# Multi-Level Collaborative Spectrum Sensing in Nakagami Fading Channels

Omkalthoum El-Bashir Hamed
Department of Electrical Engineering
The Petroleum Institute
Abu-Dhabi, United Arab Emirates
e-mail: ohamed@pi.ac.ae

Mohammed Abdel-Hafez
Department of Electrical Engineering
United Arab Emirates University
Al-Ain, United Arab Emirates
e-mail: mhafez@uaeu.ac.ae

***Abstract*— This paper is to investigate the problem of spectrum scarcity and underutilization with particular attention to the performance of opportunistic spectrum access in fading channels. In this paper we studied the energy detector in collaborative and non-collaborative sensing modes when the channels between the primary and the sensors are generalized Nakagami-*m* fading channel. Soft combining techniques perform well enough, but require that each spectrum sensor sends complete signal information to the band manager. Sending the signal information introduces unnecessary complexity. Moreover, it is more complicated and time consuming for the band manager to handle. To reduce the communication overhead, hard decision techniques can be used. In this paper, two techniques will be studied. The first is the simple hard decision technique, and the second is the use of multi-threshold decision technique. The results of this study show that the multi-threshold technique outperforms the single one with slight increment in the cost. The performances of all these techniques are evaluated in terms of probability of false alarm and probability of detection. Although soft decision techniques give less probability of miss detection at certain value of probability of false alarm, the hard techniques are simpler to implement. It is also found that the multi-threshold works better than the single threshold especially in low SNRs.**

***Keywords - Spectrum sensing; opportunistic access; cognitive radio; Nakagami-m fading channel; square law combining; maximum selection combining; hard decision combining.***

## I. INTRODUCTION

The underutilization of the spectrum leads to thinking in managing the spectrum in more flexible way by allowing second level of spectrum usage. So, some users called “secondary users” are allowed to access spectrum holes, a band of frequency assigned to a primary user, but at a particular time and specific geographic location, the band is not being utilized by that user [1, 2]. Spectrum utilization can be improved significantly by making it possible for secondary users to access spectrum holes.

The telecommunication sector debates the reallocation of frequencies used for GSM, plans for digital TV switchover, formulates policies for cognitive radio and considers options for dealing with the wireless data explosion [3]. So, applying a sensing technique in the opportunistic wireless networks is needed. A simple sensing technique that can be used for this purpose is the energy detection. One of the simplest energy detectors is presented in [4,5]. This energy detector measures the energies of $2N$ samples of a received signal over a flat band-limited Gaussian noise channel. Then, it combines these Gaussian samples for comparison with a certain threshold. The result of the comparison can be defined by two hypotheses, either signal or no signal. Accordingly, the secondary user will decide on whether or not to access the spectrum band. Relying on chi-square statistics of the resulting sum of squared Gaussian random variables, Urkowitz [6] derived both probability of detection and probability of false alarm in Gaussian channel. Since we need to have enough protection for the primary user, we have to set the threshold such that it provides some protection level to the primary user. However it is found that one sensor cannot provide reliable sensing system specially in real fading channels. Collaborative sensing techniques had been studied in [8,9,10] with local energy detectors to improve the sensing system performance in fading channels. Zou et al. [8] investigated the effect of user collaboration in Rayleigh fading channel. It showed that using more collaborative users increases the performance significantly and improves the spectrum utilization. The researcher used the equal gain and maximum selection as soft decision combining techniques to combine the local measurements and finalize the decision. Although, soft combining techniques perform well, it consumes high bandwidth for sharing information. Simple hard decision combining technique was used by [8] where each user shares his vote with the controller using only binary 0 for empty and binary 1 for occupied band. Then, the controller makes the decision according to the collaborative users votes. As a comparison between the soft and hard decision combining techniques, soft techniques give much better decisions, but the cost is in the network overload by the overhead used to share their knowledge about the signal to noise ratios.

