Finite-State Markov Chain Approximation for Geometric Mean of MIMO Eigenmodes

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***Abstract*—Time-varying nature of mobile fading channels can affect the design of wireless systems in many different aspects. This paper investigates the dynamic behavior of intrinsic spatial links in multiple-input multiple-output (MIMO) transceivers based on Geometric mean decomposition (GMD). In particular, we suggest using a finite-state Markov chain (FSMC) to model the fluctuations of both eigenmodes geometric mean and capacity for MIMO-GMD schemes in Rayleigh fading channels. In order to compute the transition probabilities analytically, a number of related statistical distributions are developed and employed. Simulation results have illustrated that FSMC can provide accurate approximations.**

Keywords-MIMO; Geometric mean decomposition; Rayleigh fading; Markov model.

I. INTRODUCTION

Spatial multiplexing-based multiple-input multiple-output (MIMO) transceiver scheme has been regarded as one of the key technologies to increase the spectral efficiency of future wireless communication systems. By using multiple antennas at both transmitter and receiver side, multiple independent data streams can be transmitted within the same time-frequency resource with appropriate signal processing techniques [1]. In some common MIMO transceivers such as schemes based on singular value decomposition (SVD) [2], the gains of multiple parallel spatial links are different, which may lead to high complexity in power distribution and bit-allocation algorithms in spatial domain. In order to reduce the complexity, it is possible to use the same modulation format on all subchannels. However, this may result in the degradation in error performance since the BER (bit error rate) is dominated by the sub-channel with the lowest power gain. In general, the minimum singular value of the MIMO channel matrix can affect the system performance in many ways [3].

An alternative MIMO transceiver architecture termed as Geometric Mean Decomposition (GMD) has been proposed by Jiang *et al.* [4], which is able to realize multiple parallel spatial links with identical gains from a MIMO channel. Specifically, the gains of all these parallel links are equal to the geometric mean of channel eigenmodes. Thus, bit and power allocation can be undertaken more conveniently as the modulation/coding scheme on all subchannels should be the same. In recent years, MIMO-GMD transceivers have received the attention from many researchers in both academia and industry. In particular, both LTE-A and WiMAX (802.16m), two of the most important standards for the next-generation wireless broadband, have considered MIMO-GMD in the standardization process [5], [6].

Nevertheless, to the best of our knowledge, statistical properties of the link gains realized by GMD in MIMO fading channels have not been characterized. In this paper, we are particularly interested in the dynamic behavior of the link gain and channel capacity in MIMO-GMD, which is important to the design of various adaptive transmission mechanisms. For instance, the adaptation/feedback rate for MIMO-GMD with adaptive modulation could be set in a more judicious manner if one could gauge the variation of link gain and capacity. On the other hand, successive interference cancelation (SIC) is usually used in the detection of a MIMO-GMD transceiver, in which the reliability of a data stream is dependent on previously-decoded data streams. If the receiver fails to detect a data stream successfully, error propagation is very likely to occur as the receiver is not able to correctly decode the data on the remaining streams. This might happen when the spatial link gain is too weak, and the characterization on channel variation may provide some crucial information on how frequently an error propagation event would occur.

Finite-state Markov chain (FSMC) is a commonly-used tool to model a time-varying process. It generally quantizes the process into multiple distinct states, and the time-varying behavior is characterized by the transition probabilities among these states. Some researchers have designed the feedback mechanisms for adaptive modulation systems with channels modeled by FSMC [7]. In this paper, FSMC is used to model both link gain and capacity processes of MIMO-GMD systems in fading channels. We provide several univariate and bivariate probability density functions (PDFs) of spatial links and capacities for a MIMO-GMD scheme in this paper, and demonstrate the methods of transition-

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probability computations using these distribution functions. Note that the univariate PDFs have already been derived in [8]. In order to construct FSMC, this paper applies the results obtained in [8], as well as extending the derivations for bivariate PDFs.

The remainder of the paper is organized as following: In Section II, the channel model for our analysis is described, and the concept of MIMO-GMD architecture and FSMC are briefly reviewed. In Section III, univariate PDFs are developed based on Gamma approximations. Then, bivariate PDFs for eigenvalue geometric mean and channel capacity are given in Section IV. In Section V, we calculate transition probabilities for the geometric mean and capacity for the FSMC. Finally, conclusions are drawn in Section VI.

