Performance Analysis of Pilot Patterns in Coded OFDMA Systems over Multi-user Channels

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***Abstract*—This paper presents a systematic approach for analyzing the bit error probability of a coded orthogonal frequency-division multiple access (OFDMA) system in multi-user channels. A comparative analysis with conventional pilot patterns, the overlapped pilot pattern and the interlaced pilot pattern, is developed. Simulation results show that the interlaced pilot pattern is more sensitive to the effect of interference, but also it outperforms the overlapped pilot pattern with serious interference scenarios.**

***Keywords*-OFDMA, interference, pilot pattern.**

I. INTRODUCTION

Multi-user orthogonal frequency division multiple access (OFDMA) is a strong candidate transceiver scheme for high-speed packet-based multi-user wireless communication systems, due to its flexibility on subcarrier allocation and multi-user diversity utilization over multi-path fading channel [1]. A dynamic channel estimation is necessary before the demodulation of OFDMA signals since the radio channel is frequency selective and time-varying for wideband mobile communication systems [2]. The channel estimation can be performed by either inserting pilot tones into same positions or different positions for each user. In the rest of the paper, we call them as the overlapped pilot pattern and the interlaced pilot pattern respectively. After channel estimation is performed in the pilot locations, the channel frequency response is typically determined by interpolating the response between samples obtained using pilot tones. Thus, the design of pilot pattern is a critical factor in determining the performance of the OFDMA system. It is well-known that to achieve theoretically perfect channel estimation, pilots must be inserted into transmitted OFDMA symbols densely enough to fulfil the Shannon-Nyquist sampling theorem both in the frequency domain and in the time domain [3] [4]. Conversely, for a specific OFDMA configuration and pilot placement in the frequency domain, the channel propagation delay dispersion is limited to a specific range if theoretically perfect channel estimation is expected. For channels with larger delay dispersion, there will be irrecoverable channel estimation error, whose the magnitude is dependent on the distribution of power conveyed by the channel within and outside the acceptable propagation delay range [5] [6]. As justified above, in designing modern OFDMA systems, it is crucial to be able to estimate the magnitude of such unavoidable errors in realistic channel conditions.

In this work our objective is to analyze the performance of the coded OFDMA system in multi-user channels. The analysis and results can be extended to the design of pilot patterns for mesh network [7].

The rest of the paper has been organized as follows. Section II contains the system model and pilot assisted channel estimation for the coded OFDMA in multi-user channels. Section III provides the bit error probability (BEP) analysis for the coded OFDMA system. Simulation results are in Section IV. Finally, Section V summarizes our main results.

II. SYSTEM MODEL

In the discrete time domain, the nth sample of the lth output symbol of the OFDMA transmitter is given by the N point complex modulation sequence

$$x_l[n] = \sum_{k=0}^{N-1} X_l[k] e^{-j2\pi kn/N}, \tag{1}$$

where $X_l[k]$ is a data signal and k is a subcarrier index.

The received OFDMA signal can be written as

$$\begin{aligned} Y_l[k] &= \sum_{n=0}^{N-1} \left\{ \sum_{m=0}^{\infty} h_l[m] x_l[n-m] + z_l[n] \right\} e^{-j2\pi kn/N} \\ &= \sum_{n=0}^{N-1} \left\{ \sum_{m=0}^{\infty} h_l[m] \left\{ \frac{1}{N} \sum_{i=0}^{N-1} X_l[i] e^{j2\pi kn/N} \right\} \right\} \\ &\cdot e^{-j2\pi kn/N} + Z_l[k] \\ &= H_l[k] \cdot X_l[k] + Z_l[k], \end{aligned} \tag{2}$$

where

$$H_l[k] = \sum_{m=0}^{N_p-1} h_l[m] e^{-i2\pi k\tau_m/N}, \tag{3}$$

$$Z_l[k] = I_l[k] + N_l[k], \tag{4}$$

$H_l[k]$ is the channel response of the kth signal in the lth OFDMA symbol and $Z_l[k]$ is the summation of interference

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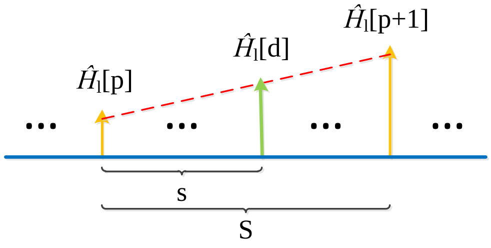

Figure 1. Linear interpolation.

