Finally conclusions are presented in Section VI.

II. SYSTEM MODEL AND ASSUMPTIONS

Similar to [3], we consider that there are N channels available, and these channels can be accessed by primary and secondary users, with primary users having priority over the secondary users. Primary and secondary calls arrive with rates λ_1 and λ_2 respectively, and are completed with rates μ_1 and μ_2, respectively. In order to find the same theoretical solutions as in [3], we use the exact same assumptions. These assumptions are:

- When the primary user arrives to a channel occupied by a secondary user, the secondary user will always notice the primary user and will leave the channel. After this, the secondary user starts to search for a new channel. During this phase, the secondary user will perform detection on the remaining channels in random order until it finds a free channel or all channels are determined to be busy. A free channel is decided to be occupied with false alarm probability P_F and an occupied channel is determined to be free with misdetection probability $P_M = 1- P_D$, where P_D is the detection probability, which refers to a probability that an occupied channel is correctly detected as busy.

 ISBN: 978-1-61208-450-3

- The sensing time and the time to perform the spectrum handover is considered negligible. So, all states transitions are instantaneous.
- A secondary user always knows the channels occupied by other secondary users and will not sense or use them. Therefore, there is no collision between secondary users.
- A primary user knows the channels occupied by other primary users so that there will be no collision between primary users.
- In case of collision between primary and secondary users, both colliding users withdraw from the channel. This means that both transmissions will be lost. Note that the collision when primary user comes to a channel used by a secondary user is assumed to be short and does not cause the primary user to leave the channel.
- The search order for new free channel is random and equally probably. The search stops after an idle channel is found or all channels are found to be occupied.

III. CTMC ANALYSIS

A two-dimensional Markov chain is used to model the system. Let $x=\{i,j\}$ be the general state representation, where i represents the number of primary users' calls in the system and j the number of secondary users' calls in the system. For example, $x=\{2,1\}$, refers to the state with two primary users and one secondary user. Let N be the number of channels available in the system. Therefore, the set of feasible states of the system is denoted as $S=\{x|\ 0 \le i \le N,\ 0 \le j \le N,\ 0 \le i + j \le N\}$.

A. State Transition Rate

The state-dependent transition rates are derived considering all possible events that can trigger state transition and all possible sensing decisions.

For example, the transition from the state $x=\{1,0\}$ to the state $x=\{0,0\}$, considering 3 channels [3], occurs in four different scenarios:

1) First, it can happen if the primary user regularly completes its call with rate μ_1;

2) It can happen if a secondary user arrives to the channel occupied by the primary user, fails to detect the presence of the primary user (with probability P_M) and transmits on that channel resulting in a collision. In this case, both transmissions are lost taking the chain to the state $x=\{0,0\}$. Since the secondary user searches for a new channel randomly and with equal probability, the transition rate in this scenario is $1/3\lambda_2 PM$;

3) Another possibility is if the secondary user arrives in one of the two idle channels (with 2/3 probability), but, after sensing, it decide that the channel is busy (with probability P_F). In this case, if the secondary user goes to the channel occupied by the primary user (with probability 0.5) and fails to detect the presence of the primary user (with probability P_M) and transmits on that channel, the secondary user will collide with the primary user and take the Markov chain to the state $x=\{0,0\}$, with rate $2/3\lambda_2 PF1/2PD$.

4) The last possibility is if the secondary user arrives in one of the two idle channels (with 2/3 probability), but, after sensing, it decides that the channel is busy (with probability P_F), then, the secondary user searches another channel and goes to the other idle channel (with probability 0.5) and decides that this channel is also busy (with probability P_F), in this case, the secondary continues his search and goes to the channel that is occupied by the primary user, fails to detect the presence of the primary user (with probability P_M) and transmits on that channel resulting in collision and both users withdraw from the channel and the Markov chain will go to state $x=\{0,0\}$ with rate $2/3\lambda_2 P_F 1/_2 P_F P_M$. So, the transition from state $x=\{1,0\}$ to state $x=\{0,0\}$ will be:

$$i\mu_1 + \lambda_2 \frac{1}{3} P_M + \lambda_2 \frac{2}{3} P_F \frac{1}{2} P_M + \lambda_2 \frac{2}{3} P_F \frac{1}{2} P_F P_M \qquad (1)$$

Figure 1 shows the Markov chain with its explicit transition rate for the case of three channels reproduced from [3].

