Performance Improvement of Differential Codebooks with Noisy Feedback Channels

Xun Li, NoeYoon Park and YoungJu Kim
College of Electrical and Computer Engineering
Chungbuk National University
Chungbuk 361-763 Rep. Korea
Email: {lixun, nypark, yjkim}@cbnu.ac.kr

***Abstract*—In this paper, a differential codebook indexing scheme is proposed for limited feedback system over noisy feedback channels. A lot of research has been done for limited feedback system assuming error free feedback. In practical systems, the feedback information experiences noisy feedback channels which cause feedback information partially all totally useless. Prior research about differential precoding focuses on the codebook design criterion which minimize the quantization distortion. The proposed scheme focuses on how to minimize the effect due to feedback errors. The relationship of feedback errors and limited feedback system performance is analyzed in this paper. Using the analytical results, an optimal differential codebook indexing scheme is proposed to improve the system performances when the feedback bits less than 3 bits and exceed 4 bits, respectively. From some selected numerical results, the proposed differential codebook indexing scheme provides non-negligible performance improvements in terms of average bit error rate than the systems without indexing.**

Keywords-Indexing, Differential codebook, Temporal correlation, Limited feedback.

I. INTRODUCTION

Transmit beamforming for multiple-input multiple-output (MIMO), which is also known as precoding, has been widely adopted in wireless communication standards (WiMAX, 3GPP-LTE [1]). It uses some type of quantized channel state information (CSI) at the transmitter to offer good trade-off between performance gain and the required amount of feedback bits [2]–[4]. The accuracy of CSI at the transmitter depends on the feedback bits used. For block to block fading channel model, the channel realization is considered to change independently. But in low mobility scenarios, the temporal correlation always existed between adjacent channel realizations. Quantized differential feedback improves the quantization resolution utilizing the temporal correlation of the channels [5], [6]. In temporally correlated channels, the channel realization is changed slowly, as well as the optimal precoder. Thereby, quantizing the whole channel space is waste the feedback resource. Quantized differential feedback scheme quantizes the specific channel subspace instead of whole space, and the codebook for current time instant is various over time and depends on the previous precoder and the channel long term statistic [7]–[9]. The proposed schemes indicate that quantized differential feedback scheme can greatly improve the system performance.

In prior researches, the feedback channel is assumed to be error free and delay free for simplicity. In this case, the indexes are arbitrarily assigned to the set of codewords. However, the feedback error cannot be avoided although many techniques (lower modulation order, high channel coding redundancy, etc.) are used in feedback transmission [1]. The feedback errors causes that the transmitter applies precoding with undesired precoder. The effects of feedback error to the performance of general codebook based precoding system have been analyzed, and the principle of codebook index algorithms have been proposed in [10], [11]. And index assignment scheme for beamforming system are proposed to minimize the effects from feedback errors. These algorithms demonstrated the procedures of codebook index which have no consideration of computation complexity.

In this paper, a complex reduced codebook index algorithm is proposed when the feedback information is more than 4 bits. The proposed algorithm can be realized with low complexity circuit. Also, the index assignment scheme is applied in quantized differential feedback system. The differential precoding system with proposed codebook index algorithms effectively lowers the error floor introduced by the feedback errors. We analyze the effects of feedback errors to the limited feedback system, the performance of proposed scheme is evaluated and compared with the performance of the long term evolution (LTE) codebook with or without noisy feedback channels, respectively. In order to make the index assignment is applicable when the number of feedback bits is more than 4 bits, a suboptimal index assignment scheme is proposed which shows more flexible trade-off between performance and calculation complexity. Without loss of the generality, we compared the performance of the LTE codebook with the differential codebook proposed in [8], which preserves per-antenna equal power constraint property like the former codebook.

The rest of this paper is organized as follows: Section II shows the system overview of general limited feedback system and quantized differential feedback system. Section III introduces the index assignment schemes in codebook based limited feedback systems. Section IV illustrates the application of index assignment scheme to the quantized differential feedback system. And the simulation results are shown in this part. Finally, the conclusions are shown in

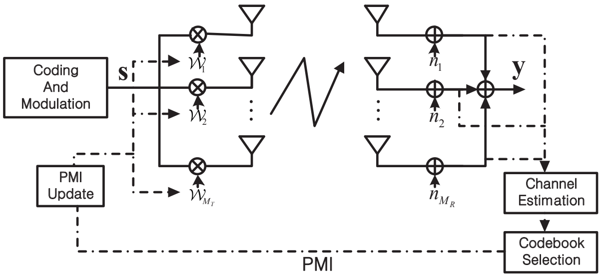

Figure 1. Block diagram of limited feedback system.

