The Mathematical Relationship Between Maximum Access Delay and the R.M.S Delay Spread

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***Abstract*—Currently, the orthogonal frequency division multiplexing (OFDM) systems use a predetermined cyclic prefix (CP) that is conservatively designed for the longest anticipated delay spread to overcome the multipath propagation delays. The most important parameter for determining the CP is power delay profile which is widely accepted to be following a negative exponentially decaying pattern. In this paper, the key parameter root mean square (r.m.s) delay spread of the power delay profile is mathematically derived based on the exponentially decaying power delay profile. The 3rd Generation Partnership Project (3GPP) power delay profiles are fitted into the exponentially decaying power delay profile (EDPDP). The performance measure bit error rate (BER) is used to evaluate the effectiveness of the EDPDP and its r.m.s delay spread. The findings show that EDPDP can be used to characterize the most of the power delay profiles. Subsequently, a mathematical formula to calculate CP estimation is derived. As a result, it is found that the CP is the natural logarithm of the ration of maximum power to minimum power of a particular power delay profile multiplied by its r.m.s delay spread. This finding gives the relationship between the maximum access delay and r.m.s delay spread as far as CP is concerned.**

***Index Terms*—Ofdm; cyclic prefix; delay spread.**

I. INTRODUCTION

New generation of wireless mobile radio systems aims to provide higher data rates to the mobile users as well as serving many users. Therefore, adaptation methods are becoming popular for optimizing mobile radio system transmission and reception at the physical layer as well as at the higher layers of protocol stack. These adaptive algorithms offer performance improvement, better radio coverage and high data rates with low power consumption. Several adaptation schemes require a form of measurement or estimation of one or more variables that may change over time [1]. Therefore, one way of increasing the spectral efficiency is to adapt the length of the cyclic prefix (CP) which varies depending on the radio environment [2]. The most important parameter for determining the CP is power delay profile which is widely accepted to follow a negative exponentially decaying pattern [3, 4]. Considering the fact that the multipath effect is highly dependent on the deployed environment in which the wireless system operates, the width of a CP is chosen in such a way that it is larger than the maximum access delay of the propagation channel. The CP is determined by the maximum access delay or by the root mean square (r.m.s) delay spread of that environment multiplied by a constant in the range between two to four as a rule of thumb [5].

The cyclic prefix adaptation in this study is composed of two parts. The first part is to prove that the exponentially decaying power delay profile (EDPDP) can be used to characterize power delay profiles. Then, the EDPDP model has been used to derive a mathematical formula for estimating the cyclic prefix in OFDM system. The second part of the study focuses on how the channel impulse response (CIR) can be obtained during one OFDM frame. This can be achieved by transmitting predetermined bits in the OFDM pilot subcarriers in the transmitter part. On the receiver part, the minimum mean square error (MMSE) or least square (LS) channel estimators can be used to estimate the CIR. Subsequently, the power delay profile (i.e., the energy taps) can be directly obtained from the CIR [6, 7].

In order to gauge the necessary adaptive CP, the power delay profile (energy taps) firstly should be fitted into the EDPDP model. Consequently, the CP model which is derived in this paper can be used to get the expected CP. This predicted CP can be used by the receiver in the subsequent transmitted frame. Therefore, by using this technique, it is expected to adapt the CP for each OFDM frames dynamically. Assuming that the mobile user is under low mobility condition; consequently, the power delay profile and its r.m.s delay spread are assumed to be stationary during one OFDM frame time.

This paper will focus on the first part of the study. In order to prove the validity of the proposed models, the 3rd Generation Partnership Project (3GPP) channel models are fitted into the EDPDP model. Besides, the decaying constant of the EDPDP is estimated for each power delay profiles. Subsequently, OFDM physical layer simulation is used to validate the accuracy of the proposed models (i.e., the EDPDP and the CP models) in terms of bit error rate (BER). In the transmitter part, the CP is increased in steps in order to reduce the effects of the 3GPP power delay profiles which are used to characterize the effects of the multipath signal. Next, the BER performance is plotted for these different CP values. Lastly, the CP which gives the minimum BER is compared with the

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estimated CP. It is found that estimated CP is always in the vicinity of the optimum CP value.