The same approach should be taken to determine the transition rate from any pair of states directly connected. As we can see, it can be an exhaustive work, especially if the number of channels is large. In this paper, we developed generic expressions which can be used to determine the transition rate from one state to another for any number of channels N. These expressions are presented in Table I.

B. Transition Matrix and Stationary Probability

Let Q be the transition matrix of the CTMC. We determine the total transition rate from state a to state b, which is the summation of transition rate from state a to state b considering all possible user activities for all a,b ($a \neq b$) ϵ S namely q_{ab}. The diagonal elements in Q, i.e., q_{aa}, $a \epsilon$ S are found as [8]:

$$q_{aa} = -\sum_{b \in S, b \neq a} q_{ab} \qquad (2)$$

The stationary probabilities, $\pi(x)$ can be computed from the global balance equations and the normalization equation, given as:

$$\pi Q = 0, \quad \sum_{x \in S} \pi(x) = 1 \qquad (3)$$

where π is the steady state probability vector.

Once the steady state probabilities are determined, the performance of the system can be evaluated by different parameters, as presented in the next section.

IV. PERFORMANCE ANALYSIS

The analysis presented in [3] considers only two parameters, the primary user termination probability and the probability

ISBN: 978-1-61208-450-3

TABLE I. TRANSITIONS RATE FROM A GENERIC STATE (i,j)

Dest. state	Conditions	Transition Rate
$(i+1, j)$	$i<N$	$\frac{\alpha}{N-i}\lambda_1+\frac{j}{N-i}\lambda_1\left[\sum_{t=1}^{\alpha}\binom{\alpha}{t}(1-P_F)^t P_F^{\alpha-t}\left(\sum_{r=0}^{i}\binom{i}{r}P_D^{\,r}P_M^{\,i-r}\frac{t}{t+(i-r)}\right)\right]$
$(i-1, j)$	$i>0$	$i\mu_1+\lambda_2\left[\sum_{t=1}^{i}\binom{i}{t}P_M^{\,t}P_D^{\,(i-t)}\left(\sum_{r=0}^{\alpha}\binom{\alpha}{r}P_F^{\,r}(1-P_F)^{\alpha-r}\frac{t}{t+(\alpha-r)}\right)\right]$
$(i, j+1)$	$(i+j)<N$	$\lambda_2\left[\sum_{t=1}^{\alpha}\binom{\alpha}{t}(1-P_F)^t P_F^{[\alpha-t]}\left(\sum_{r=0}^{i}\binom{i}{r}P_D^{\,r}P_M^{\,i-r}\frac{t}{t+(i-r)}\right)\right]$
$(i, j-1)$	$j>0, i=0$	$j\mu_2$
$(i, j-1)$	$j>0, i>0$	$j\mu_2+\frac{j}{N-i}\lambda_1\left[\sum_{t=1}^{i}\binom{i}{t}P_M^{\,t}P_D^{\,i-t}\left(\sum_{r=0}^{\alpha}\binom{\alpha}{r}P_F^{\,r}(1-P_F)^{\alpha-r}\frac{t}{t+\alpha-r}\right)\right]$
$(i+1, j-1)$	$i<N, j>0$	$\frac{j}{N-i}\lambda_1 P_F^{\,\alpha}P_D^{\,i}$

$\alpha=\left[N-(i+j)\right]$

that a secondary call is normally terminated, and only for a particular case (N=3).