Yilmaz et al. [7] extended the work by applying a new combining technique which is collaborative sensing with a decision vector that uses a uniform quantization. It consists of multiple thresholds and a weight vector for global decision. Each operating secondary user should sense the channel locally and decide on one of the designed levels according to the measured signal to noise ratio. Then the secondary user should send his decision to the fusion center. The fusion center then makes final decisions according to the different users’ votes using a special weighted sum decision rule. This method performed better than the single threshold hard decision studied in [8] with little addition to the overhead. Moreover, as much as the number of levels increased, the decision becomes better and the overhead increases.

In [9], Liang et al. came up with a closed form solution for the probability of detection in Nakagami-$m$ channel with

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only integer fading parameters in non-collaborative mode. Since Nakagami-$m$ fading channel can have a non-integer fading parameter, and to our best knowledge, this hasn't been studied this before. Therefore we started the work by using new approach to study soft combining collaboration techniques in sensing in Nakagami-$m$ fading channel with any real parameter. In this paper, we extended the work of [11] to study the hard single and multi-thresholds combining techniques for collaborative sensing in Nakagami fading channel with any real fading parameters.

The rest of this paper is organized as follows: Section 2 presents the system model followed by the analysis of spectrum sensing in Additive White Gaussian Noise (AWGN) channel in Section 3. Section 4 addresses the spectrum sensing in Nakagami fading channel. Single threshold hard decision detection technique is introduced in Section 5. In Section 6, multi-levels hard decision technique is introduced and the effect of collaborative sensing is studied. Section 7 shows the results of some numerical examples. Concluding remarks are presented in Section 8.

## II. SYSTEM MODEL

Figure 1 shows the suggested model for sensing scenario in our opportunistic spectrum access system. Infrastructure-based sensors are distributed in the model to sense the primary user signal in a certain band. The channel between the sensor and the use is assumed to be generalized Nakagami-$m$ fading channel with instantaneous signal to noise ratio $\gamma$. The band manager at data fusion center will then decide based on the information reported from $k$ sensors with one of combining techniques will be studied in this paper.

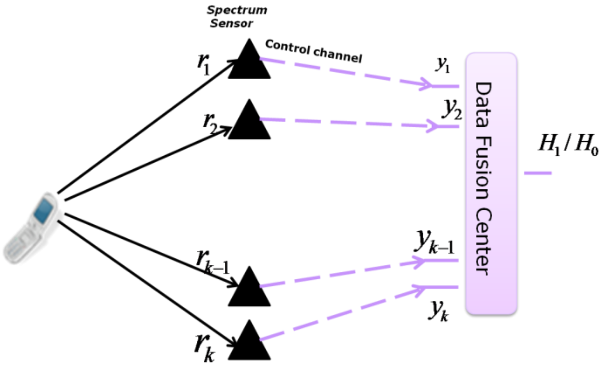

Figure 1. System model.

## III. SPECTRUM SENSING IN NON-COLLABORATIVE MODE IN AWGN

AWGN channel is the ideal case of wireless channel where the noise is only due to the additive noise at the receiver. The performance in AWGN channel is studied in terms of the probability of detection and probability of false alarm. The probability of detection, $P_d$, and probability of false alarm, $P_f$, in AWGN channel were studied in [6]. Then it is revisited by [9] and studied for the sampled version of the signal. It is found that these probabilities can be expressed as,

$$P_d = Q_N\left(\sqrt{\frac{2\gamma}{\sigma^2}}, \sqrt{\frac{\lambda}{\sigma^2}}\right) \tag{1}$$

and

$$P_f = \frac{\Gamma(N, \lambda/2\sigma^2)}{\Gamma(N)} \triangleq G_N(\lambda) \tag{2}$$

where $\gamma$ is the $SNR$, $\sigma^2$ is the variance of the channel. For simplicity and without loss of generality, $\sigma^2$can be assumed to be unity, $\Gamma(.,.)$ is incomplete gamma function [12], $N$ is the half number of samples. $Q_N(.,.)$ is the Generalized Marcum $Q$ function [13].