Section V.

II. SYSTEM OVERVIEW

In this section, the limited feedback system and quantized differential feedback system are introduced. The codebook design criterion is also introduced.

A. Limited Feedback System

A MIMO system employing M_t transmit antennas and M_r receive antennas is assumed in this paper. The block diagram is shown in Fig. 1. The transmit symbols at the time instant τ are denoted by $\mathbf{s}_\tau = [s_{\tau,1}, \cdots, s_{\tau,V}]^T$, where V denotes the number of data streams (also called transmission rank), and $1 \le V \le min\{M_t, M_r\}$. The received signal is represented by

$$\mathbf{y}_\tau = \sqrt{\frac{\rho}{V}} \mathbf{H}_\tau \mathbf{F}_\tau \mathbf{s}_\tau + \mathbf{n}_\tau, \tag{1}$$

where ρ is signal-to-noise ratio (SNR), $\mathbf{F}_\tau \in \mathbb{C}^{M_t \times V}$ denotes the precoder at time instant τ. Without loss of generality, we assume that each column of $\mathbf{F}$ is normalized. $\mathbf{n}_\tau$ denotes the additive white Gaussian noise (AWGN) vector at time instant τ with distribution of $\mathcal{CN}(0,1)$. The matrix $\mathbf{H}_\tau \in \mathbb{C}^{M_r \times M_t}$ represents a spatially uncorrelated but temporally correlated Rayleigh fading channel, which is modeled by the first-order Gauss-Markov process

$$\mathbf{H}_\tau = \epsilon \mathbf{H}_{\tau-1} + \sqrt{1-\epsilon^2} \mathbf{G}_\tau, \tag{2}$$

where $\mathbf{G}_\tau$ has the same size of $\mathbf{H}_\tau$ with i.i.d entries and represents the evolution of $\mathbf{H}_\tau$. The $\epsilon \in [0,1]$ denotes the time correlation between the channel coefficient of adjacent time instants. In this paper, the ϵ obeys Jakes' model [12], [13].

Assuming perfect channel knowledge of the current channels at the receiver, the mutual information is known to be

$$I(\mathbf{F}_\tau) = \log_2 \left(\det \left(\mathbf{I}_V + \frac{\rho}{V} \mathbf{F}_\tau^* \mathbf{H}_\tau^* \mathbf{H}_\tau \mathbf{F}_\tau \right) \right). \tag{3}$$

The optimal precoder without quantization can be obtained via singular value decomposition (SVD) of channel. In limited feedback systems, a codebook is known by the transmitter and receiver, the precoder $\mathbf{F}_\tau$ is selected from the codebook $\mathcal{F}_\tau = \{\mathbf{F}_{\tau,i}\}_{i=1}^N$, where N denotes the codebook size. The receiver selects favorite precoder from the codebook and sends the index back to transmitter. There is no argument on that precoder selection is based on mutual information maximization criterion, which is shown as the following

$$\mathbf{F}_\tau = \underset{\mathbf{F}_{\tau,i} \in \mathcal{F}_\tau}{\arg\max} \{I(\mathbf{F}_{\tau,i})\}. \tag{4}$$

B. Differential Feedback Framework

In temporal correlated channels, the quantized differential feedback can virtually increase the codebook size. Quantizing the specific subspace instead of the whole channel space improves the quantization resolution. The differential codebook generates points on Grassmann manifold which are centered by the previous precoder. The differential codebook is shared by the transmitter and receiver, as well as the codebook update criterion. A quasi-diagonal differential codebook is proposed in [5]. The spherical cap differential codebook with adaptive cap radius is proposed in [7]. These differential codebooks can be categorized into total power constraint codebook since they change the power and phase of each antenna to achieve the maximum throughput. In LTE standard and its advanced version (LTE-A), the equal gain transmission property is considered to be the basic requirement of the precoding scheme. In order to fairly compare the performance degradation of limited feedback system and quantized differential feedback system over noisy feedback channels, we use the differential equal gain transmission (DEGT) codebook proposed in [8].