In this paper, we introduced four new parameters to analyze the performance of the system:

a) Secondary blocking probability

b) Primary network capacity

c) Secondary network capacity

d) Spectrum utilization

Also, using the generic transition rates presented in Table I, we can compute the performance of the system (for our new four parameters and the two parameters presented in [3]) for any number of channels.

In this section, we presented the expressions to compute each performance parameter. Numerical results and discussions are presented in Section V.

A. Primary Network Performance Analysis

1) Primary user termination probability: As defined in [3], the primary user termination probability is the probability that a primary user call, which has not been blocked at start, is terminated due to collision with secondary users because of misdetection. This probability is computed by [3]:

$$P_{PT}=\frac{\left[\sum_{i=1}^{N}\sum_{j=0}^{N-1}\pi(i,j)\left(T_{(i-1,j)}^{(i,j)}-i\mu_1\right)+\sum_{i=1}^{N-1}\sum_{j=1}^{N-1}\pi(i,j)\left(T_{(i,j-1)}^{(i,j)}-i\mu_2\right)\right]}{\lambda_1\left(1-P_{b1}\right)} \quad (4)$$

where $\pi(i,j)$ is the state probability of the state (i,j) and $T_{(i-1,j)}^{(i,j)}$ is the transition from state (i,j) to state $(i-1,j)$, and P_{b1} is the primary user blocking probability given by:

$$P_{b1}=\pi(\mathrm{N},0) \quad (5)$$

2) *Primary network capacity*: We define the term capacity as the average number of calls completed per unit time. Either (5) or (6) can compute the primary network capacity:

$$\rho_1=\lambda_1(1-P_{b1})(1-P_{PT}) \quad (6)$$

$$\rho_1=\sum_{x\in S} i\mu_1\pi(x) \quad (7)$$

where i is the number of primary user's calls in state x.

B. Secondary Network Performance analysis

1) Secondary User Blocking probability: A secondary user's call is blocked if a secondary user arrival does not take the system to a state (i,j+1). Therefore, the blocking probability can be expressed as:

$$P_{b2}=\sum_{x\in S}\pi(x)\left(1-\frac{T_{(1,j+1)}^{(\mathrm{i,j})}}{\lambda_2}\right) \quad (8)$$