II. Background

A. Channel Model and Assumptions

In this paper, we consider an (N_t, N_r) MIMO system with N_t and N_r antennas at transmitter and receiver respectively. The MIMO channel matrix, $\boldsymbol{H}$, is therefore an $N_r \times N_t$ matrix. Here we denote $m = \min(N_t, N_r)$, $n = \max(N_t, N_r)$ and $l = n - m$. Each of the channel entries is modeled as a complex Gaussian random variable with zero mean and unit variance (Rayleigh fading). The eigenvalues of the channel correlation matrix ($\boldsymbol{H}^\dagger \boldsymbol{H}$), represent the power gains of spatial links intrinsic to the SVD-based MIMO systems (the so called eigenmodes), and their joint statistics are governed by central Wishart distribution [9]:

$$f(\lambda_1, \ldots, \lambda_m) = \frac{\exp\left(-\sum_{i=1}^{m} \lambda_i\right)}{m! \prod_{i=1}^{m} (n-i)!(m-i)!} \times \prod_{i=1}^{m} \lambda_i^l \prod_{i<j} (\lambda_i - \lambda_j)^2. \quad (1)$$

On the other hand, we presume that the channel evolves over time in accordance to:

$$\boldsymbol{H}(t+\tau) = J_0(2\pi f_D \tau)\, \boldsymbol{H}(t) + \sqrt{1 - J_0(2\pi f_D \tau)^2}\, \boldsymbol{\Psi}, \quad (2)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function and f_D and τ are the Doppler frequency and time displacement, respectively. $\boldsymbol{\Psi}$ is a $N_r \times N_t$ random matrix with zero-mean complex Gaussian elements $\mathcal{CN}(0, 1)$. For simplicity, we denote the auto-correlation $\zeta = J_0(2\pi f_D \tau)$ in the rest of the paper.

B. Review of Geometric Mean Decomposition

The general input-output relationship of a MIMO transmission in flat-fading channel can be written as:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{z} \quad (3)$$

where $\boldsymbol{y}$ is an $N_r \times 1$ vector of received symbol, $\boldsymbol{x}$ is an $N_t \times 1$ vector of transmitted data, and $\boldsymbol{z}$ is a Gaussian noise vector with N_r elements. Based on (3), the authors of [4] have suggested to design MIMO transceivers using GMD, in which the MIMO channel is decomposed as:

$$\boldsymbol{H} = \boldsymbol{Q}\,\boldsymbol{R}\,\boldsymbol{P}^\dagger \quad (4)$$

where $\boldsymbol{P}$ and $\boldsymbol{Q}$ are semi-unitary matrices and $\boldsymbol{R}$ represents an $m \times m$ upper triangular matrix with identical diagonal elements. By pre-coding the data signal $\boldsymbol{x}$ by $\boldsymbol{P}$ at the transmitter, and multiplying $\boldsymbol{y}$ by $\boldsymbol{Q}^\dagger$ at the receiver, the transmitted signal can be extracted and decoded using the *nulling and cancelation* procedure as in BLAST schemes [1]. Hence, without considering the effects of error propagation, the data signals are sent on m subchannels. We denote the diagonal elements of $\boldsymbol{R}$ by g_σ and

$$g_\sigma = \left(\prod_{i=1}^{m} \sigma_i\right)^{\frac{1}{m}}. \quad (5)$$

where σ_i represents the i^{th} singular value of the MIMO channel $\boldsymbol{H}$. Hence, the sub-channel envelops are identical and are equivalent to the geometric mean of MIMO channel singular values. Remarkably, as m subchannels have identical gains, the performance of MIMO-GMD schemes does not suffer from "worst subchannel" problems as in some other transceiver architectures. Presuming that the transmission power is uniformly allocated on all subchannels, the resultant channel capacity is

$$C = m\,\log_2(1 + \gamma\, g_s^2) = m\,\log_2(1 + \gamma\, g), \quad (6)$$

where γ denotes the SNR on each sub-channel and

$$g = g_s^2 = \left(\prod_{i=1}^{m} \lambda_i\right)^{\frac{1}{m}} \quad (7)$$

is the sub-channel power gain, which is equivalent to the geometric mean of eigenmodes.