The rest of the paper is structured as follows; in Section II, a formula for r.m.s delay spread is derived based on EDPDP model. The concept of curve fitting techique is explained in Section III. A new CP formula is adopted in Section IV. The OFDM physical layer simulation which is used to validate the effectiveness of the proposed models is clearly explained in Section V. The Performance of the EDPDP and CP models in terms of BER are discussed in Section VI. Meanwhile the main findings are elucidated in Section VII. Finally, the conclusion derived from this study is stated in Section IX.

II. THE R.M.S DELAY SPREAD OF THE EDPDP

The delay spread is a type of distortion that is caused when identical signals arrive at different times at their destination. The signals usually arrive via multiple paths with different angles of arrival. The time difference between the moment of arrival of the first multipath component (typically the line-of-sight component) and the last multipath component is called the maximum access delay. Additionally, the r.m.s delay spread $(\tau_{r.m.s})$ is defined as the square root of the second central moment of the power delay profile $P(\tau)$. The formula for calculating the delay spread for multipath signals is defined as [8]:

$$\tau_{rms} = \sqrt{\frac{\sum_{i=1}^{N} P_i \tau_i^2}{\sum_{i=1}^{N} P_i} - \frac{(\sum_{i=1}^{N} P_i \tau_i)^2}{(\sum_{i=1}^{N} P_i)^2}} \quad (1)$$

where τ_i and P_i are the arrival time and power of i_{th} path respectively. In the case of continuous power delay profile, the $(\tau_{r.m.s})$ is defined as [8]:

$$\tau_{rms} = \sqrt{\frac{\int_0^\infty (\tau - m)^2 P_h(\tau) d\tau}{\int_0^\infty P_h(\tau) d\tau}} \quad (2)$$

where m is the mean of access delay which is also defined as:

$$m = \frac{\int_{\tau_{min}}^{\tau_{max}} \tau P(\tau)\, d\tau}{\int_{\tau_{min}}^{\tau_{max}} P(\tau)\, d\tau} \quad (3)$$

where τ_{min} and τ_{max} are the arrival times of the first path and the last path respectively. As agreed by some researchers, the power delay profile is negatively exponentially decaying [3,4], it is possible to formulate the power delay profile as:

$$P = P_0 e^{-\alpha\tau} \quad (4)$$

where P_0 is the power when $\tau = 0$, α is the decaying constant and τ is the multipath signal arrival time. The mean of the power delay profile (m_1) can be calculated as:

$$m_1 = \int_{\tau_{min}}^{\tau_{max}} \tau P_0 e^{-\alpha\tau} d\tau \quad (5)$$

In turn, Equation 5 is solved to produce:

$$m_1 = \left[P_0 e^{-\alpha\tau} \left(\frac{\tau}{-\alpha} - \frac{1}{\alpha^2} \right) \right]_{\tau_{min}}^{\tau_{max}} \quad (6)$$

$$m_1 = \left[-P_0 e^{-\alpha\tau} \left(\frac{\tau}{\alpha} + \frac{1}{\alpha^2} \right) \right]_{\tau_{min}}^{\tau_{max}} \quad (7)$$

$$m_1 = -\frac{1}{\alpha} \left[P_0 e^{-\alpha\tau} \left(\tau + \frac{1}{\alpha} \right) \right]_{\tau_{min}}^{\tau_{max}} \quad (8)$$

Now, τ_{mean} is defined as the mean time which can be obtained by normalizing the mean of power delay profile (m_1) by the total sum of all arrived powers (I_0) which can be defined as:

$$I_0 = \int_{\tau_{min}}^{\tau_{max}} P_0 e^{-\alpha\tau} d\tau = \left[\frac{-P_0}{\alpha} e^{-\alpha\tau} \right]_{\tau_{min}}^{\tau_{max}} \quad (9)$$

From the aforementioned definitions, the mean time τ_{mean} can be articulated as:

$$\tau_{mean} = \frac{m_1}{I_0} \quad (10)$$

$$\tau_{mean} = \frac{-\frac{1}{\alpha} \left[P_0 e^{-\alpha\tau} (\tau + \frac{1}{\alpha}) \right]_{\tau_{min}}^{\tau_{max}}}{\left[\frac{-P_0}{\alpha} e^{-\alpha\tau} \right]_{\tau_{min}}^{\tau_{max}}} \quad (11)$$

$$\tau_{mean} = \left[(\tau + \frac{1}{\alpha}) \right]_{\tau_{min}}^{\tau_{max}} \quad (12)$$