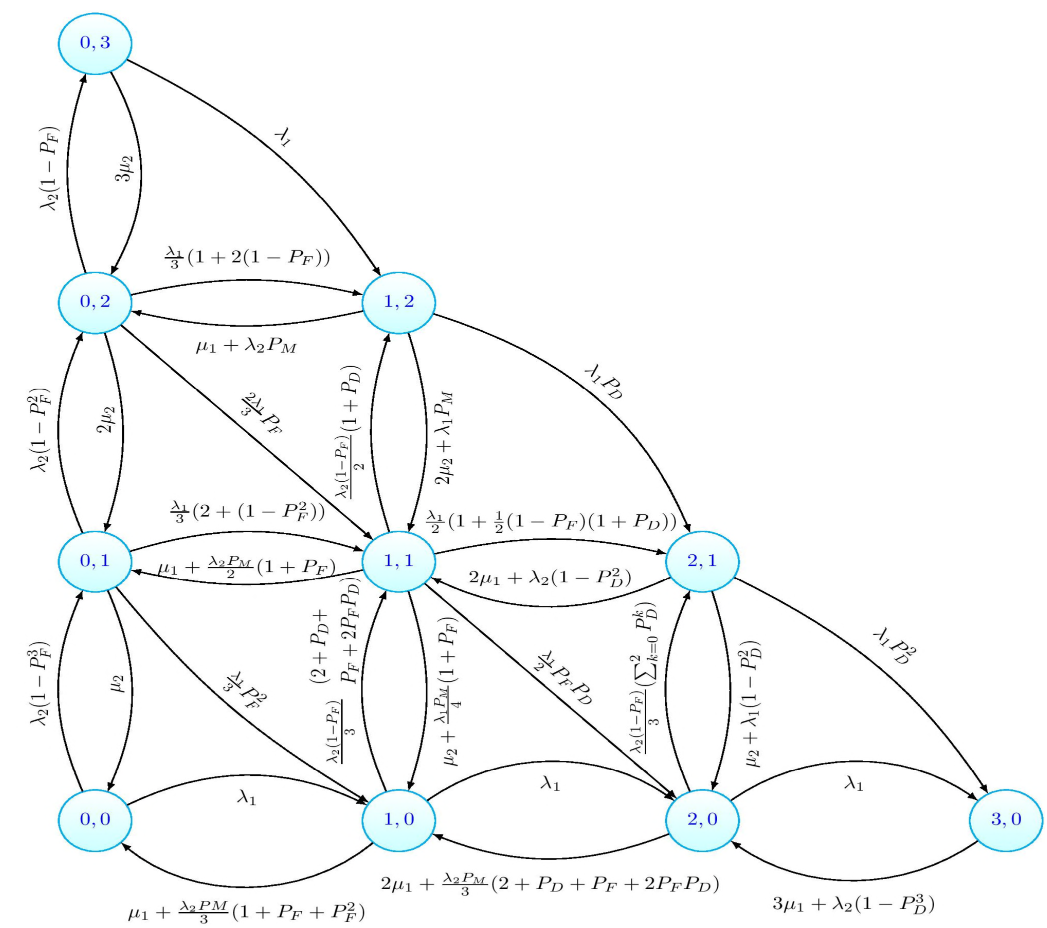

Figure 1 State diagram

2) Secondary user termination probability: We define the secondary user termination probability as the probability that a secondary user call, which has not been blocked at start, is forced to terminate before its transmission is finished. So, the secondary user termination probability can be expressed as:

$$P_{ST} = \frac{R_{int}}{\lambda_2(1-P_{b2})} \tag{9}$$

where R_{int} is the rate that a primary user preempts a secondary user's call in a way that the secondary user is forced to terminate.

Than, we have:

$$R_{int} = \left[\sum_{x\in S} \pi(x)(T^{(i,j)}_{(i,j-1)} - j\mu_2) + \sum_{x\in S} \pi(x) T^{(i,j)}_{(i+1,j-1)} \right] \tag{10}$$

Therefore, the probability that a secondary call is normally terminated can be expressed as:

$$P_{SNT} = (1-P_{b2})(1-P_{ST}) \tag{11}$$

3) Secondary network Capacity: We define this parameter as the average number of secondary user´s call completion per time unit. Either (10) or (11) can compute the secondary network capacity. Therefore, the secondary network capacity will be:

$$\rho_2 = \lambda_2(1-P_{b2})(1-P_{ST}) \tag{12}$$

$$\rho_2 = \sum_{x\in S} j\mu_2 \pi(x) \tag{13}$$

where j is the number of primary user´s calls in state x.

C. Spectrum Utilization

We define the spectrum utilization as the average number of utilized channels over the total number of channels. The spectrum utilization of the cognitive radio network can be expressed as:

$$U = \sum_{x\in S} \pi(x) \frac{(i+j)}{N} \tag{14}$$

 ISBN: 978-1-61208-450-3

V. Numerical Results

In this section, numerical results are presented to illustrate the performance of the opportunistic spectrum access with respect to parameters presented in the previous section, taking into account the effect of the imperfect sensing.

In order to compare the results presented here with the results presented in [3], we opted to use the same values used in [3], for the following parameters: $\lambda_1 = 7$, $\lambda_2 = 3.5$, $\mu_1 = \mu_2 = 4$, $P_F = 0.15$ and $P_D = 0.713$.

In our analysis, we can define the number of channels as any value. To illustrate the results, we opted to use N=5 and N=10. We also used N=3 for the sake of comparison with the results in [3]. N=3 is the only value presented in [3].

1) Primary user termination probability: Figure 2 shows the termination probability of a primary user as the P_D varies. It can be seen that when the $P_D = 1$, the termination probability is zero ($P_D = 1$ means perfect detection and so there is no interruption from the secondary network). Figure 2 also shows the primary user termination probability for N=3 presented in [3] and reproduced here.