C. Finite-State Markov Chain

FSMC has been widely used for modeling time-varying processes in various contexts such as signal processing and wireless communications. In order to model the fluctuation of a random process using an FSMC, the process shall be firstly quantized into a number of discrete states. Then, the dynamic behavior is characterized by the transition probabilities among these states. In this paper, we only consider the first-order FSMC, in which the transition probability is solely dependent on the preceding state. In our case, we partition the processes of both eigenmode geometric mean and capacity based on their magnitudes. That is, we simply partition the random process of interest into S states by setting $S-1$ threshold levels. In the first state (denoted as $\mathcal{S}_0$), the process has a value smaller than the lowest threshold level T_0. In the final state ($\mathcal{S}_S$), the process is larger than the highest threshold level T_{S-1}. Otherwise, the process is said to be in the i^{th} state (denoted as $\mathcal{S}_i$) if its value falls in the range between threshold levels T_{i-1}

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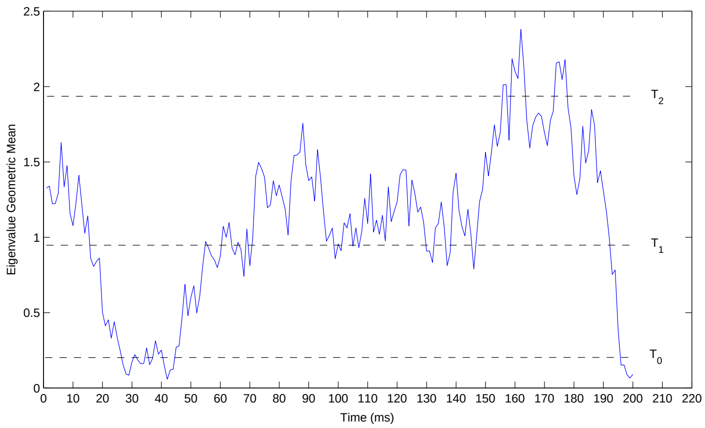

Figure 1. An example of a Markov state quantization. The time-varying eigenmodes geometric mean process in a (2,2) MIMO channel is partitioned into four states using three threshold levels: T_0, T_1 and T_2.

and T_i. Fig 1 shows an example in which the eigenmode geometric mean process is partitioned into four states. The main objective of this paper is to provide analytical methods of calculations for transition probabilities, which requires the univariate and bivariate statistical distributions for the target random processes (eigenmode geometric mean and capacity in MIMO-GMD schemes). We develop these distribution functions in subsequent sections.

III. Univariate Distributions

For MIMO-GMD schemes with two subchannels ($m = 2$), the exact PDFs for the eigenmodes geometric mean can be written as described in [8]:

$$f(g) = \frac{4\, g^{2(n-1)}}{(n-1)!(n-2)!} K_1(2\,g), \tag{8}$$

where $K_1(\cdot)$ represents the Modified Bessel Function of the Second Kind of order 1. Moreover, the closed-form PDF for the channel capacity in MIMO-GMD scheme with $m = 2$ has also been derived as:

$$f(C) = \frac{2\,\ln(2)\,\sqrt{2}^{C}\,\left(2^{\frac{C}{2}}-1\right)^{2(n-1)}}{\gamma^{(n-0.75)}\,(n-1)!\,(n-2)!} K_1\left[\frac{2}{\gamma}\left(2^{\frac{C}{2}}-1\right)\right]. \tag{9}$$

Nonetheless, for MIMO-GMD schemes with $m > 2$ subchannels, the PDFs for both eigenmode geometric mean and channel capacity are difficult, if not impossible, to derive due to high computational complexity. Fortunately, some previous studies have concluded that MIMO eigenmodes $(\lambda_1, \lambda_2, \ldots, \lambda_m)$ can be very accurately approximated by a Gamma random variables [10]. In addition, [11] claims that the geometric mean of multiple independent Gamma random variables could be either a Gamma or a mixture of Gamma distributions. Although MIMO eigenvalues are not independent processes, they are weakly correlated. So we may simply approximate the eigenvalue geometric mean g by Gamma distribution, the PDF which is widely-known.

