

Efficient Rate Adaptive Resource Allocation Scheme in Uplink OFDMA Wireless Systems

Vasileios D. Papoutsis and Stavros A. Kotsopoulos

Wireless Telecommunications Laboratory, Department of Electrical and Computer Engineering,
University of Patras, Rio, Greece 265 00,

Email: vpapoutsis@ece.upatras.gr, kotsop@ece.upatras.gr

Abstract—The problem of resource allocation for the uplink of wireless SISO-OFDMA systems is investigated. To relieve heavy computational burden, a suboptimal, but efficient scheme is devised which maximizes the sum of the users' data rates subject to constraints on the per user transmitted power and minimum data rate requirements among users. Simulation results indicate that the proposed scheme can satisfy minimum data rate constraints, distributing sum data rate fairly and flexibly among users. In addition, the proposed scheme is complexity effective, and can be applied to latest-generation wireless systems that provide Quality-of-Service (QoS) guarantees.

Index Terms—OFDMA, resource allocation, multiuser diversity.

I. INTRODUCTION

Orthogonal Frequency Division Multiple Access (OFDMA) [1] has developed into a popular scheme for wideband wireless digital communication. In OFDMA, multiple access is achieved by first dividing the spectrum of interest into a number of subcarriers and then assigning subsets of the subcarriers to individual users. OFDMA helps exploit multiuser diversity in frequency-selective channels, since it is very likely that some subcarriers that are “bad” for a user are “good” for at least one of the other users. Because of its superior performance in frequency-selective fading wireless channels, OFDMA is the modulation and multiple access scheme used in latest wireless systems such as IEEE 802.16e (Mobile WiMAX).

There are fixed and adaptive allocations to allocate subcarriers. Fixed allocations use Time Division Multiple Access (TDMA) or Frequency Division Multiple Access (FDMA) as multiple access schemes to allocate each user a predetermined time slot or frequency band for transmission. While applying fixed allocation the system neglects the channel diversity and does not use the deep faded subcarriers for other users which do not seem as deep faded to them. In [2], these two fixed allocation schemes are discussed and compared in much detail. On the other hand, adaptive allocations allocate resources to users based on their channel gains. Due to the time-varying nature of the wireless channel, dynamic resource allocation makes full use of the multiuser diversity to achieve higher performance.

Most work on resource allocation has been done for the downlink OFDMA systems. In [3] [4], total transmit power is minimized. In [5] [6], it is proved that the downlink system capacity is maximized when each subcarrier is exclusively

assigned to the user with the best subcarrier gain, eliminating the intra-cell interference (ICI), and power is then distributed by the water-filling algorithm [7]. In [8], the minimum data rate among users is maximized and in [9]-[12], a proportional fairness criterion is employed. In [13] [14], the fulfillment of every user's data rate constraints is guaranteed and in [15], per time-slot resource allocation is introduced with “self-noise” and phase noise. In [16], long term access proportional fairness is introduced. Finally, in [17], the sum of the users' data rates is maximized but the resource allocation unit is not the subcarrier, as in previous algorithms [3]-[16], but a time/frequency unit (slot), in accordance with WiMAX systems.

Recently, uplink resource allocation has received some attention in literature. A practical low-complexity algorithm for a two-user case is proposed in [18]. In [19], the optimality of OFDMA in uplink transmissions has been studied while in [20], a non-iterative and near-optimal joint subcarrier and power allocation scheme is proposed. In [21], the results of [20] are generalized by considering the utility maximization in one time-slot, where the utility is a function of the instantaneous data rate in this time-slot. Another work that focused on per time-slot fairness is [22]. In [23], the uplink resource allocation problem is approached using a gradient scheduler but considers long-term total utility maximization which depends on the average data rate or queue sizes. Finally, in [24], resource allocation algorithms are proposed to find a Nash Bargaining solution according to Game theory.

In this paper, the resource allocation problem in uplink OFDMA systems is investigated. We focus on single antenna systems where at most one user can be assigned per subcarrier. The objective is to maximize the sum of the users' data rates subject to constraints on per user power and minimum data rates among users. The proposed scheme, which is also complexity effective, consists of three algorithms; an algorithm that determines the number of subcarriers for each user, a subcarrier allocation algorithm by dividing the users in two groups and the water-filling algorithm [7]. The first two algorithms assign the available subcarriers to the users of the system and the third one allocates the available power of each user.