Similarly, the second moment (m_2) of the power delay profile can be defined as:

$$m_2 = \int_{\tau_{min}}^{\tau_{max}} \tau^2 P_0 e^{-\alpha\tau} d\tau \quad (13)$$

$$m_2 = \left[-P_0 \alpha e^{-\alpha\tau} \left(\frac{\tau^2}{-\alpha} - \frac{2\tau}{\alpha^2} + \frac{2}{-\alpha^3} \right) \right]_{\tau_{min}}^{\tau_{max}} \quad (14)$$

$$m_2 = \left[-P_0 \alpha e^{-\alpha\tau} \left(-\frac{\tau^2}{\alpha} - \frac{2\tau}{\alpha^2} - \frac{2}{\alpha^3} \right) \right]_{\tau_{min}}^{\tau_{max}} \quad (15)$$

$$m_2 = -\frac{P_0}{\alpha} \left[\alpha e^{-\alpha\tau} \left(\tau^2 + \frac{2\tau}{\alpha} + \frac{2}{\alpha^2} \right) \right]_{\tau_{min}}^{\tau_{max}} \quad (16)$$

Accordingly, the second moment of the arrival times $\tau_{s-moment}$ can also be expressed as:

$$\tau_{s-moment} = \frac{m_2}{I_0} \quad (17)$$

$$\tau_{s-moment} = \frac{-\frac{P_0}{\alpha} \left[\alpha e^{-\alpha\tau} \left(\tau^2 + \frac{2\tau}{\alpha} + \frac{2}{\alpha^2} \right) \right]_{\tau_{min}}^{\tau_{max}}}{\left[\frac{-P_0}{\alpha} e^{-\alpha\tau} \right]_{\tau_{min}}^{\tau_{max}}} \quad (18)$$

$$\tau_{s-moment} = \left[\tau^2 + \frac{2\tau}{\alpha} + \frac{2}{\alpha^2} \right]_{\tau_{min}}^{\tau_{max}} \quad (19)$$

The r.m.s delay spread (τ_{rms}) is defined as the square root of the second central moment of the arrival times which is given as [8]:

$$\tau_{rms} = \sqrt{\tau_{s-moment} - \tau_{mean}^2} \quad (20)$$

$$\tau_{rms} = \sqrt{\left[\tau^2 + \frac{2\tau}{\alpha} + \frac{2}{\alpha^2} - \left(\tau^2 + \frac{2\tau}{\alpha} + \frac{1}{\alpha^2} \right) \right]_{\tau_{min}}^{\tau_{max}}} \quad (21)$$

$$\tau_{rms} = \sqrt{\left[\frac{1}{\alpha^2}\right]_{\tau_{min}}^{\tau_{max}}} \tag{22}$$

$$\tau_{rms} = \frac{1}{\alpha} \tag{23}$$

As can be seen from Equation (23), the r.m.s delay spread is inversely proportional to the decaying constant. Subsequently, the r.m.s delay spread which is just derived from the EDPDP model will be identified as $(\hat{\tau}_{rms})$ in this chapter in the following sections.

III. Fitting the Power Delay Profiles into the EDPDP Model

Since it is required to get the value of r.m.s delay spread (i.e., α), and the power delay profile is always a discrete profile, there is a need to use the least squares curve fitting technique to fit the power delay profile into the EDPDP model. As a case study, the power delay profiles are obtained from the 3GPP channel models as shown in Table I. These channel models are fitted into the EDPDP model; as a result, the EDPDP coefficients P_0 and α are estimated. For the sake of simplicity, the P , P_0 , α and τ in Equation (4) are substituted by y ,A ,$-B$ and x respectively, which finally gives the EDPDP model as:

$$y = Ae^{Bx} \tag{24}$$

taking the natural logarithm for both sides:

$$lny = lnA + Bx \tag{25}$$

let$a = lnA$ and $b = B$, then the sum of all the errors squared $\left(\epsilon^2\right)$ can be expressed as:

$$\epsilon^2 = \sum_{i=1}^{n} y_i(lny_i - a - bx_i)^2 \tag{26}$$

applying the least squares fitting technique gives:

$$a\sum_{i=1}^{n} y_i + b\sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} y_i lny_i \tag{27}$$

$$a\sum_{i=1}^{n} x_i y_i + b\sum_{i=1}^{n} x_i^2 y_i = \sum_{i=1}^{n} x_i y_i lny_i \tag{28}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i^2 y_i \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} y_i lny_i \\ \sum_{i=1}^{n} x_i y_i lny_i \end{bmatrix} \tag{29}$$