## IV. SPECTRUM SENSING IN NAKAGAMI FADING CHANNELS

This section focuses on the performance of spectrum sensing in the Nakagami fading channel. The performance of spectrum sensing in the Rayleigh fading channel is a special case when fading parameter $m = 1$. The performance is formulated in terms of the probability of detection and probability of false alarm.

For the Nakagami fading channel, the $pdf$ of the signal to noise ratio $SNR$, $\gamma$, has the following gamma distribution,

$$f_\gamma(\gamma) = \frac{1}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1}\exp\left(\frac{-m}{\bar{\gamma}}\gamma\right), \gamma \geq 0 \tag{3}$$

where $\bar{\gamma}$ is the average signal to noise power ratio in the fading channel and $m$ is the Nakagami fading parameter.

The probability of false alarm is independent of $\gamma$ because it is the probability of the received energy being above the threshold with the absence of the primary user. Since, under $H_0$, no primary user's signal exists, $P_f$ is not affected by fading. On the other hand, the probability of detection over Nakagami fading channel, $P_{d,Nak}$, can be found by averaging (1) over (3) as,

$$P_{d,Nak} = \frac{1}{\Gamma(m)}\int_0^\infty Q_N\left(\sqrt{(2\gamma)}, \sqrt{\lambda}\right)\left(\frac{m}{\bar{\gamma}}\right)^m . \gamma^{m-1}\exp\left(\frac{-m}{\bar{\gamma}}\gamma\right) d\gamma \tag{4}$$

This integration can be simplified using the following steps: Let $x = \sqrt{(2\gamma)}$, $b = \sqrt{\lambda}$, and $p^2 = \frac{m}{\bar{\gamma}}$, then (4) can be expressed as,

$$P_{d,Nak} = \frac{2}{\Gamma(m)}\left(\frac{p^2}{2}\right)^m \int_0^\infty Q_N(x,b)\, x^{2m-1} . \exp\left(-\frac{p^2x^2}{2}\right) dx \tag{5}$$

Marcum $Q$-function defined by [12] is used and defined by,

$$Q_N(x,b) = \int_b^\infty \frac{\alpha^N}{x^{N-1}}\, e^{-\left(\frac{x^2+\alpha^2}{2}\right)} I_{N-1}(\alpha x)\, d\alpha \qquad (6)$$

where α is a dummy variable. After some manipulations and using the formulas in (p720 and p1059, [15]), the probability of detection in Nakagami channel can be expressed as,

$$P_{d,Nak} = \frac{2^{-N+1}}{\Gamma(N)} \left(\frac{p^2}{1+p^2}\right)^m \cdot \int_b^\infty \alpha^{2N-1}\, e^{-\frac{\alpha^2}{2}}\, \varphi\left(m, N, \frac{\alpha^2}{2(1+p^2)}\right) d\alpha \qquad (7)$$

where $\varphi(.,.,)$ is the degenerate hyper geometric function defined in [12].

The probability of miss detection, can be expressed as,

$$P_{m,Nak} = 1 - P_{d,Nak} \qquad (8)$$

or

$$P_{m,Nak} = \frac{2^{-N+1}}{\Gamma(N)} \left(\frac{p^2}{1+p^2}\right)^m \cdot \int_0^b \alpha^{2N-1}\, e^{-\frac{\alpha^2}{2}}\, \varphi\left(m, N; \frac{\alpha^2}{2(1+p^2)}\right) d\alpha \qquad (9)$$

reverting to the original terms and constants in (9), we arrive at the following,

$$P_{m,Nak} = \frac{2^{-N+1}}{\Gamma(N)} \left(\frac{m}{\bar{\gamma}+m}\right)^m \cdot \int_0^{\sqrt{\lambda}} \alpha^{2N-1}\, e^{-\frac{\alpha^2}{2}}\, \varphi\left(m, N; \frac{\alpha^2\bar{\gamma}}{2(\bar{\gamma}+m)}\right) d\alpha \qquad (10)$$