The remainder of the paper is organized as follows. The problem of sum data rate maximization using minimum data rate constraints and power constraint for each user is formu-

lated in Section II. The proposed scheme is introduced in Section III and Section IV contains the complexity analysis of the proposed scheme and a complexity comparison with other schemes. Simulation results and a comparison between the proposed scheme and other existing schemes are provided in Section V. Finally, Section VI contains concluding remarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The following assumptions are used in this paper: (i) the time-varying channels between different users and the Base Station (BS) are assumed to be frequency selective wireless channels with independent Rayleigh fading and the channel can be regarded as constant during the resource allocation period; (ii) the ISI is completely removed by exploiting OFDM techniques, *i.e.* the width of each subcarrier is much smaller than the coherence bandwidth of the channel. Thus, each user experiences flat fading in each subcarrier; (iii) the Channel State Information (CSI) is perfectly known by the receiver, and the BS feedbacks a certain form of channel information correctly to each user; (iv) each subcarrier can be used by only one user at each time.

The BS decides uplink resource transmission parameters for all available users based on the feedback CSI. The resource allocation parameters are then sent to each user through a dedicated control channel. Then, each user loads its data into the allocated subcarriers and the BS decodes the data sent from all users. The resource allocation scheme is updated as soon as the channel information is collected and also the resource allocation information is sent to BS for detecting.

Consider an OFDMA uplink transmission in a single cell with K active users and N subcarriers. P_k is the transmit power of each user $k = 1, 2, \dots, K$ and the channel gain for user k in subcarrier n is denoted by $g_{k,n}$. Each subcarrier n of user k is assigned a power $p_{k,n}$. With the noise power spectral density being N_0 and the total bandwidth of all subcarriers being B , the additive white noise power is $\sigma^2 = \frac{N_0 B}{N}$. Therefore, the subcarrier SNR can be expressed as $h_{k,n} = \frac{g_{k,n}}{\sigma^2}$ and the transmitted SNR of user k in subcarrier n is $\gamma_{k,n} = p_{k,n} h_{k,n}$.

Each of the user's bits are modulated into N M-level QAM symbols, which are subsequently combined using the IFFT into an OFDMA symbol. For a square M-level QAM using Gray bit mapping as a function of transmitted SNR $\gamma_{k,n}$ and number of bits of user k in subcarrier n $r_{k,n}$, the BER can be approximated to within 1 dB for $r_{k,n} \geq 4$ and $\text{BER} \leq 10^{-3}$ as [25]

$$\text{BER}_{\text{MQAM}}(\gamma_{k,n}) \approx \frac{1}{5} \exp \left[\frac{-1.6\gamma_{k,n}}{2^{r_{k,n}} - 1} \right] \quad (1)$$

By solving (1), $r_{k,n}$ is

$$r_{k,n} = \log_2 \left(1 + \frac{\gamma_{k,n}}{\Gamma} \right) = \log_2 (1 + p_{k,n} H_{k,n}) \quad (2)$$

where $\Gamma = -\ln(5\text{BER})/1.6$ and $H_{k,n} = \frac{h_{k,n}}{\Gamma}$ is the effective subcarrier SNR of user k in subcarrier n .

Taking into account the conclusions above, the optimization problem is formulated as:

$$\max_{c_{k,n}, p_{k,n}} \frac{B}{N} \sum_{k=1}^K \sum_{n=1}^N c_{k,n} r_{k,n} \quad (3)$$

subject to

$$c_{k,n} \in \{0, 1\}, \forall k, n \quad (4)$$

$$p_{k,n} \geq 0, \forall k, n \quad (5)$$

$$\sum_{k=1}^K c_{k,n} = 1, \forall n \quad (6)$$

$$\sum_{n=1}^N p_{k,n} \leq P_k, \forall k \quad (7)$$

$$\sum_{n=1}^N c_{k,n} r_{k,n} \geq m r_k, \forall k \quad (8)$$

where $c_{k,n}$ is the subcarrier allocation indicator such that $c_{k,n} = 1$ if subcarrier n is assigned to user k and $c_{k,n} = 0$ if not. Constraints (4) and (5) ensure the correct values for the subcarrier allocation indicator and the power, respectively. Constraint (6) restricts the assignment of each subcarrier to only one user and (7) is the individual power constraint. Finally, (8) is the minimum data rate constraint. The total data rate for user k , denoted as R_k , is defined as

$$R_k = \frac{B}{N} \sum_{n=1}^N c_{k,n} r_{k,n} \quad (9)$$

where $r_{k,n}$ is given by (2) and $m r_k$ in (8) is the minimum data rate of each user.