The DEGT has flexible trade-off between performance and codebook design complexity. Assuming the scalar design scheme which utilizes the structure of initial codebook, the DEGT codebook $\mathcal{F}' = \{\mathbf{F}_i'\}_{i=1}^N$ can be designed as the following

$$\mathbf{F}_i' = e^{j\alpha \angle \mathbf{F}_{i,0}} \tag{5}$$

where $\exp(\cdot)$ denotes the exponential function, $j = \sqrt{-1}$, $\angle \mathbf{A}$ denotes the phase matrix of $\mathbf{A}$, and $\mathbf{F}_{i,0}$ denotes the i-th codeword of initial codebook $\mathcal{F}_0$. The scalar factor α decides the range of differential codebook covered the Grassmann manifold. It should be designed appropriately according to the channel temporal correlation and can be determined using iterative simulation. The capacity-α relationship for different channel temporal correlation is illustrated in Fig. 2. The codebook update criterion can be expressed as follows

$$\mathbf{F}_{i,\tau} = \mathbf{F}_{i,\tau-1} \circ \mathbf{F}_i' \tag{6}$$

where $\circ$ denotes the matrix element-wise multiplication. Note that, the codeword index in $\mathcal{F}_\tau$ selected by the receiver also is the differential codeword index in $\mathcal{F}'$. Thereby, the whole codebook $\mathcal{F}_\tau$ is not necessary to be constructed at the transmitter.

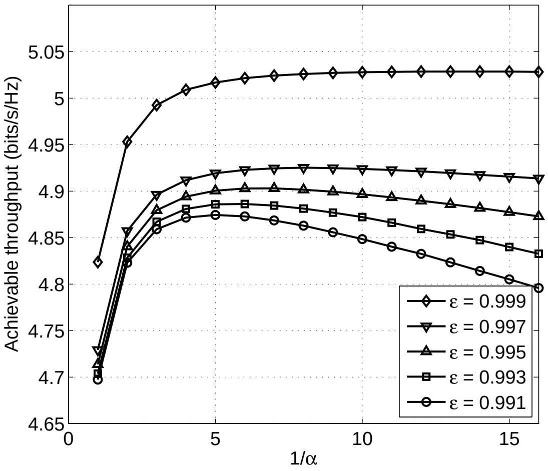

Figure 2. The optimal scalar factor value for different mobile speed.

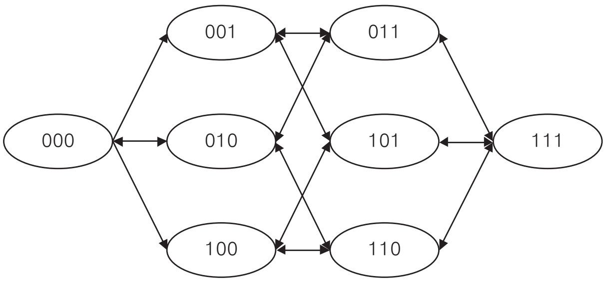

Figure 3. Example of 1 bit error flow chart (3 bits feedback).

III. INDEX ASSIGNMENT FOR DIFFERENTIAL CODEBOOK

In this part, the codebook index assignment scheme is introduced. The codebook within 3 bits feedback can be calculated directly. For the codebook over 4 bits feedback, the computation complexity is too huge to calculate directly. We illustrate a grouping index assignment to reduce the complexity.

A. Codebook Index Assignment within 3 bits

The illustration of index variation when 1 bit error occurs in 3 bits feedback sequence is shown in Fig. 3. In this paper, we assume that 1 bit error occurs per feedback at most, since it already is a high probability. Based on the relationship shown in Fig. 3, the codebook index assignment procedure can be applied as follows

1) Generate all possible combinations of the N codewords in the codebook with different sort. The number of the possible combinations is $N!$.
2) Calculate the total Hamming distance between pair of different codewords of each combination. This is shown in follows
$$C = \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} \Psi\left\{d\left(I_i, I_j\right)\right\} \left\| \mathbf{F}_i'^{H} \mathbf{F}_j' \right\|_F^2, \quad (7)$$
where I_i denotes the binary format of i-th codeword index. $d(x,y)$ denotes the Hamming distance between binary sequence x and y. The function $\Psi\cdot$ is shown as follows
$$\Psi\left\{a\right\} = \left\{ \begin{array}{ll} 1 & \text{if } a = 1 \\ 0 & \text{otherwise} \end{array} \right. , \quad (8)$$
The optimal codebook can be found by searching for the largest C.