The integration in (10) is limited and can be evaluated easily by using the Monte Carlo integration method [16]. The advantage here is that $P_{m,Nak}$ can be evaluated for (integer and non-integer) fading parameters $m$. At this point, the probability of miss detection in Rayleigh fading channel can be found by simply setting $m = 1$ as special case. This approach of finding the probability of miss detection in Nakagami-$m$ fading channel is suggested and confirmed by comparing its results with some results in the literature. Digham and Alouini [4] found a closed form formula for the average probability of detection in Nakagami for only integer values of $m$ values.

## V. SINGLE THRESHOLD HARD DECISION TECHNIQUE

Using this technique, the spectrum sensor sends only one bit information as an individual decision. It sends 0 if the locally detected signal energy is less than the threshold to decide on $H_0$. Otherwise, it sends 1 to decide on $H_1$. Then, the band manager finalizes the decision using votes according to the "$n$ out of $k$" rule, where $n$ is the required number of voters necessary to decide on the existence of the primary signal.

Given that all the sensors are independent, and applying the Neyman-Pearson criterion (which is based on fixing the probability of false alarm to an acceptable value to find a test threshold that maximizes the probability of detection), results in the following combining rule [5]

$$\sum_{i=1}^{n} S_i \log_e \left[\frac{P_{d_i}\left(1-P_{f_i}\right)}{\left(1-P_{d_i}\right)P_{f_i}}\right] \lessgtr_{H_0}^{H_1} \Lambda \qquad (11)$$

where $S_i$ is the $i^{\text{th}}$ sensor decision. $P_{d_i}$ and $P_{f_i}$ are the individual probabilities of detections and false alarm, respectively. The band manager decides by comparing the weighted sum of the individual decisions to a threshold $\Lambda$; where $\Lambda$ is a global threshold with discrete value in this technique. In this study, iid sensors are assumed to simplify the analysis. So, $P_{d_i}$ and $P_{f_i}$ are assumed to be equal for all the sensors as a result of identical path loss and fading. It is also assumed that all the users imply the same threshold λ in their local decision for simple implementation. Thus based on the other chosen global threshold $\Lambda$, the data fusion center implements an "$n$ out of $k$" voting rule. It decides $H_1$ if $n$ or more vote to $H_1$, otherwise, it will decide on $H_0$. The average probabilities of detection and false alarm for the $n$ out of $k$ rule are related to their single user probabilities through binomial distribution. The AND and the OR decision rules are considered as special cases from the general $n$ out of $k$ rule. By using AND rule, the band manager will decide on $H_1$ if all the sensors agree on deciding on the primary user existence. On the other hand, by the OR rule, the band manager will decide on $H_1$ when at least one sensor has decided locally on the primary user existence [5]:

$$P_{f_{Nak,HD}} = \sum_{i=n}^{k} \binom{k}{i} P_f^{\,i} \left(1-P_f\right)^{\mathrm{k-i}} \qquad (12)$$

and

$$P_{d_{Nak,HD}} = \sum_{i=n}^{k} \binom{k}{i} P_{d,nak}^{\,i} \left(1-P_{d,nak}\right)^{\mathrm{k-i}} \qquad (13)$$

where $P_{d,Nak}$ and $P_f$ are the individual probabilities of detection and false alarm as defined by (7) and (2), respectively.

## VI. MULTI-THRESHOLD HARD DECISION TECHNIQUE

The simple hard decision algorithm introduced in Section 5 was based on single threshold. In the single threshold detection method, the decision $H_0$ or $H_1$ depends only on one local threshold λ. $P_d$ and $P_f$ for a single secondary user can be calculated using λ for a selected channel model and using the exact formulas given by (12) and (13).