Table I
Parameters of Gamma approximation for eigenvalue geometric mean, g

	(4,4)	(4,8)	(8,8)
$E(g)$	1.8075	6.0279	3.2150
$Var(g)$	0.5280	1.5306	0.5507
k	6.1880	23.7393	18.7687
θ	0.2921	0.2539	0.1713

Therefore, we may make a hypothesis stating that

$$f(g) \approx \text{Gamma}(k,\theta) = \frac{g^{k-1}\,\exp(\frac{-g}{\theta})}{\Gamma(k)\,\theta^k}, \tag{10}$$

where $k = \text{E}(g)/\theta$ and $\theta = \text{Var}(g)/\text{E}(g)$ are shape factor and scale factor of Gamma distribution, respectively. The computations for $\text{Var}(g)$ requires the first two moments of g as $\text{Var}(g) = \text{E}(g^2) - \text{E}(g)^2$. Note that the v^{th} moment of g can be calculated as:

$$\text{E}(g^v) = \int_0^\infty \cdots \int_0^\infty \left(\prod_{i=1}^{m} \lambda_i\right)^{\frac{v}{m}} f(\lambda_1, \ldots, \lambda_m)\, d\lambda_1, \ldots, d\lambda_m, \tag{11}$$

where $f(\lambda_1, \ldots, \lambda_m)$ has been given as (1). To make this paper self-contained, we have tabulated the numerical values of $\text{E}(g)$, $\text{Var}(g)$, k and θ for (4,4), (4,8) and (8,8) cases in Table I. Hence, one may construct the PDFs for g in these cases accordingly.

Utilizing Gamma approximation of (10), we can extend it to find the results on capacity PDF. With simple transformations, we have

$$\begin{aligned} f(C) &= \left|\frac{\partial\, g}{\partial\, C}\right| \times f(g)|_{g \to g(C)} \\ &= \frac{\ln(2)\, 2^{\frac{C}{m}}\, \left(2^{\frac{C}{m}}-1\right)^{k-1}}{m\,\gamma^k\,\Gamma(k)\,\theta^k} \exp\left(\frac{1-2^{\frac{C}{m}}}{\gamma\,\theta}\right). \end{aligned} \tag{12}$$

Here we justify the suitability of Gamma approximations for the eigenmode geometric mean via simulations. In Fig 2, we can see that Gamma distributions provide excellent approximation for eigenvalue geometric means. Although we did not show here, it has been found that Gamma distributions also fit the simulation data in $m = 2$ cases. Hence, we can claim that, for practical MIMO systems ($m \leq 8$) at least, the distributions of eigenmode geometric means can be approximated by Gamma random variables. For channel capacity, we compare the distributions of simulation samples with the capacity PDF (12), which was derived based on Gamma approximation of g. The comparisons are shown in Fig 3, and we can see that simulation data and computed results highly agree with each other.

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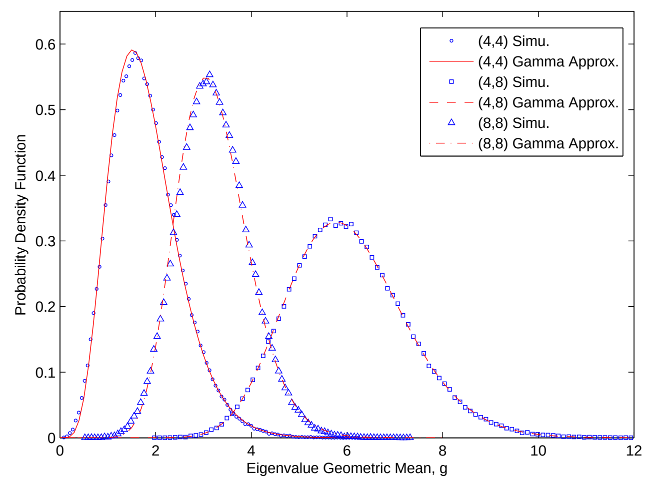

Figure 2. Comparison between simulation and Gamma approximation results for distributions of eigenvalue geometric mean (sub-channel power gain) in (4,4), (4,8) and (8,8) MIMO Rayleigh fading channels.