Note that problem (3) is an NP-hard combinatorial optimization problem [26] with non-linear constraints. In a system with K users and N subcarriers, there are K^N possible subcarrier assignments, since it is assumed that no subcarrier can be used by more than one user. For a certain subcarrier assignment, a per user power distribution can be used to maximize the sum of the users' data rates, while guaranteeing minimum data rate constraints. The maximum sum data rate over all K^N subcarrier assignment schemes is the global maximum and the corresponding subcarrier assignment and per user power distribution is the optimal resource allocation scheme. However, it is difficult to obtain an optimal solution within any reasonable time frame. As a result, a novel and cost-effective resource allocation scheme is formulated to solve this problem.

III. THE PROPOSED RESOURCE ALLOCATION SCHEME

Ideally, power of each user and subcarriers should be allocated jointly to solve optimization problem (3) optimally. This process has a prohibitive computational complexity. In the following, a suboptimal resource allocation scheme is proposed which consists of three algorithms and assures a low complexity performance:

A. Number of subcarriers of each user.

In this algorithm, the number of subcarriers N_k , to be initially assigned to each user, is determined. This process is based on the average effective subcarrier SNR of each user which is calculated by

$$\bar{H}_k = \frac{1}{N} \sum_{n=1}^N H_{k,n}, \quad \forall k = 1, 2, \dots, K \quad (10)$$

The approximate data rate of each user is

$$\bar{R}_k = N_k \frac{B}{N} \log_2 (1 + \bar{H}_k \bar{P}_k), \quad \forall k = 1, 2, \dots, K \quad (11)$$

where \bar{P}_k is the equal power allocation of each user among respective subcarriers. At each iteration, the user with the minimum difference $\bar{R}_k - mr_k$ has the option to be assigned one more subcarrier. When all the available subcarriers are assigned to K users of the system, the approximate number of subcarriers N_k for each user is got. This algorithm is as follows

1) Initialization:

- Set $mr_k, \forall k = 1, 2, \dots, K$, the minimum data rate constraints.
- Set the initial number of subcarriers $N_k = \lfloor N \frac{mr_k}{\sum_{k=1}^K mr_k} \rfloor, \forall k = 1, 2, \dots, K$ and $N_{al} = \sum_{k=1}^K N_k$.
- Get the average effective subcarrier SNR for each user using equation (10).

2) Approximate data rate:

- Get the equal power to each allocated subcarrier $\bar{P}_k = \frac{P_k}{N_k}, \forall k = 1, 2, \dots, K$.
- Calculate $\bar{R}_k, \forall k = 1, 2, \dots, K$, using equation (11).

3) While $N_{al} < N$:

- Find $k^* = \operatorname{argmin}_{k=1,2,\dots,K} \{\bar{R}_k - mr_k\}$. For the found k^* , let $N_{k^*} = N_{k^*} + 1$ and $N_{al} = N_{al} + 1$.
- Get the equal power to each allocated subcarrier $\bar{P}_k = \frac{P_k}{N_k}, \forall k = 1, 2, \dots, K$.
- Calculate $\bar{R}_k, \forall k = 1, 2, \dots, K$, using equation (11).

In initialization step, $N \frac{mr_k}{\sum_{k=1}^K mr_k}$ is approximated to the lower integer because N_k should be an integer. Hence, it is not sure that $N_{al} = N$; there might be some remaining subcarriers. That is the reason why step 3) of the algorithm is necessary.

B. Subcarrier assignment to available users.

In this subsection the $N_k, \forall k = 1, 2, \dots, K$, subcarriers are allocated to available users in order to maximize the sum of the users' data rates while guaranteeing minimum data rates of K users. The algorithm is described below.