$$a = \frac{\sum_{i=1}^{n}(y_i lny_i)\sum_{i=1}^{n}(x_i^2 y_i) - \sum_{i=1}^{n}(x_i y_i)\sum_{i=1}^{n}(x_i y_i lny_i)}{\sum_{i=1}^{n} y_i \sum_{i=1}^{n}(x_i^2 y_i) - (\sum_{i=1}^{n} x_i y_i)^2} \tag{30}$$

$$b = \frac{\sum_{i=1}^{n} y_i \sum_{i=1}^{n}(x_i y_i lny_i) - \sum_{i=1}^{n}(x_i y_i)\sum_{i=1}^{n}(y_i lny_i)}{\sum_{i=1}^{n} y_i \sum_{i=1}^{n}(x_i^2 y_i) - (\sum_{i=1}^{n} x_i y_i)^2} \tag{31}$$

TABLE I
Power Delay Profiles of 3GPP Channel Models

RAx		TUx		HTx	
(dB)	time (ns)	(dB)	time (ns)	(dB)	time (ns)
-5.2	0	-5.7	0	-3.6	0
-6.4	42	-7.6	217	-8.9	356
-8.4	101	-10.1	512	-10.2	441
-9.3	129	-10.2	514	-11.5	528
-10.0	149	-10.2	517	-11.8	546
-13.1	245	-11.5	674	-12.7	609
-15.3	312	-13.4	882	-13.0	625
-18.5	410	-16.3	1230	-16.2	842
-20.4	469	-16.9	1287	-17.3	916
-22.4	528	-17.1	1311	-17.7	941
		-17.4	1349	-17.6	15000
		-19.0	1533	-22.7	16172
		-19.0	1535	-24.1	16492
		-19.8	1622	-25.8	16876
		-21.5	1818	-25.8	16882
		-21.6	1836	-26.2	16978
		-22.1	1884	-29.0	17615
		-22.6	1943	-29.9	17827
		-23.5	2048	-30.0	17849
		-24.3	2140	-30.7	18016

TABLE II
The r.m.s Delay Spread Values Using the Two Calculation Methods for 3GPPT Channel Model

τ_{rms}	RAx	TUx	HTx
Eq (1) (μs)	0.10001	0.50006	3.038
Eq (23) (μs)	0.13362	0.50048	4.220

where $b = B$, $A = exp(a)$ and $\alpha = -B = -b$ as previously assumed.

In Table II, the values of the r.m.s delay spread are calculated by using two methods; the first method is the direct r.m.s delay spread formula as stated in Equation (1), and the second method is as given in Equation (23).

IV. Cyclic Prefix Formulation

In this section, a mathematical formula is derived to estimate the CP based on the basic assumption that the channel power profile is exponentially decaying, that is:

$$P = P_0 e^{-\alpha\tau} \tag{32}$$

taking the natural logarithm of both sides and making τ as the subject of the formula:

$$\tau = \frac{ln(P/P_0)}{-\alpha} \tag{33}$$

substituting of Equation (23) into Equation (33) yields:

$$\tau = -\tau_{rms} ln(P/P_0) \tag{34}$$

The graph in Figure (1) shows that τ_{max} is the arrival time of the last path which has a power level of β , from this definition,τ_{max} can be expressed as:

$$\tau_{max} = -\tau_{rms} ln(\beta/P_0) \tag{35}$$

meanwhile the value of τ_{min} is the arrival time of the first path which has a power level of γ , it can also be expressed as:

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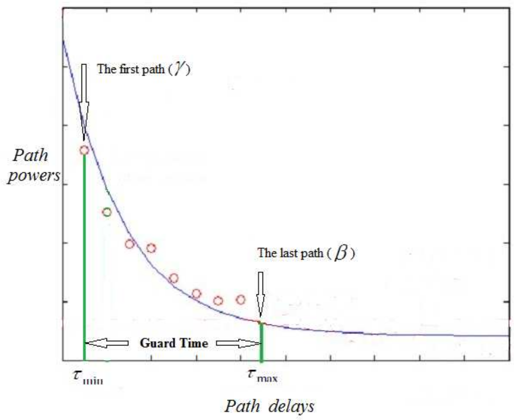