Figure 2 shows four-thresholds as an example of the method proposed by [7], namely, $\lambda_0$ , $\lambda_1$ , $\lambda_2$ and $\lambda_3$. The distance between the center thresholds $\lambda_1$ and $\lambda_2$ and the other thresholds is fixed and is equal to Δ, and there is a $\Delta_c$ distance among the center thresholds themselves. Each sensor determines the quantization bin from the bins vector $[B_0, B_1, B_2, B_3, B_4]$ locally according to its measurement and the thresholds given. For example, if the measured energy value is between the values of $\lambda_0$ and $\lambda_1$ , the sensor will decide on $B_1$ bin. When the measurement is in the region between thresholds $\lambda_1$ and $\lambda_2$ the sensor shouldn't send its decision. This technique censors some sensors from sending their information because of its low importance. The measurements in the "no decision" region are not important because it is in the middle of the range. In other words, it is not high enough to vote for the primary user presence, nor low enough to vote for its absence. The idea behind censoring is avoiding overloading the band with unnecessary data by having sensors send their decision to the fusion center only if this decision is considered to be "informative" Or, only if they are sure enough about it.

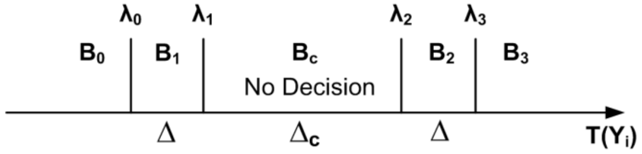

Figure 2. Multi threshold Energy detector with four thresholds.

The sensors send their softened decisions or quantized measurements in 2-bit formats for 4-threshold case and 3-bit formats for 8-threshold cases and so on. The fusion center gives a weight vector to the quantization bins. So, deciding on each bin has special weighting determined by the fusion center to change the rule of the decision used. For example $\vec{w} = [-1\ -1\ 0\ 1\ 1]$ is equivalent to the majority rule. The fusion center receives the softened decisions and counts the number of users in each quantization bin and forms a vector $\vec{B}$ that lists how many sensors reported in each bin. Then, if the inner product of the two vectors $\vec{w}$ and $\vec{B}$ is > 0, $\delta_{\vec{w}}(\vec{B})$ is considered 1, otherwise, it will be considered as 0.

To quantify the performance of this method, probability of detection and probability of false alarm are calculated using the formulas below [7]:

$$P_d = \sum_{\vec{B}\in\beta} \delta_{\vec{w}}(\vec{B})\, C(k,n_0)C(k-n_0,n_1)C(k-n_1-n_0,n_2)$$
$$C(k-n_2-n_1-n_0,n_3)\left(P_{B_0,H_1}\right)^{n_0}\left(P_{B_1,H_1}\right)^{n_1}\left(P_{B_c,H_1}\right)^{n_c}$$
$$\left(P_{B_2,H_1}\right)^{n_2}\left(P_{B_3,H_1}\right)^{n_3} \tag{14}$$

and

$$P_f = \sum_{\vec{B}\in\beta} \delta_{\vec{w}}(\vec{B})\, C(k,n_0)C(k-n_0,n_1)C(k-n_1-n_0,n_2)$$
$$C(k-n_2-n_1-n_0,n_3)\left(P_{B_0,H_0}\right)^{n_0}\left(P_{B_1,H_0}\right)^{n_1}\left(P_{B_c,H_0}\right)^{n_c}$$
$$\left(P_{B_2,H_0}\right)^{n_2}\left(P_{B_3,H_0}\right)^{n_3} \tag{15}$$

where $\beta$ represents all combinations of number of users distributed in quantization bins, $C(k;\, n)$ represents $n$ combinations out of $k$, $n_i$ represents the number of users in $B_i$ , and $P_{B_i,H_j}$ represents the probability of the received energy being in $B_i$ conditioned on $H_j$ and under AWGN channel. Similar formulas can be obtained for fading channels by calculating local probabilities according to fading channel formulas. This model is used in [7] to evaluate ROC in AWGN and in Rayleigh channels. In this thesis, only a special case of this model is studied and applied as per the Nakagami-$m$ channel with any real fading parameter.