$I_l[k]$=$H_l^{'}[k]\cdot X_l^{'}[k]$ according to other users using same resource and Gaussian noise $N_l[k]$.

A. Channel Model

We consider the wide sense stationary uncorrelated scattering (WSSUS) Rayleigh fading channel. The channel impulse response is represented as

$$h(t) = \sum_{m=0}^{N_p-1} c_m(t)\, \delta(t-\tau_m), \tag{5}$$

where N_p is the number of multiple paths, τ_m and $c_m(t)$ are the time delay and the channel response of the mth multipath respectively.

B. Pilot Assisted Channel Estimation

For the channel estimation, the known symbol, so-called pilot signal, is usually employed. Before the transmission, pilot signals are uniformly inserted into the data stream. Upon receiving the corrupted pilot signals at the receiver, the channel impulse response at pilot locations is estimated. The channel impulse response at data locations can then be obtained through interpolation with the pilot channel estimations. Here, we consider the least-squares (LS) estimate for the channel estimation at a pilot location. The lth estimated channel response can be obtained as follow:

$$\hat{H}_l[p] = \frac{Y_l[p]}{X_l[p]} = H_l[p] + \frac{Z_l[p]}{X_l[p]} = H_l[p] + V_l[p], \tag{6}$$

where p denotes the index of pilot subcarrier, $X_l[p]$ is the pth pilot signal at the lth OFDMA symbol and $Y_l[p]$ is the received symbol corresponding to pilot signal $X_l[p]$.

For the data location, the channel response can be estimated by taking interpolation between the pilot channel estimate. There are several forms to interpolate: uniform, spline interpolation, and 2D Wiener interpolation etc. Here, linear interpolation is used [4]. In the linear interpolation, the data channel estimate is given by

$$\hat{H}_l[d] = \left(1-\frac{s}{S}\right)\hat{H}_l[p] + \frac{s}{S}\hat{H}_l[p+1], \tag{7}$$

where d denotes the index of data subcarrier, S is the interval between pilot subcarriers and s is the distance between the pth pilot subcarrier and the dth data subcarrier as shown in Figure 1.

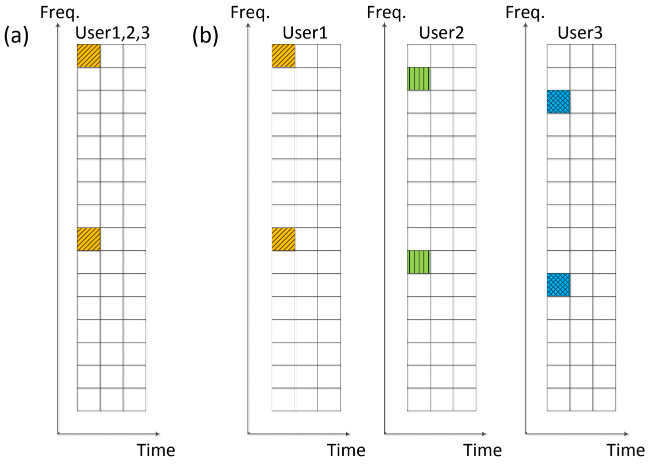

Figure 2. Pilot Patterns for OFDMA systems: (a) overlapped pilot pattern, (b) interlaced pilot pattern

Thus, the transmitted data signal at the dth data subcarrier in the lth OFDMA symbol can be estimated by

$$\hat{X}_l[d] = \frac{Y_l[d]}{\hat{H}_l[d]}. \tag{8}$$

For two patterns, which are the overlapped pilot pattern and the interlaced pilot pattern as shown Figure 2, the above formula can be re-written as follows:

1) Overlapped pilot pattern:

$$\hat{X}_l[d] = \frac{Y_l[d]}{\hat{H}_l[d]} = X_l[d] + \frac{Z_l[d]}{\hat{H}_l[d]} - \frac{V_l[d]X_l[d]}{\hat{H}_l[d]}, \tag{9}$$

$$\hat{Z}_l[d] = I_l[d] + N_l[d], \tag{10}$$

$$I_l[d] = \sum_{i\in\kappa} H_l^{(i)}[d]X_l^{(i)}[d], \tag{11}$$

where $I_l[d]$ is the interference of the dth data signal in the lth OFDMA symbol, $H_l^{(i)}[d]$ and $X_l^{(i)}[d]$ are the channel response and the data signal of the ith user respectively and κ is the set of neighbor users.

2) Interlaced pilot pattern:

$$\hat{X}_l[d_1] = \frac{Y_l[d_1]}{\hat{H}_l[d_1]} = X_l[d_1] + \frac{Z_l[d_1]}{\hat{H}_l[d_1]} - \frac{V_l[d_1]X_l[d_1]}{\hat{H}_l[d_1]}, \tag{12}$$

$$\hat{Z}_l[d_1] = I_l[d_1] + N_l[d_1], \tag{13}$$

$$I_l[d_1] = \sum_{i\in\kappa, i\neq j} H_l^{(i)}[d_1]X_l^{(i)}[d_1] + H_l^{(j)}[p]X_l^{(j)}[p], \tag{14}$$

$$\hat{X}_l[d_2] = \frac{Y_l[d_2]}{\hat{H}_l[d_2]} = X_l[d_2] + \frac{Z_l[d_2]}{\hat{H}_l[d_2]} - \frac{V_l[d_2]X_l[d_2]}{\hat{H}_l[d_2]}, \tag{15}$$

$$\hat{Z}_l[d_2] = I_l[d_2] + N_l[d_2], \tag{16}$$

$$I_l[d_2] = \sum_{i \in \kappa} H_l^{(i)}[d_2] X_l^{(i)}[d_2], \tag{17}$$

where j is the user index that the jth user's pilot signal is the interference.

For the interlaced pilot pattern, we can divide the data signals into two parts. The first part is the region that one of neighbor user's pilot signal and another neighbor users' data signals are interference. The second part is the region that all of neighbor users' data signals are interference. The first part gets more interference than second part because the pilot power is boosted for accurate channel estimation. At the above equations, we denote d_1 as data signal index in the first part and d_2 as data signal index in the second part.

C. Pilot Assisted Channel Estimation for Multi-user

For a single user case, it's obvious that more pilot signals leads to better performance but with sacrificing in symbol rate. Therefore the number of pilots is a trade-off between channel estimation accuracy and bandwidth efficiency. However, for multi-user case, it's not obvious that more pilot signals outperform less pilot signals due to interference between users. If pilot signals are corrupted, data fail to be demodulated irrespective of correcting processes such as despreading and decoding. Also, signal to interference ratio remains the same in the overlapped pilot pattern regardless of pilot power boost-up. The solution is to make the pilots from different users not to collide each other because pilot signals are relatively stronger than spread data signals. The channel performance estimation is improved by reducing the number of pilot signal collisions. However better channel performance estimation dose not always guarantee better data detection performance. The interference of pilot region and the interference of data region is trade-off.

III. PERFORMANCE ANALYSIS OF CODED OFDMA SYSTEMS

A. Coded OFDMA System

Error correcting coding is an essential part of an OFDMA system for mobile communication. OFDMA in a fading environment is almost always used with coding to improve its performance and as such is often referred to as Coded OFDMA or COFDMA. For an uncoded OFDMA system in a frequency selective Rayleigh-fading environment, each OFDMA subcarrier has a flat-fading channel. Accordingly, the average probability of error for an uncoded OFDMA system is the same as that for a flat-fading single-carrier system with the same average geometric mean of SNR.