B. Codebook Index Assignment over 4 bits

The number of combinations of the codebook with more than 4 bits feedback is too large to calculate by computer. We proposed a suboptimal codebook index assignment scheme by dividing the codewords into several groups called reference codebook. This shows flexible trade-off between calculation complexity and performance. Suppose the size of original codebook is N. The processes of reference codebook generation can be applied as follows

1) Generate $N = card\left(\mathcal{F}'\right)$ reference codebooks with $N-1$ codewords in each reference codebook by deleting one codeword from the original codebook. The function $card(\cdot)$ denotes the cardinality of a set.
2) Find the codebook with maximized minimum Chordal distance between each pair of codewords in the reference codebook as shown in the following
$$\mathcal{W}_t = \underset{\mathcal{W}_k(1\leq k\leq N)}{\arg\max} \left\{ \min_{\mathbf{F}_i', \mathbf{F}_j' \in \mathcal{F}_k'} d\left(\mathbf{F}_i', \mathbf{F}_j'\right) \right\}_{i=1}^{N}, \quad (9)$$
$$d\left(\mathbf{F}_i', \mathbf{F}_j'\right) = \sqrt{M - \left\| \mathbf{F}_i' \mathbf{F}_j'^{H} \right\|_F^2} \quad (10)$$
3) Repeat the step 1) and 2) until the size of reference codebooks are R which is possible to use the algorithm introduced in Section III-A. The optimal reference codebook $\mathcal{W} = \left\{\mathbf{W}_i\right\}_{i=1}^{R}$ is obtained by sorting the reference codebook with that algorithm. We assume the set $\mathcal{W}' = \mathcal{F}' - \mathcal{W}$ which contains the deleted codewords $\mathcal{W}' = \left\{\mathbf{W}_i'\right\}_{i=1}^{N-R}$. We considered the optimal reference codebook has R groups and each group has single element at the first.
4) Move the codewords to the groups from $\mathcal{W}'$. The group index can be determined as the following
$$r = \underset{1\leq i\leq R, 1\leq j\leq N-R}{\arg\max} \left\{ \left\| \mathbf{W}_i{}^{H} \mathbf{W}_j' \right\|_F \right\}, \quad (11)$$
5) Repeat step 4) until all elements in $\mathcal{W}'$ is moved to $\mathcal{W}$. Sort the each group using the algorithm in Section III-A. Appending the index sequence of the codewords in group to the group index sequence. The suboptimal

codebook index assignment can be finished by these procedures.

IV. SIMULATION RESULTS AND DISCUSSIONS

Monte-Carlo simulation is employed to obtain the performances of proposed scheme and conventional schemes. A MIMO system is considered which has 4 transmit antennas and 2 receive antennas. The number of data stream is one. The channel is modeled with first order Gaussian Markov process as described in Section II-A. The number of feedback bits is set to be 4 to consistent with LTE standard. The temporal correlation factor ϵ is assumed to be 0.991 and 0.997 which approximate to be 10 km/h and 3 km/h for LTE system. Correspondingly, the differential codebook scalar factor is set to be 5 and 8 respectively.

For quantized differential feedback system, once the error takes place in feedback, the codebook saved at transmitter and receiver becomes different and the difference will be accumulated. The initial codebook will be launched per 100 iterations in the simulation to break the accumulation.

Fig. 4 shows the comparison of LTE codebook and the differential codebook introduced in Section II-B. We assume the user equipment mobility is 10 km/h, and the scalar factor is 5. The bit error rate (BER) of feedback channel is assumed to be 10^{-3}.The performances are same for LTE codebook with or without index assignment. But the differential codebook shows performance improvement by using index assignment. Note that, the LTE codebook outperformed DEGT codebook after 9 dB since the error accumulation problem degrades the performance of DEGT scheme.

Figs. 5 and 6 show the comparison of DEGT codebook with or without index assignment when feedback error is 10^{-3} and 10^{-4}, respectively. The mobility of user equipment is 10 km/h in Fig. 5 and that is 3 km/h in Fig. 6. The index assignment scheme provides significant performance improvement in both low mobility and high mobility scenario.

Fig. 7 illustrates the relationship between the BER performance and initial codebook launching interval. The SNR is 10 dB, the mobility of user equipment is 10 km/h. The BER of feedback channel is assumed to be 10^{-3}. The index assignment scheme can increase the initial codebook launching interval.

V. CONCLUSIONS

In this paper, an index assignment algorithm for quantized differential feedback scheme is proposed. By using the index assignment to minimize the effects from the feedback error, the proposed scheme significantly improves the BER performance of the differential precoding system in the noisy feedback channels. For the codebook with more than 4 feedback bits, we also introduced a practical algorithm to make index assignment realizable.