1) Initialization:

- Set $\mathcal{S} = \{1, 2, \dots, N\}$, $R_k = 0, \forall k = 1, 2, \dots, K$, $c_{k,n} = 0, \forall k = 1, 2, \dots, K$ and $n \in \mathcal{S}$.

- Sort the K users by average effective subcarrier SNR, i.e., $\bar{H}_1 \leq \dots \leq \bar{H}_m \leq \dots \leq \bar{H}_K$ without loss of generality.
- Divide the K users in bad effective subcarrier SNR group ($\text{user}_b = \{1, 2, \dots, m\}$) and good effective subcarrier SNR group ($\text{user}_g = \{m + 1, m + 2, \dots, K\}$).

2) For $k = 1, 2, \dots, m$:

- Find n satisfying $H_{k,n} \geq H_{k,j}, \forall j \in \mathcal{S}$.
- For the found n , set $c_{k,n} = 1, N_k = N_k - 1, \mathcal{S} = \mathcal{S} - \{n\}$ and update R_k according to equation (9). In equation (9), $p_{k,n} = \frac{P_k}{\sum_{n=1}^N c_{k,n}}$.

3) While $|\text{user}_b| \neq \emptyset$:

- Find $k^* = \operatorname{argmin}_{k \in \text{user}_b} \{R_k - mr_k\}$.
- For the found k^* , if $N_{k^*} > 0$
 - Find n satisfying $H_{k^*,n} \geq H_{k^*,j}, \forall j \in \mathcal{S}$.
 - Set $c_{k^*,n} = 1, N_{k^*} = N_{k^*} - 1, \mathcal{S} = \mathcal{S} - \{n\}$ and update R_{k^*} according to equation (9). In equation (9), $p_{k^*,n} = \frac{P_{k^*}}{\sum_{n=1}^N c_{k^*,n}}$.
- else
 - $\text{user}_b = \text{user}_b - \{k^*\}$.

4) Redo:

- Steps 2,3 for the good effective subcarrier SNR group, i.e., for users $\in \text{user}_g$.

In step 1) of the subcarrier assignment to available users algorithm, all the variables are initialized. \mathcal{S} is the set of available subcarriers, $c_{k,n}$ is the subcarrier allocation indicator and R_k is a vector which keeps track of the data rate of each user $k = 1, 2, \dots, K$. Then, users are divided in two groups according to parameter m ; the user_b and the user_g , the group of users with bad average effective subcarrier SNR and the group with good average effective subcarrier SNR respectively. Parameter m is chosen in such a way that the two user groups contain the same number of users, if K is an even number. Otherwise, if K is an odd number, one of the two user groups would contain one more user than the other group.

In step 2), each user of user_b group is assigned the available subcarrier on which he has the largest effective subcarrier SNR. Note that an inherent advantage is gained by the fact that users of user_b group choose their best subcarrier earlier than the users of the other group.

In step 3), subcarriers are assigned to available users until each user gets his allotment of N_k subcarriers. The user who has the minimum difference between its data rate and respective minimum data rate constraint has the priority to choose his best subcarrier. The best subcarrier is that on which he has the largest effective subcarrier SNR. The user, who gets his allotment of N_k subcarriers, can no longer be assigned any more subcarriers. $|\text{user}_b|$ here denotes the cardinality of set user_b .

In step 4), the same procedure takes place but for the user_g group; the group of users with good average effective subcarrier SNR. The condition $k = 1, 2, \dots, m$, changes to $k = m + 1, m + 2, \dots, K$.

C. Power allocation.

In subcarrier assignment to available users algorithm available power is distributed uniformly among subcarriers. In order to further enhance the sum of the users' data rates, in power allocation algorithm, the subcarrier allocation is kept, but the available power P_k of each user is assigned to subcarriers of each user using the water-filling [7] algorithm:

$$p_{k,n} = \left(\lambda_k - \frac{1}{H_{k,n}} \right)^+$$

where $p_{k,n}$ is the allocated power in each subcarrier, $(\cdot)^+ = \max(0, \cdot)$, and λ_k satisfies

$$\sum_{n=1}^N p_{k,n} = P_k, \forall k = 1, 2, \dots, K.$$

IV. COMPLEXITY ANALYSIS

In this section, the computational complexity of the proposed resource allocation scheme is analyzed and compared with that of [5], [20]-[22]. Recall that K refers to the total number of users in the system and N refers to the number of subcarriers, which is much larger than K . As mentioned in Section II, for the exhaustive search algorithm, there are K^N possible subcarrier assignments which require $O(K^N)$ time.