Just as we can introduce time diversity through coding and interleaving in a flat-fading single-carrier system, we can introduce frequency diversity through coding and interleaving across subcarriers in an OFDMA system. However, since OFDMA in itself does not increase the system bandwidth it can never introduce frequency diversity on flat fading channels.

With coding and interleaving across subcarriers, the strong subcarriers help the weak ones as the block fading effect is decreased. Thus overall data detection performance is dependent on the ratio of strong part and weak part such as coding rate.

B. Bit Error Probability Analysis with Simple Bound

To compare the performance of the overlapped pilot pattern and the interlaced pilot pattern, we consider the simple bound derived in [8]. Here, we use (n, k) block code that maps each k information bits into n codeword bits.

Then the simple bound on the bit-error rate is given by

$$P_b(E) = \sum_{h=h_{min}}^{n-k+1} min \left\{ e^{-nE(c,h)}, e^{nr(\delta)} Q\left(\sqrt{2ch}\right) \right\}, \tag{18}$$

and the exponent of the simple bound is expressed by

$$E(c,h) = \begin{cases} \frac{1}{2} ln[1 - 2c_0(\delta)] + \frac{cf(c,\delta)}{1 + f(c,\delta)}, \\ \qquad\qquad \text{if } c_0(\delta) < c < \frac{e^{2r(\delta)} - 1}{2\delta(1-\delta)}, \\ -r(\delta) + \delta c, \quad \text{otherwise,} \end{cases}$$

with $\delta = \frac{h}{n}$, $r(\delta) \triangleq \frac{ln \sum_w [w/k] A_{w,h}}{n}$, $c_0(\delta) = \left(1 - e^{-2r(\delta)}\right) \frac{1-\delta}{2\delta}$, $c = \overline{\gamma}_b(k/n)$, and $f(c,\delta) = \sqrt{\frac{c}{c_0(\delta)} + 2c + c^2} - c - 1$. w is the input Hamming weight, h is the output Hamming weight, and $A_{w,h}$ is the input-output weight distribution. For (n, k) random block code, the code spectrum $A_{w,h}$ is known to follow a binomial distribution:

$$A_{w,h} = \frac{\binom{k}{w}\binom{n}{h}}{2^n}.$$

The average signal to interference noise ratio(SINR) per bit defined by

$$\overline{\gamma}_b = \frac{E[|HX|^2]}{KE[|Z|^2]}, \tag{19}$$

where K denotes the number of bits represented by one symbol.

For the overlapped and interlaced pilot patterns, the average SINR per bit defined by (19) becomes

1) Overlapped pilot pattern:

$$\overline{\gamma}_b = \frac{2\sigma_1^2}{2\left(\sigma_N^2 + \sum_{i \in \kappa} \sigma_{D,i}^2\right)}. \tag{20}$$

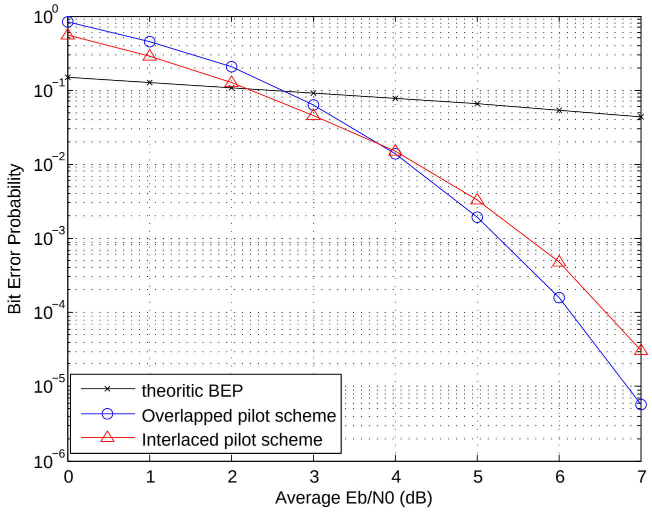

Figure 3. Pilot Patterns for 2 Users, pilot power boosting 4dB, pilot gap 20: (a) overlapped pilot pattern, (b) interlaced pilot pattern.