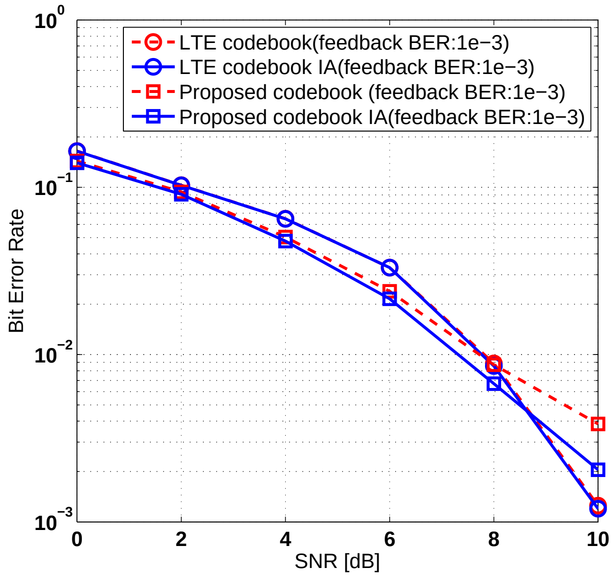

Figure 4. Performance comparison of differential codebook and LTE codebook.

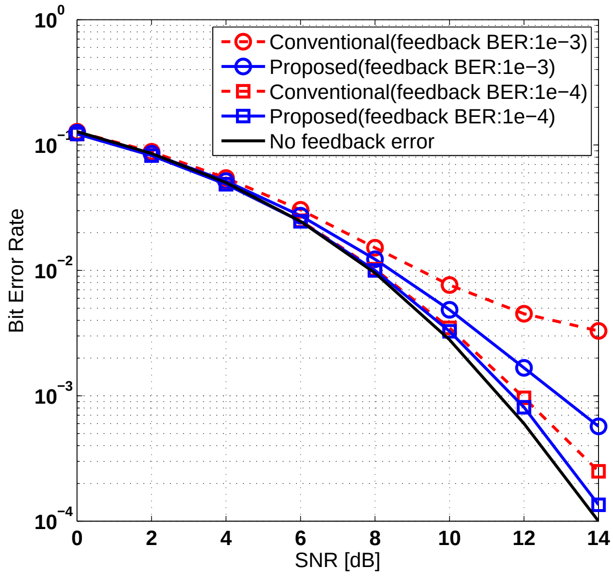

Figure 5. Performance comparison of index reassigned codebook ($f = 5$).

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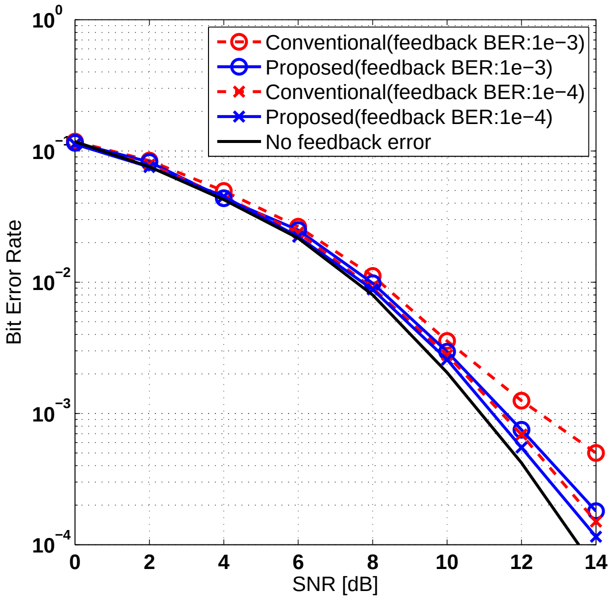

Figure 6. Performance comparison of index reassigned codebook ($f = 8$).

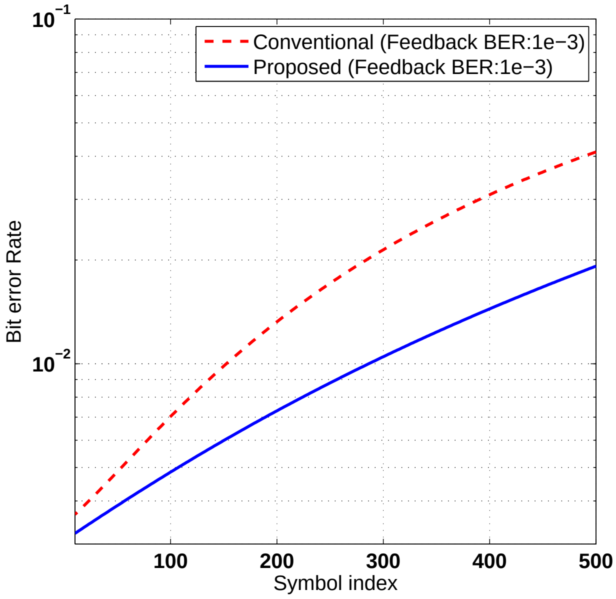

Figure 7. Performance comparison of retransmission timing of initial codebook ($f = 5$, SNR=10 dB).

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