Initialization step of the first algorithm of the proposed scheme requires K multiplications to set the initial number of subcarriers N_k , and also average effective subcarrier SNR is calculated K times. Thus, the complexity of this initialization step is $O(K)$. In second step of the same algorithm, \bar{R}_k is calculated K times which is $O(K)$. In third step, the user with the minimum $\bar{R}_k - mr_k$ among K users is found and \bar{R}_k is calculated for $k = 1, 2, \dots, K$. This is repeated until $N_{al} = N$. Thus, this step requires $O(K(N - N_{al}))$ complexity which is also the overall complexity of the first algorithm of the proposed resource allocation scheme.

In initialization step of the second algorithm of the proposed resource allocation scheme, K users are sorted by average effective subcarrier SNR which has $O(K \log_2 K)$ complexity. Then the complexity of the division of K users in two groups is $O(K)$. Thus, the complexity of this initialization step is $O(K \log_2 K)$. In the second step, for each user of one group, the best subcarrier is found which has complexity $O(KN)$ because in our simulations the two groups contain equal number of users, i.e., $m = \frac{K}{2}$. In step three of this algorithm, subcarriers are allocated to users of one group until each user gets his allotment of N_k subcarriers. In worst case scenario, the complexity of this step is $O(KN)$. In step four, the same procedure takes place but for the other group of users. Thus, because $K \ll N$ and $\log_2 K \ll N$ the complexity of the second algorithm of the proposed resource allocation scheme is $O(KN)$.

Finally, in the third algorithm of the proposed resource allocation scheme, after subcarrier allocation is found, the water-filling power allocation algorithm is implemented which requires to find λ_k . The update of λ_k can be done by using a

simple bisection method until the power of each user converges [27]. Thus, in order to perform water-filling power allocation for each user the overall time complexity is $O(KN)$. Consequently the complexity of the proposed resource allocation scheme is $O(K(N - N_{al}) + KN + KN) \approx O(KN)$.

In the resource allocation algorithm proposed in [5], each subcarrier is allocated to the user with the maximum effective subcarrier SNR. Then, after the subcarrier allocation, either water-filling [7] or equal power allocation is applied for each user. Thus, the complexity of [5] is $O(KN)$. Algorithm in [5] is optimal in the downlink scenario but not in the uplink because in the latter there are individual power constraints.

In algorithms proposed in [20] [21], N iterations are required to allocate N subcarriers to available users. In each iteration, water-filling [7] is performed for each user with time complexity $O(N)$. This means that the time complexity of one iteration is $O(KN)$ and for all iterations is $O(KN^2)$. In [21], a fast implementation method is introduced which is based on binary tree data structure and has $O(KN \log_2 N)$ time complexity but it requires greater storage memory in order to store all the required information in each node of the binary tree.

The algorithm proposed in [22] consists of the step of initial subcarrier allocation and the step of residual subcarrier allocation. For both steps the complexity is $O(KN)$ as described extensively in [22].

It is easily observed that the proposed resource allocation scheme has a very dramatic reduction in complexity compared to $O(K^N)$ required by the exhaustive search. In addition it has similar complexity to [5] [22] and smaller than [20] [21], without using the binary search tree introduced in [21].

V. SIMULATION RESULTS

In this section, the performance of the proposed uplink resource allocation scheme is evaluated using simulation. In all simulations presented in this section, the frequency-selective channel consists of six independent Rayleigh multipath components (taps). As in [9], an exponentially decaying power delay profile is assumed, the ratio of the energy of the l th tap to the first tap being equal to e^{-2l} . For each channel realization the proposed scheme is used to perform resource allocation, and the data rates for each user are computed. A maximum delay spread of $5 \mu\text{s}$ and maximum doppler of 30 Hz is assumed. The channel information is sampled every 0.5 ms to update the resource allocation. As in [9], the total available bandwidth is equal to $B = 1$ MHz, the number of subcarriers of an OFDM symbol is $N = 64$, variance of the additive noise is equal to $N_0 = -80$ dB-W/Hz (single-sided PSD), and BER = 10^{-7} . Minimum data rate constraints are $mr_k = 1$ bit/s/Hz for $k = 1, 2, \dots, K$, the number of channel realizations is equal to 10^5 and parameter $m = \frac{K}{2}$.