2) Interlaced pilot pattern:

$$\overline{\gamma}_{b,1} = \frac{2\sigma_1^2}{2\left(\sigma_N^2+\sigma_{P,j}^2+\sum_{i\in\kappa,i\neq j}\sigma_{D,i}^2\right)}, \tag{21}$$

$$\overline{\gamma}_{b,2} = \frac{2\sigma_1^2}{2\left(\sigma_N^2+\sum_{i\in\kappa}\sigma_{D,i}^2\right)}, \tag{22}$$

$$P_b(E) = \frac{P_{b,1}(E)+\epsilon P_{b,2}(E)}{1+\epsilon}, \tag{23}$$

where $\epsilon = \epsilon_D/\epsilon_P$, $\epsilon_D(\epsilon_P)$ is the number of symbols that affected by data(pilot) symbols of another users as an interference.

IV. SIMULATION RESULTS

For the simulation, we use (7,4) hamming code as an error correcting code. The channel estimation is performed by using LS estimator and the flat fading channel model is used.

Figures 3-7 give the BEP performance of two coded patterns using overlapped pilots and interlaced pilots for channel estimation, and uncoded BPSK in [3]. Eb/N0 is the ratio of Energy per Bit (Eb) to the Spectral Noise Density (N0). These results show that the BEP performance of the overlapped pilot pattern is relatively less susceptible to the number of users, but the BEP performance of the interlaced pilot pattern can be sensitive to the number of users, mainly due to the effect of interference.

The effect of pilot power boosting can be observed from Figure 3 and Figure 6. For the overlapped pilot pattern, the performance dose not improved as pilot power increase. The reason for this is that the pilot power of interference users are also increased. Also, Figures show that the region whose the performance of the interlaced pilot pattern is better than that of the overlapped pilot pattern is increased as the pilot gap and the number of user are increased.

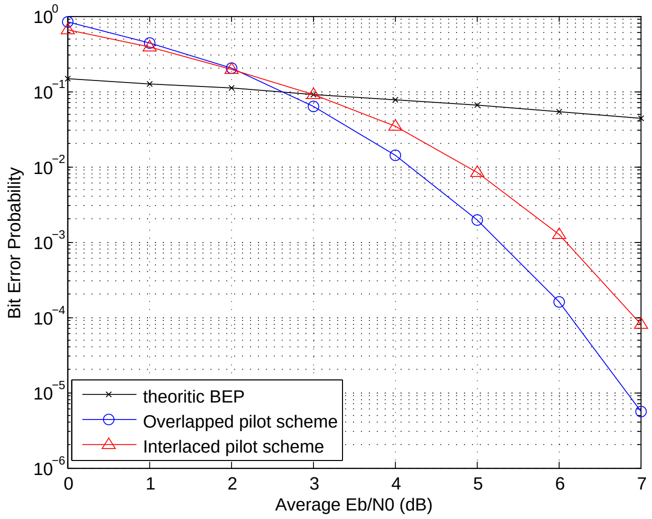

Figure 4. Pilot Patterns for 2 Users, pilot power boosting 4dB, pilot gap 8: (a) overlapped pilot pattern, (b) interlaced pilot pattern.