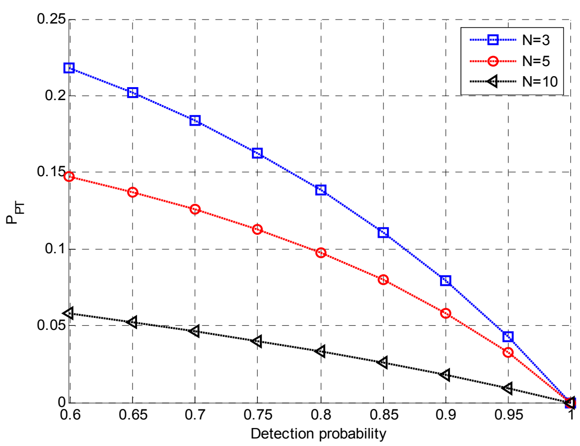

Figure 2. Primary user termination probability

2) Primary Network Capacity: Figure 3 shows that the capacity decreases when there is less accuracy on the detection of the primary presence. If the P_D is low, then there will be more interruptions and fewer calls will have the chance to regularly finish their transmission.

3) Secondary User Blocking probability: Figure 4 shows that, if the arrival rate of primary users' calls increases, then the secondary user blocking probability also increases. This occurs because there will be less transmission opportunity for the secondary network, resulting in more blocked secondary users' calls.

4) Probability that a secondary call is normally terminated: Figure 5 shows that, with the increase of the arrival rate of the primary network, secondary users' services will be interrupted more often. Therefore, the probability that a secondary user´s call successfully ends decrease. Figure 5 also shows the probability that a secondary call is normally terminated for different values of μ_2 and N=3 presented in [3] and reproduced here.