## VII. NUMERICAL RESULTS

Figure 3 shows the Complementary Region Of Convergence (CROC) for combined $iid\ k$ spectrum sensors in Nakagami fading channel with average $SNR = 20\ dB$ and fading parameter $m = 1.8$ . The figure shows the performance improvement when using more than one sensor to detect the channel when the hard decision combining technique is used. The decision rule is based on "$n$ out of $k$" rule. Figure 3 shows the OR rule has the least probability of miss detection at a certain value of probability of false alarm among all the other values of $n$. The AND rule has the most probability of detection at a certain value of probability of false alarm among the other voting rules. This means that by using the OR rule, we can guarantee a better level of QoS for the primary user. However, the down side is degradation in utilization. Therefore, the AND rule gives better utilization, but it decreases the QoS of the primary user. Performance of all other "$n$ out of $k$" schemes is in between the two extreme cases, the OR and the AND rules. The network designer should be aware of the required level of primary user QoS and the additional utilization required in deciding which rule to use.

By comparing this technique with the soft decision technique presented in [11], we can consider the performance of the least probability of miss detection case (OR curve) with the soft decision cases when the number of collaborated users is $k = 4$ . In general, soft decision technique outperforms hard decision but hard decision technique is much simpler. The complexity of the soft decision combining techniques arises from different factors. The first is implementation needs; for example MSC needs channel gain estimation. Moreover, in the soft decision combining technique, the process of gathering information from sensors is complicated. In addition, the sensors share their measurements rather than their decisions with the fusion center. This needs more bandwidth to carry all the information, especially when there is large number of sensors. This will affect the utilization of the scarce spectrum, because the hard decision uses only one bit (0 or 1) to report its final decision to the band manager, after which the band manager simply applies one of the discussed voting rules to finalize the decision. So, a very simple receiver can carry out the process of finalizing the decision.

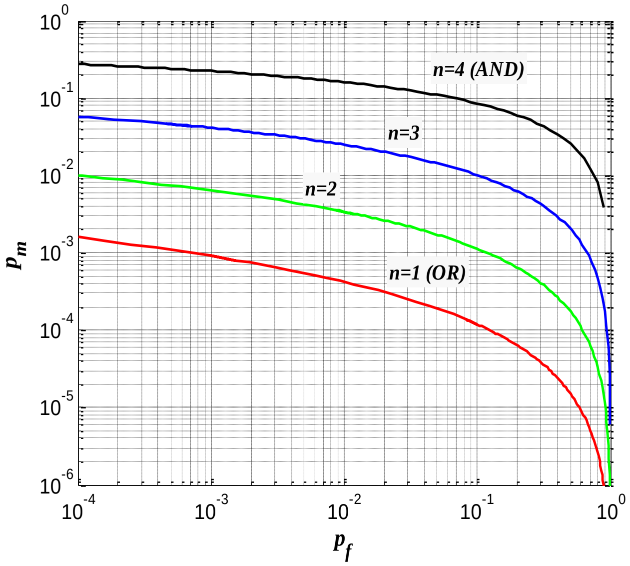

Figure 3. CROC for diversity in Nakagami for $k = 4$ iid sensors using Hard Decision combining technique for average $SNR = 20dB, N = 5$, and $m = 1.8$.