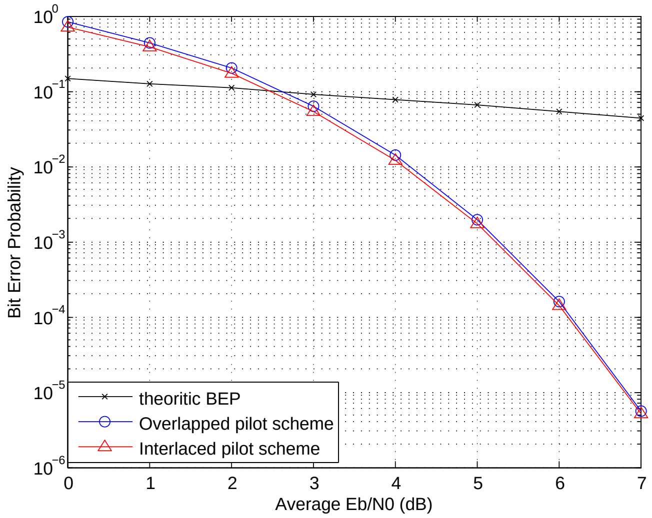

Figure 5. Pilot Patterns for 5 Users, pilot power boosting 4dB, pilot gap 8: (a) overlapped pilot pattern, (b) interlaced pilot pattern.

V. CONCLUSION

In this paper, we analyze the performance of the overlapped pilot pattern and the interlaced pilot pattern for channel estimation. This comparison is of special interest since the pilot pattern affects the performance of OFDMA systems. Simulation results in terms of BEP corroborate our theoretical analysis. We notice that the interlaced pilot pattern is more suitable for the multi-user networks like mesh network in which serious interference exists. We

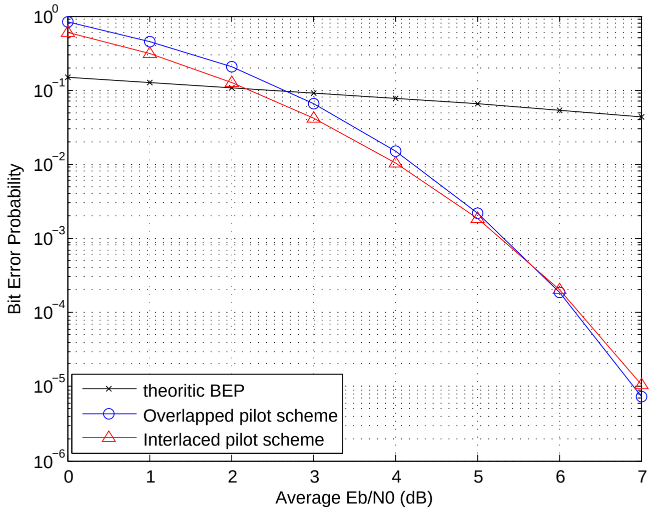

Figure 6. Pilot Patterns for 2 Users, pilot power boosting 3dB, pilot gap 20: (a) overlapped pilot pattern, (b) interlaced pilot pattern.

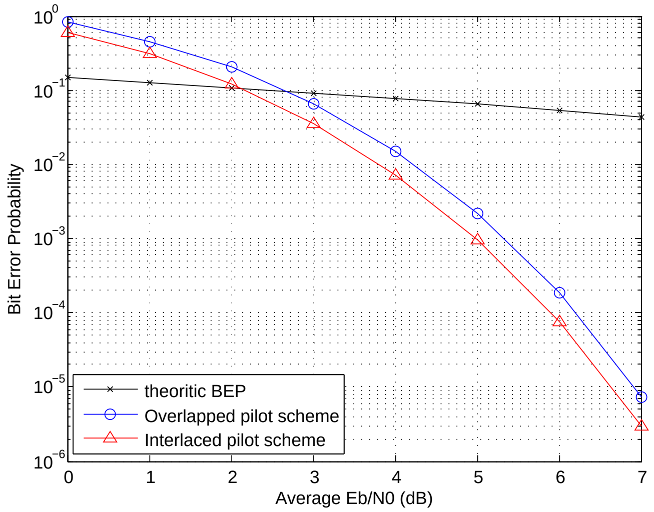

Figure 7. Pilot Patterns for 5 Users, pilot power boosting 3dB, pilot gap 20: (a) overlapped pilot pattern, (b) interlaced pilot pattern.

expect that the results obtained here can be directly applied to evaluate the performance of coded OFDMA systems in mesh network as well.

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