Fig. 1. Illustration Diagram Shows the Maximum Access Delay and CP Formulation Concept

$$\tau_{min} = -\tau_{rms} ln(\gamma/P_0) \tag{36}$$

Now, the required CP based on maximum access delay concept is defined as:

$$CP = \tau_{max} - \tau_{min} \tag{37}$$

Substitution of Equations (35) and (36) into Equation (37) yields:

$$CP = \tau_{rms}(ln\gamma/P_0 - ln\beta/P_0) \tag{38}$$

$$CP = \tau_{rms} ln\left(\frac{\gamma}{\beta}\right) \tag{39}$$

In Equation (39), the CP is a function of γ , whereby γ is the power of the first path as defined earlier. Interestingly, the value of γ depends on the propagation distance between the mobile station and the base station. Subsequently, the propagation distance is a function in the long term fading model. Accordingly, it is possible to hypothesize that there is a connection between the proposed CP model and the long term fading model.

V. The OFDM Physical Layer System Design

In order to evaluate the performance of the proposed models, a detailed link level simulation has been developed to investigate the physical layer performance of the IEEE 802.16e air interface. The functional blocks used in the transmitter and receiver chain of the link level simulation are shown in Figure (2).

Functional blocks of the OFDM system are implemented according to IEEE 802.16-2004 and IEEE 802.16-2005 with the exception of pulse shaping, which is outside the scope of standards. The first box in the block is the Bernoulli binary generator which is the information bits generator. The function of the second box is for channel coding which is composed of Forward Error Correction (FEC) and interleaving. For FEC, the conventional encoder is used, which is the only mandatory coding scheme according to IEEE 802.16e-2005 specification. The conventional encoder mother code rate (R0) is and its memory size is 6. After bit interleaving is used, the system uses the 16 PSK mapping scheme. At the input of the Inversed Fast Fourier Transform (IFFT), the data selector is used to form the OFDM symbol which consists of 1024 subcarriers (i.e., $Nfft = 1024$) . This OFDM symbol is constructed from 720 data subcarriers, 120 pilot subcarriers, 183 guard subcarriers and one direct current (DC) subcarrier which is used as a center frequency. Before transmitting the OFDM symbol, CP is added it in order to mitigate the effect of the multipath. This CP is added at the front of the OFDM symbol which is a duplication of the tail of the useful symbol with ratios of $G = 1/512, 1/256, 1/128, 1/64, 1/32, 1/16, 1/8, 1/4, 1/2$; where G is the CP to useful symbol time ratio. The other system parameters are the channel bandwidth $BW = 10MHz$, sampling factor $n = 28/25$, sampling frequency $Fs = (n \times BW/8000) \times 8000 = 11.2MHz$, subcarrier spacing $f = Fs/Nfft = 10.9375$, useful symbol time $T_b = 1/f = 91.42857\mu s$,$CP = G \times T_b$ and OFDM symbol time $Ts = T_b + CP$. The value CP and T_s depend on the values of G as shown in Table (III).

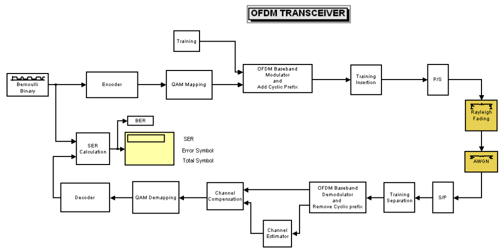

Fig. 2. The OFDM transceiver block diagram

TABLE III
THE RELATIONSHIP BETWEEN THE PARAMETER G AND AND THE CP

The ratio G	Cyclic Prefix (μs)	OFDM symbol time (T_s (μs))
1/512	0.17	91.6
1/256	0.35	91.7
1/128	0.7	92
1/64	1.4	93
1/32	2.8	94
1/16	5.7	97
1/8	11.4	102
1/4	22.8	114
1/2	45.7	137