Figure 4 shows the effect of the number of collaborated sensors on detection performance. In this figure, $k = 8$ iid sensors are used. The figure shows "$n$ out of $k$" voting rule with especial cases OR, and AND. The improvement in the performance due to larger number of collaborative sensors is very clear in this instance. So, to get good detection without degrading the utilization, the designer can use more sensors and use any of the $n$ out of $k$. For example, instead of using the $4$ out of $4$ rule, we can use the $4$ out of $8$ rule to get better performance without degrading the utilization. The cost of this improvement is the cost of the extra sensors and a slight addition in band consumption (1 bit/sensor).

Figure 5 shows the CROC curve for the single and 4-threshold Hard Decision combining technique model system. The figure is generated for the two cases with $k = 5$ collaborative sensors, average $SNR = 20\ dB$, and majority rule. The 4-threshold majority rule is chosen with a weighting vector $\vec{w}$ =[-1 -1 0 1 1]. For the single threshold, the majority rule is considered when $n = 3$ in the "$n$ out of $k$" rule and is compared with the performance of the single threshold hard decision combining technique. It is found that this method significantly outperforms the single threshold hard decision technique. The cost of this improvement is only a slight increment in the overhead coming from sending two bits instead of one bit for each sensor.

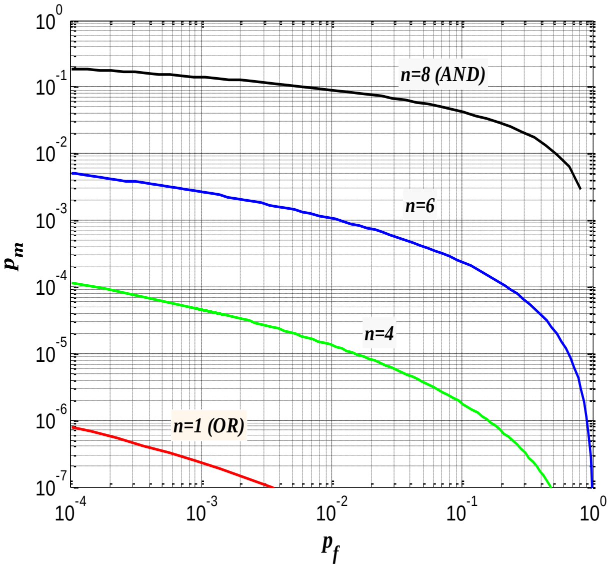

Figure 4. CROC for diversity in Nakagami for $k = 8$ iid sensors using Hard Decision combining technique for average $SNR = 20dB, N = 5$, and $m = 1.8$.

This combining technique can be considered to be the best among all the combining techniques studied, because it gives the designer the chance to make a trade-off between the cost and the detection accuracy of the system. Then accordingly, the number of collaborative sensors and number of thresholds can be chosen to fit the need.

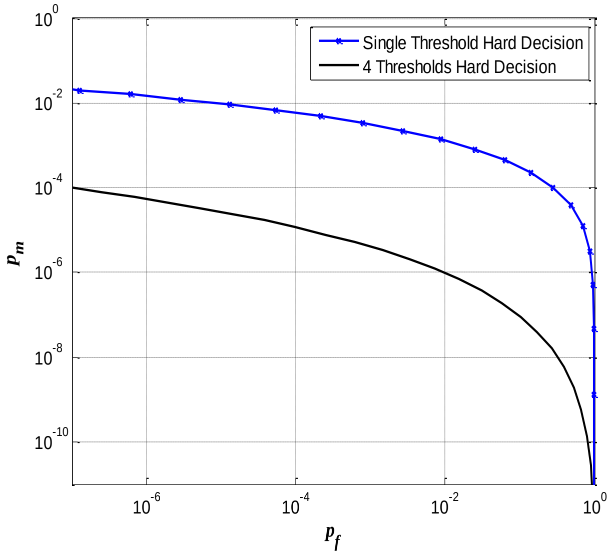

Figure 5. CROC for 4 thresholds hard decision combining technique compared to the single threshold, majority rule in the two cases,average $SNR = 20\ dB, m = 1.8, \Delta = 3, \Delta_c = 4, k = 5$.