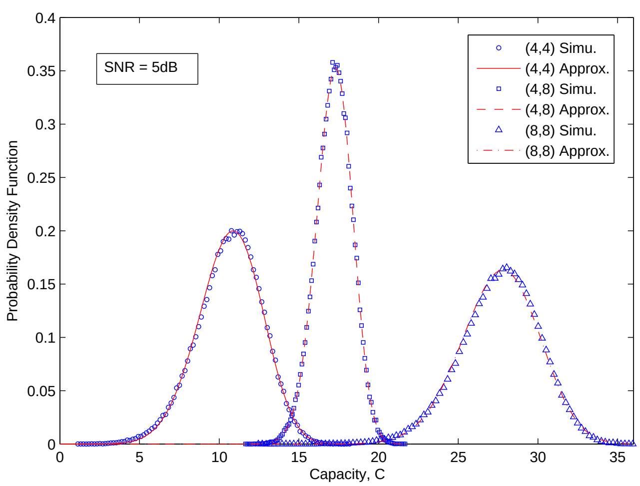

Figure 3. Comparison between simulation and calculation results for distributions of system capacity using GMD in (4,4), (4,8) and (8,8) MIMO Rayleigh fading channels. Assuming $\gamma = 5dB$.

IV. Bivariate Distributions

In the previous section, we have developed PDFs for the eigenmodes geometric mean and channel capacity based on Gamma approximations. In order to compute the transition probabilities for FSMC, the joint probability density for the process at two adjacent time points is also required. Since we have observed that the eigenmode geometric mean can be accurately approximated by a Gamma random variable, the joint density of eigenmode geometric mean at two correlated time instants, $g(t)$ and $g(t+\tau)$, could be described by the bivariate Gamma PDFs. Thus, by modifying the bivariate Gamma PDF in [12], we have:

$$f(g,\widehat{g}) \approx \frac{(g\,\widehat{g})^{\frac{k-1}{2}}}{\theta^{k+1}\Gamma(k)(1-\rho)\rho^{\frac{k-1}{2}}} \exp\left\{\frac{-(g+\widehat{g})}{\theta(1-\rho)}\right\} \times I_{r-1}\left(\frac{2}{\theta(1-\rho)}\sqrt{\rho\, g\, \widehat{g}}\right), \tag{13}$$

where $\widehat{g} = g(t+\tau)$ and ρ represents the correlation coefficient between g and $\widehat{g}$, which is defined as

$$\rho = \frac{\mathrm{E}[g(t)\,g(t+\tau)] - \mathrm{E}[g(t)]\,\mathrm{E}[g(t+\tau)]}{\sqrt{\mathrm{Var}[g(t)]\,\mathrm{Var}[g(t+\tau)]}} = \frac{\mathrm{E}(g\,\widehat{g}) - \mathrm{E}(g)^2}{\mathrm{Var}(g)}. \tag{14}$$

In (14), both $\mathrm{E}(g)$ and $\mathrm{Var}(g)$ can be calculated using (1). On the other hand, the computation for the joint moment, $\mathrm{E}(g\,\widehat{g})$, requires the joint density for eigenmodes at two time instants [10]:

$$f(\lambda_1,\ldots,\lambda_m,\widehat{\lambda}_1,\ldots,\widehat{\lambda}_m) = \frac{\prod_{i<j}[\frac{1}{\alpha^2}(\widehat{\lambda}_i-\widehat{\lambda}_j)]\prod_{i<j}(\lambda_i-\lambda_j)\prod_{i=1}^{m}\lambda_i^l}{\alpha^{2m}\prod_{i=1}^{m}[(n-i)!(m-i)!]} \times \exp\left(-\sum_{i=1}^{m}\lambda_i\right) \times G(\boldsymbol{\lambda}), \tag{15}$$

where

$$G(\boldsymbol{\lambda}) = \det\begin{bmatrix} f(\widehat{\lambda}_1|\lambda_1) & f(\widehat{\lambda}_2|\lambda_1) & \cdots & f(\widehat{\lambda}_m|\lambda_1) \\ f(\widehat{\lambda}_1|\lambda_2) & f(\widehat{\lambda}_2|\lambda_2) & & \vdots \\ \vdots & & \ddots & \vdots \\ f(\widehat{\lambda}_1|\lambda_m) & \cdots & \cdots & f(\widehat{\lambda}_m|\lambda_m) \end{bmatrix}$$