VI. The Performance of the Cyclic Prefix Model

The purpose of this section is to validate the efficiency of the CP model using the 3GPP channel models which consist of three power delay profiles. The first power delay profile is the Rural Area channel model (RAx), the second one is the Typical Urban channel model (TUx) and the last one is the Hilly Terrain channel model (HTx) as shown in Table (I). The 3GPP channel models are fitted into the EDPDP and the r.m.s delay spread is calculated using the standard formula and the proposed formula as shown in Table (II). The curves in Figure (3) show the BER of RAx power delay profile. It can be clearly seen that the minimum BER occurs when $G = 1/128$ and its corresponding $CP = 0.71429\mu s$ as shown in Table (III). On the other hand, the estimated (CP) is $CP = \hat{\tau}_{rms} ln\left(\frac{\gamma}{\beta}\right) = 0.528\mu s$ as shown in Table (IV). Additionally, it can be observed that when the CP increases (i.e., G increases to $1/64$), there is no enhancement in BER performance. This means that the optimal CP occurs when $G = 1/128$. Moreover, it is observed that when the CP increases (G increases), the BER performance degrades. This is due to the fact that the BER is influenced by two factors. The first factor is inter-symbol-interference (ISI) and the second factor is the power consumption. When the CP increases, the effect of ISI is eliminated. Nevertheless, since the CP is considered as an additional transmitted data, transmitting these additional data needs to increase the transmitted power in order to maintain the same BER performance. In other words, increasing the CP size needs to increase transmitted power in order to keep the same BER. Therefore, increasing CP size while keeping the same amount of power for the OFDM frame, this will leads to a degradation in BER performance.

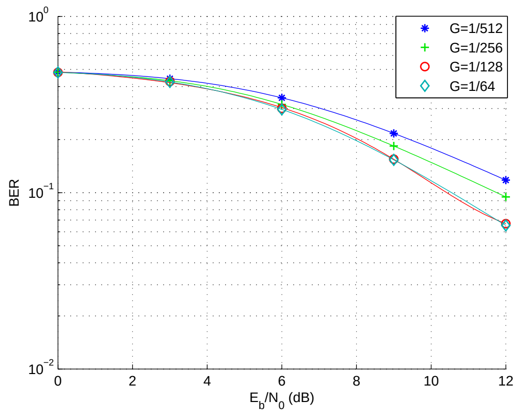

Fig. 3. BER Performance of 3rd Generation Partnership Project (3GPP)-Rural Area Channel Model (RAx) for Different CP Values

Focusing now on Figure (4), it can be clearly seen that the minimum BER of TUx occurs when $G = 1/32$ and its corresponding $CP = 2.8\mu s$ as shown in Table (III), meanwhile the estimated CP is $CP = \hat{\tau}_{rms} ln\left(\frac{\gamma}{\beta}\right) = 2.14\mu s$ as illustrated in Table (IV).

Moving on, the curves in Figure (5) show that the minimum BER for HTx occurs when $G = 1/4$ and its corresponding $CP = 22.8\mu s$ as shown in Table (III), meanwhile the estimated $CP = \hat{\tau}_{rms} ln\left(\frac{\gamma}{\beta}\right) = 18.01\mu s$ as depicted in Table (IV). Although there are some differences between the values of the r.m.s delay spread standard formula and the values of the proposed formula, the RAx and HTx estimated CP values are still positioned in the right range. On the other hand, the value of r.m.s delay spread of the standard formula

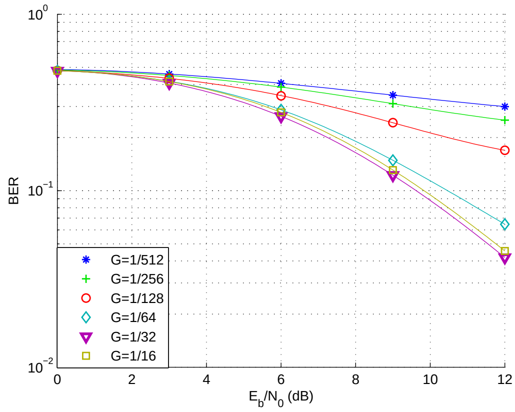

Fig. 4. BER Performance of 3rd Generation Partnership Project (3GPP)-Typical Urban Channel Model (TUx) for Different CP Values

and the value of r.m.s delay spread of the proposed formula for TUx almost have the same values. This is due to the fact that TUx measured data is smoothly decaying, meanwhile there is a slight divergence in RAx and HTx measured data as it can be clearly seen in Table (I).