## VIII. CONCLUSION AND FUTURE WORKS

In conclusion, collaborative spectrum sensing model is studied in this paper when the sensing channel is a general Nakagami fading channel. Hard decision combining techniques were used to combine the collaborated sensors signal where each sensor shares only one or few bits to represent its sensing results. Single and multi-thresholds techniques are considered. Results show that both of them improved the system sensing performance significantly. It is also found that the multi-threshold works better than the single threshold especially in low SNRs. Results were compared to the other soft decision techniques used in the literature. It is found that soft techniques perform better but it has high fixed cost. On the other hand, the multi-level hard techniques are simpler to implement and have the flexibility tradeoff between the performance and the cost according to the application and the channel type.

## REFERENCES

[1] http://www.portiodirect.com/productDetail.aspx?pid=49$55$51$431, Portio Research Mobile Factbook 2009, retrieved: April, 2012.

[2] http://www.sharedspectrum.com, retrieved: April, 2012.

[3] Fedral Communication Commission, “Spectrum Policy Task Force,” Rep. ET, Docket No. 02-135, Nov. 2002.

[4] F. F. Digham, M.-S. Alouini, and M. K. Simon, “On The energy Detection of Unknown Signals over Fading Channels,” IEEE Transactions on Communications, vol. 55, no.1, pp. 21-24, January 2007.

[5] A. Ghasemi and E. Sousa, “Opportunistic Spectrum Access in Fading Channels Through Collaborative Sensing,” Journal of communications, vol. 2, no. 2, pp. 71-82, March 2007.

[6] H. Urkowitz, “Energy Detection of Unknown Deterministic Signals,” Proc. IEEE, vol. 55, pp. 523-531, April 1967.

[7] H.Birkan Yilmaz, Tuna Tugcu, and Fatih Alagoz, “Uniform Quantizer for Cooperative Sensing in Cognitive Radio Networks”, PIMRC 2010, September 2010.

[8] Q. Zou, S. Zheng, and A. H. Sayed, “Cooperative Spectrum Sensing Via Coherence Detection,” 15th workshop on statistical signal processing, IEEE/SP, pp. 610 – 613, Aug. 31 2009-Sept. 3 2009.

[9] Y.-C. Liang, Y. Zeng, T. Hoang, and E. Peh, “Sensing-Throughput Trade-off for Cognetive Radio Networks,” IEEE Trans. on wireless communications, vol. 7, no. 4, pp. 1326-1337, April 2008.

[10] A. Ghasemi and E. Sousa, “Fundmental Limits of Spectrum-Sharing in Fading Environments,” IEEE Trans. on wireless communnication, vol. 6, no. 2, pp. 649-658, Feb. 2007.

[11] O. Al-Bashir and M. Abdel-Hafez, “Opportunistic Spectrum Access Using Collaborative Sensing in Nakagami-m Fading Channel with Real Fading Parameter,” 7th International Wireless Communications and Mobile Computing Conference (IWCMC), pp. 472 - 476, July 5-8, 2011, Istanbul, Turkey.

[12] A. H. Nuttal,“Some Integral Involving the $Q_M$ -Function,” IEEE Trans. Inf. Theory , vol. 21, no. 1, pp. 95-96, Jan. 1975.

[13] A. Papoulis, “Probability, Random Variables, and Stochastic Processes,” McGraw-Hill Europe; 4th edition (January 2002).

[14] C. Shannon, “A mathmatical theory of communication,” Bell Systems Technical Journal 1948, 27, pp. 623-656.

[15] I. S. Gryadshteyn and IM. Ryzhik, “Table of Integrals, Series, and Products,” 5th Edition. San Diego: Academic, 1994.

[16] W. Tranter, K. Shanmugan, T. Rappaport and K. Kosbar, “Principles of Communication Systems Simulation with Wireless Applications,” Prentice Hall, 2004.