The proposed resource allocation scheme is compared with the algorithms proposed in [5] (Jang et al.), [20] (Kim et al.), [21] (Ng et al.), [22] (Gao et al.), and a static TDMA scheme. In Figs. 1, 3, 5 the number of users of the system varies from 2 - 8 in increment of 2 and total transmitted power of each

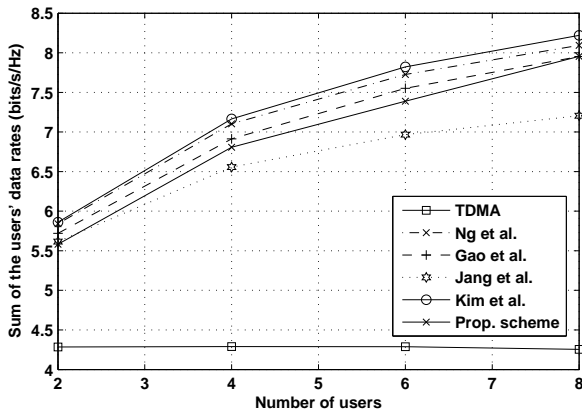


Fig. 1. Sum of the users' data rates vs number of users.

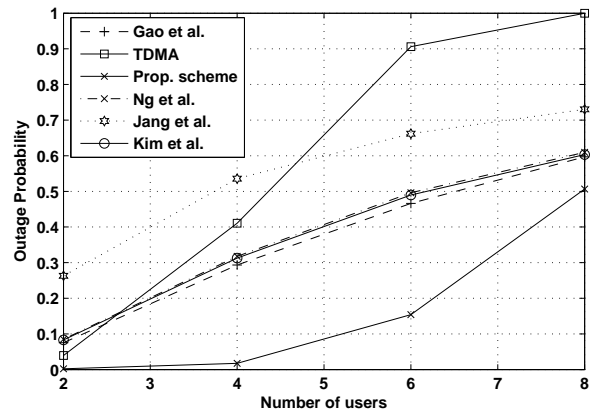


Fig. 3. Outage probability vs number of users.

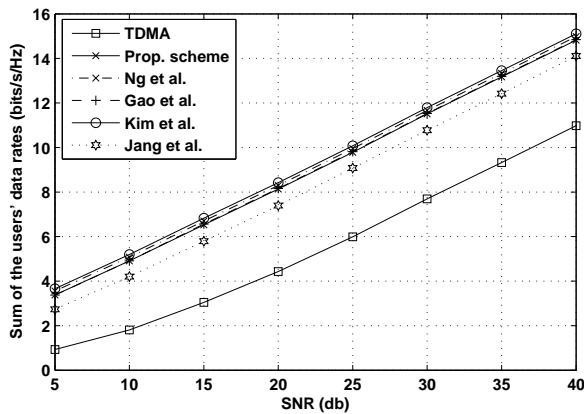


Fig. 2. Sum of the users' data rates vs SNR(db).

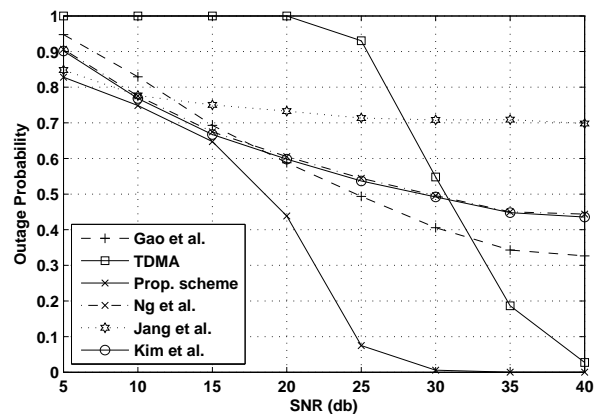


Fig. 4. Outage probability vs SNR(db).

user is equal to $P_k = 1$ W, while in Figs. 2, 4, 6 the number of users is $K = 8$ and SNR varies from 5 - 40 in increment of 5.