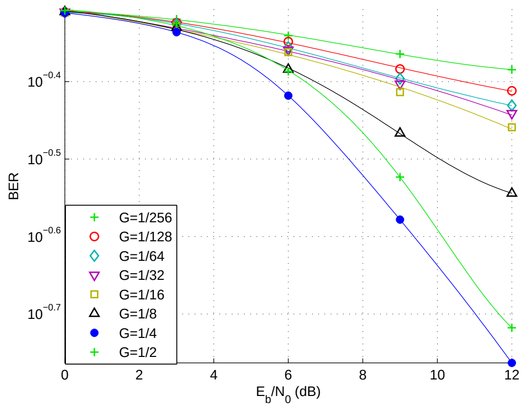

Fig. 5. BER Performance of 3rd Generation Partnership Project (3GPP)-Hilly Terrain Channel Model (HTx) for Different CP Values

TABLE IV
THE VALUES OF THE PROPOSED CP MODEL PARAMETERS FOR 3GPP CHANNEL MODELS

Parameters	Formula	RAx	TUx	HTx
P_0	$P = P_0 e^{-\alpha\tau}$	306.87	268.99	175.46
γ	$\gamma = P_0 e^{-\alpha\tau_{min}}$	306.87	268.99	175.46
β	$\beta = P_0 e^{-\alpha\tau_{max}}$	5.90	3.73	2.45
R	$R = ln\left(\frac{\gamma}{\beta}\right)$	3.95	4.27	4.26
$CP(\mu s)$	$CP = \tau_{rms} ln\left(\frac{\gamma}{\beta}\right)$	0.52	2.14	18.01

An important finding derived from this study, the ratio $R = ln\left(\frac{\gamma}{\beta}\right)$ is approximately in the vicinity of the value 4 for all the 3GPP power delay profiles as shown in Table (IV). As aforementioned, the proposed CP model is derived based on maximum access delay, and it is found that $CP = 4 \times \tau_{rms}$

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as shown in Table (IV). As a result, the proposed CP can be formulated as $CP = 4 \times \tau_{rms}$. This result of this study seemed to confirm the finding of a study by Arslan who found that the CP equals τ_{rms} multipled by a constant in the range between two to four [5]. As a result, the calculation of CP based on maximum access delay or based on r.m.s delay spread multiplied by 4 will give the same result.

VII. Findings

It is possible to consider γ as the main signal meanwhile the rest of the multipath signals can be considered as interference signals (i.e., β). Therefore, the main contribution of this paper, it gives the required span between the main signal γ and the others interference signals. This span can be used to eliminate the ISI when the r.m.s delay spread is used to calculate the CP. This finding is parallel with the finding in [10, pp. 77-78], in which the CP length is proposed to be adjusted as:

$$CP = \beta_0 \tau_{rms} \tag{40}$$

Therefore, when we compare Equation (39) with Equation (40), we will find that:

$$\beta_0 = ln\left(\frac{\gamma}{\beta}\right) \tag{41}$$

As a result, Equation (41) can be used to get the relationship between CP and required span instead of the look up table that has been proposed in [11].

VIII. Discussions of the EDPDP and CP Models

What is interesting in Equation (23) is that the r.m.s delay spread depends on the decaying constant α . The decay of the multipath signals depends on the attenuation caused by reflected and diffracted objects that surround the mobile station. This leads to an important fact that the r.m.s delay spread depends on the environment that surrounds the mobile station[5].

Next, as seen in Table (III) and Table (IV), the EDPDP model gives an acceptable level of precision for the r.m.s delay spread values indicating that all other types of power delay profiles can be represented by the EDPDP. This assumption is compatible with electromagnetic propagation phenomenon where the electromagnetic multipath signals reach the receiver after being reflected or diffracted from the surrounding walls of the buildings. In fact, the multipath signals are propagating through different propagation distances; therefore, they have different power levels. These power levels are inversely proportional to the propagation distances; as the propagation distance increases, the propagation time delay also increases. Therefore, the multipath signals should decay as their propagation distances increase. Moreover, the multipath signals encounter the same amount of power absorption loss. However, it is possible to find some paths that are reflected; for instance, from glass windows and metal doors, these paths that are reflected from such surfaces face an absorption loss slightly different from the first category. As a result, it is possible to find some paths arriving late but with power levels still higher than the ones which arrived earlier. However, the curve fitting technique will compensate for this kind of effects.

IX. Conclusion

In conclusion, the EDPDP model is proposed to characterize the power delay profiles, and its r.m.s delay spread is mathematically derived. The required CP for mitigating the influence of ISI in OFDM is also formulated. Fitting the power delay profiles into the EDPDP and using Equation (39), the necessary required adaptive CP can be achieved. In addition, there are two methods that are being used to estimate the CP; the first one is based on maximum access delay and the second is based on r.m.s delay spread multiplied by 4. The proposed CP model clearly shows that there is a mathematical relationship between these two methods. In addition, such proposed CP model confirms that these two methods lead to the same result.

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