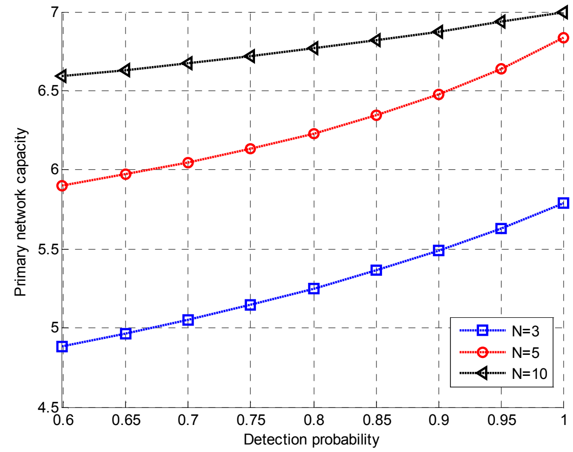

Figure 3. Primary network capacity

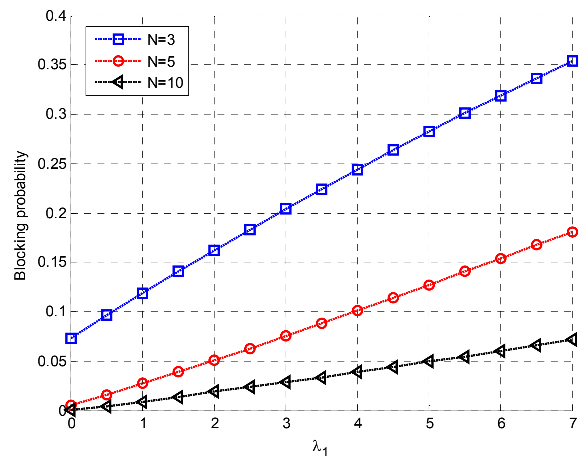

Figure 4. Secondary user blocking probability

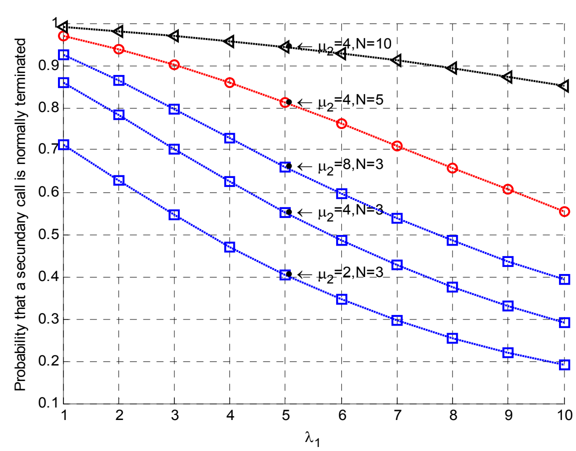

Figure 5. Probability that a secondary call is normally terminated

5) Secondary network Capacity: Figure 6 shows that low capacity is achieved if the arrival of primary users' calls are frequent. Figure 6 also shows that higher capacity is achieved when more channels are available.

6) Spectrum Utilization: Figure 7 shows that better spectrum utilization can be obtained if there are less detections errors.

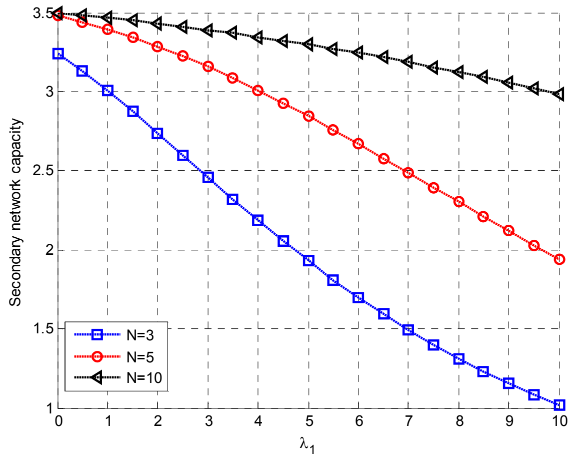

Figure 6. Secondary network capacity

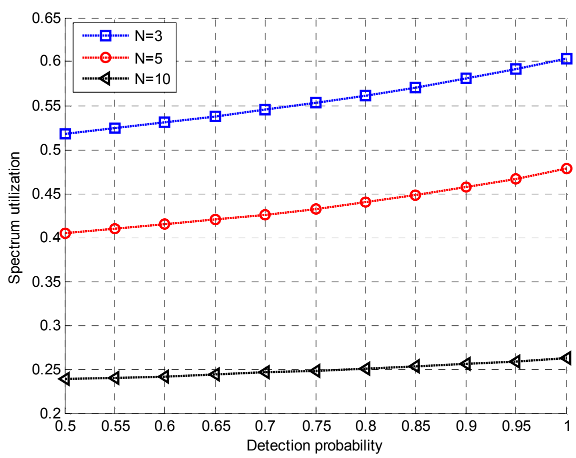

Figure 7. Spectrum utilization

In Figures 2-7, it can be seen that the performance of the opportunistic spectrum access gets better for larger number of channels N. It happens because if more channels are available, then less calls will be forced to terminate due to misdetections resulting in more capacity and better spectrum utilization.

VI. CONCLUSIONS

In this paper, an existing state diagram based approach for modeling the opportunistic access of a cognitive network with imperfect sensing is improved by allowing the state equations to be generalized to any numbers of channels. Additionally, we introduced four new parameters to evaluate the performance of such network. The developed analytical model allows us to see the effect of the imperfect sensing on the number of calls completed per time unit for both primary and secondary networks and the overall spectrum utilization for any number of channels in the system.

ACKNOWLEDGMENT

This work was partially supported by Finep, with resources from Funttel, Grant No. 01.14.0231.00, under the Radiocommunication Reference Center (Centro de Referência em Radiocomunicação) project of the National institute of telecommunications (Instituto Nacional de Telecomunicaçoes – Inatel), Bazil.

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