and

$$f(\widehat{\lambda}|\lambda) = \frac{\alpha^2}{\beta^2}\left(\frac{\widehat{\lambda}}{\alpha^2\lambda}\right)^{v/2}\exp\left[\frac{-\alpha^2\lambda-\widehat{\lambda}}{\beta^2}\right] I_l\left(2\frac{\alpha}{\beta^2}\sqrt{\lambda\widehat{\lambda}}\right)$$

with $\alpha = \zeta$, $\beta = \sqrt{1-\zeta^2}$ and $\zeta = J_0(2\pi f_D\tau)$ as before.

We can extend (13) to obtain the bivariate PDF for MIMO-GMD channel capacity process. Using Jacobian transform, $f(C,\widehat{C})$ can be written as

$$\begin{aligned} f(C,\widehat{C}) &= \det\begin{vmatrix} \frac{\partial g}{\partial C} & \frac{\partial g}{\partial \widehat{C}} \\ \frac{\partial \widehat{g}}{\partial C} & \frac{\partial \widehat{g}}{\partial \widehat{C}} \end{vmatrix} \times f(g,\widehat{g})|_{g\to g(C),\widehat{g}\to\widehat{g}(\widehat{C})} \\ &= 0.4805\,\frac{2^{\frac{C+\widehat{C}}{m}}\left[\frac{(2^{\frac{C}{m}}-1)(2^{\frac{\widehat{C}}{m}}-1)}{\gamma^2\rho}\right]^{(k-1)/2}}{m^2\,\gamma^2\,\Gamma(k)\,\theta^{k+1}\,(1+\rho)} \\ &\quad\times\exp\left[\frac{2-2^{C/m}-2^{\widehat{C}/m}}{\gamma\,\theta\,(1-\rho)}\right] \\ &\quad\times I_{k-1}\left[\frac{2\sqrt{\rho\,(2^{C/m}-1)\,(2^{\widehat{C}/m}-1)}}{\gamma\,\theta\,(1-\rho)}\right]. \end{aligned} \tag{16}$$

With PDFs given in Sections III and IV, the transition probabilities of FSMC can be computed.

V. Computations of Transition Probabilities

As aforementioned, we consider a first-order FSMC in this paper. Therefore, the transition probabilities from $\mathcal{S}_i$ to $\mathcal{S}_j$, denoted as $P_{i,j}$, for the eigenmode geometric mean process can be computed as

$$P_{i,j} = \frac{\text{Prob}(g(t) \in \mathcal{S}_i, g(t+\tau) \in \mathcal{S}_j)}{\text{Prob}(g(t) \in \mathcal{S}_i)}, \quad (17)$$

where

$$\text{Prob}(g(t) \in \mathcal{S}_i, g(t+\tau) \in \mathcal{S}_j) = \int_{\mathcal{S}_j} \int_{\mathcal{S}_i} f(g, \widehat{g})\, dg\, d\widehat{g}, \quad (18)$$

and

$$\text{Prob}(g(t) \in \mathcal{S}_i) = \int_{\mathcal{S}_i} f(g)\, dg. \quad (19)$$

Note that $f(g)$ could be (8) or (10), depending on the value of m. For sake of simplicity, we assume that the channel variation is slow enough so the process would only transit to one of the adjacent states (from $\mathcal{S}_i$ to $\mathcal{S}_{i-1}$ or $\mathcal{S}_{i+1}$) or stay in the same state (from $\mathcal{S}_i$ to $\mathcal{S}_i$) in this paper. Therefore, we have