Figs. 1, 2 depict the comparison of the sum of the users' data rates versus number of users and SNR, respectively. It can be seen, the reasonable price being paid in order to guarantee minimum data rates by using the proposed scheme. In Fig. 1, as the number of users increases, the difference in sum data rates also increases because additional multiuser diversity is available to [20] [21] that only target sum data rate maximization. On the other hand, more users put more constraints to the proposed scheme, because new users need to share the same resources. In addition, sum data rate of the proposed scheme is significantly enhanced over both [5] and static TDMA algorithm as can be seen in Figs. 1, 2.

Figs. 3, 4 depict the comparison of outage probability versus number of users and SNR, respectively. The outage probability of the proposed resource allocation scheme is significantly smaller than any of the other comparing algorithms. This point is very critical in real systems where users should satisfy the minimum data rate criterion.

Figs. 5, 6 depict the comparison of the fairness pointer

versus number of users and SNR, respectively. Fairness pointer F is the one introduced in [9], and is defined as

$$F = \frac{(\sum_{k=1}^K R_k)^2}{K \sum_{k=1}^K (R_k)^2},$$

where F is a real number in the interval $(0, 1]$ with the maximum value of 1 for the case when equal data rates are achieved among users. As can be seen in Figs. 5, 6, users' data rates are almost equal when the proposed resource allocation scheme and static TDMA algorithm are employed with the proposed scheme being more fair. Algorithm in [5] guarantees the least fairness between the users' data rates. Algorithms in [20] [21] guarantee almost the same fairness but it is much lower than that of the proposed scheme and algorithm in [22] guarantees improved fairness than [5] [20] [21] but noticeably less than the proposed scheme. In addition, it can be seen in Fig. 5 that as the number of users increases the fairness pointer increases in all algorithms except the proposed scheme and static TDMA where fairness pointer remains almost constant regardless of the increasing number of users.

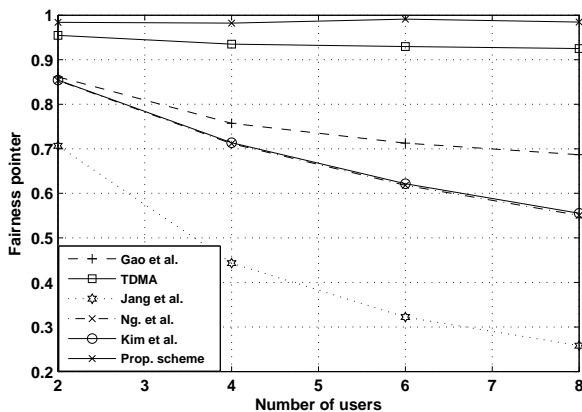


Fig. 5. Fairness pointer vs number of users.

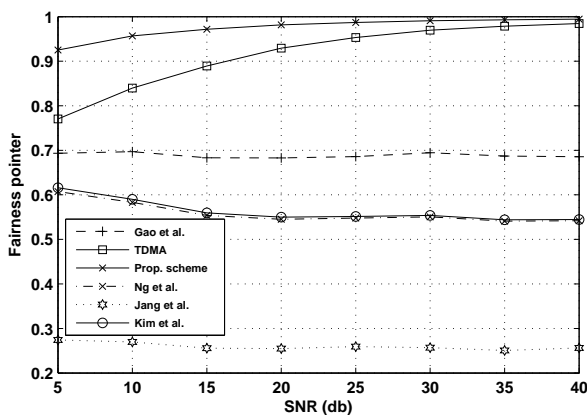


Fig. 6. Fairness pointer vs SNR(db).

VI. CONCLUSION

A resource allocation scheme for the uplink of SISO-OFDMA wireless systems was introduced which maximizes sum data rate of the system's users while guaranteeing minimum data rates mr_k among users. It is also complexity efficient and consists of three algorithms; The first algorithm determines the number of subcarriers to be initially assigned to each user, the second algorithm assigns the subcarriers to each user, and the third one allocates the available power P_k of each user to subcarriers, using the water-filling equation [7]. In addition, its innovative priority scheduling exploits more efficiently the multiuser diversity and makes it perform better than previous schemes. Finally, sum of the users' data rates is distributed more fairly among users.

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