$$P_{i,i} = 1 - P_{i,i+1} - P_{i,i-1}. \quad (20)$$

For channel capacity in MIMO-GMD scheme, the transition probabilities can be computed in a similar fashion. In order to verify that FSMC is an appropriate tool to model the fluctuation of MIMO-GMD channel, several Monte Carlo simulations have been carried out. In particular, simulation results on transition probabilities for both eigenmodes geometric mean and channel capacity are compared with our calculations by (17) and (20). Note that the threshold levels for state quantization are set arbitrarily in our simulations. In practical adaptive modulation schemes, for example, the threshold levels for state quantization could be set based on minimum-required channel gain for a target error performance. We set $f_D = 30$Hz, $\tau = 0.001$sec and $\gamma = 10$dB in all scenarios. In Fig 4, the eigenmode geometric mean is modeled as a FSMC consisting of four states with $\{T_0, T_1, T_2\} = \{1.5, 2.5, 3.5\}$, and transition probabilities from both simulations and calculations are plotted. Similarly, we approximate the MIMO-GMD capacity process in a (2,2) system using a four-state FSMC with $\{T_0, T_1, T_2\} = \{4, 7, 9\}$ in Fig 5. In both cases, it is apparent that our calculations can provide very accurate approximations.

VI. Conclusion

In this paper, we have aimed to model the channel variation in MIMO-GMD scheme using a FSMC. Since FSMC simply quantizes the process into multiple discrete states, which naturally fits to various applications in channel-dependent adaptive transmission. For example, in link adaptation mechanisms, each modulation and coding scheme usually corresponds to a range of channel gains, which can be deemed as a state in the FSMC. Additionally, we may also use FSMC of eigenmode geometric-mean to examine how likely the MIMO-GMD system would enter the state with high probability of error propagation. In order to calculate transition probabilities analytically, we have approximated the eigenmodes geometric-mean process by Gamma distribution, and developed associating bivariate PDFs based on such approximations. From the simulation results, it is apparent that our computations can give very accurate approximations. Hence, we can conclude that FSMC is

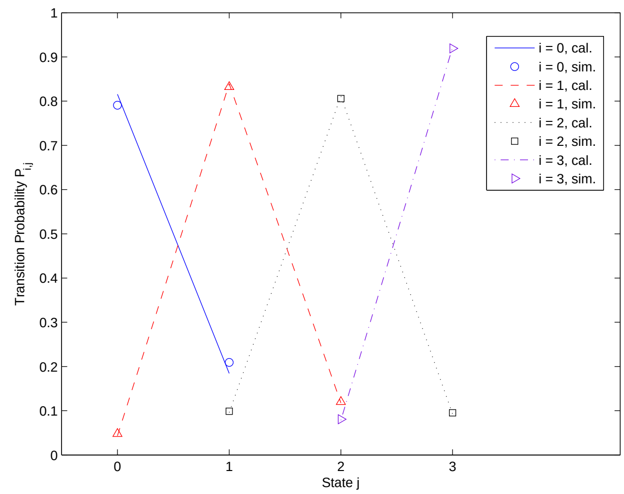

Figure 4. Comparison of transition probabilities for the eigenmodes geometric mean (with $f_D = 30$Hz, $\tau = 0.001$sec) in a (2,4) MIMO system obtained from calculations and simulations.

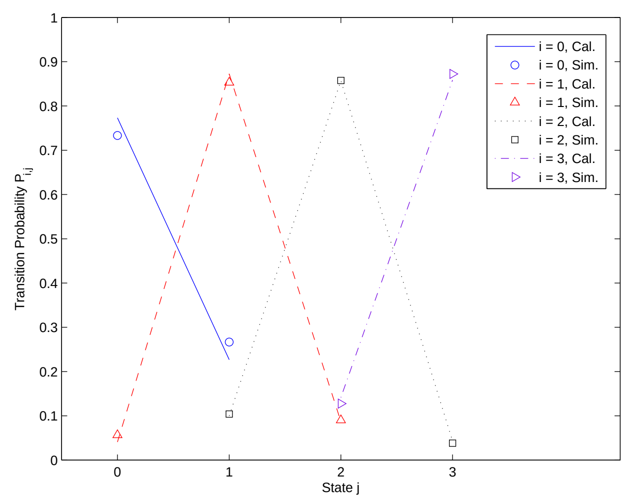

Figure 5. Comparison of transition probabilities for the MIMO-GMD channel capacity (with $f_D = 30$Hz, $\tau = 0.001$sec and $\gamma = 10$dB) in a (2,2) system obtained from calculations and simulations.

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a simple but effective approach to capture the dynamic behavior of a MIMO-GMD channel.

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