

# BER Performance of BICM-coded Cooperative Networks with Selection Decode-and-Forward Relaying over Nakagami- $m$ Fading Channels

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**Abstract**—Bit-error-rate (BER) performance of coded cooperative networks with selection decode-and-forward (S-DF) relaying is analyzed over the Nakagami- $m$  fading channels. Previously, in the literature, BER analysis was done only for the un-coded network, where an impractical symbol-based forwarding is employed. In this paper, we analyze the bit-interleaved code modulation (BICM) coded cooperative network with an easy-to-implement packet-based forwarding. In particular, a closed-form BER expression is derived for the S-DF with source re-transmission (S-DF/RT) relaying, where the source re-transmits the packet to the destination on behave of a relay if it fails to decode. The accuracy of the proposed analysis is confirmed by computer simulations.

**Keywords**—cooperative communication; bit-interleaved coded modulation; selection decode-and-forward.

## I. INTRODUCTION

Cooperative communication has recently emerged as a promising technique to combat multi-path fading in wireless systems, thanks to its ability to provide spatial diversity for the size-limited mobile terminals [1][2]. It exploits the broadcast nature of wireless communications by allowing intermediate nodes, called relays, to overhear the packet transmitted from the source and forward it to the destination. One of the commonly employed relaying methods is the fixed decode-and-forward (F-DF) [2][3], where relays decode the received packet, re-encode and forward it to the destination. However, as was shown in [2], F-DF fails to provide full diversity because the received packet is always forwarded by a relay even if it is decoded erroneously. Selection DF (S-DF) is a relaying method proposed in [2] to overcome the shortcoming of F-DF, where a relay forwards the overheard packet only when it is decoded correctly. In this way, a full diversity can be achieved.

Performance analysis of the S-DF relaying has been a topic of extensive research [2][4]-[10]. Analyses were done over the Rayleigh fading channels from the aspects of capacity [4], outage probability [2][5] and symbol-error-rate (SER) [6], respectively. Very recently, analysis has been extended to the Nakagami- $m$  fading channels in [7]-[10] for the un-coded network. In particular, in [7], SER was analyzed for a single-relay network under the correlated and uncorrelated channels, and exact SER was provided in [8] for multiple-relay networks. In [9], a closed-form expression for the moment generating function of the received signal-

to-noise ratio (SNR) at the destination was derived, and it was used to evaluate SER, outage probability and channel capacity. Lastly, in [10], SER and diversity order were investigated for the networks with inter-relay links.

In [6]-[10], the analyses were focused on the un-coded network using a symbol-based forwarding in which symbols are detected separately at a relay, and only the correct symbols are forwarded to the destination. Unfortunately, the symbol-based forwarding is not practical in real systems because whether a particular symbol is detected correctly or not is not known at the relay. In addition, a huge signaling overhead is needed for notifying exactly which symbols are forwarded.

In this paper, we analyze the bit-interleaved coded modulation (BICM) [11][12] coded cooperative network over the Nakagami- $m$  fading channels, using a packet-based forwarding which can be easily implemented by using a cyclic redundancy check (CRC). BICM has been extensively applied in real systems. To our best of knowledge, this work is the first attempt to do the performance analysis for such a system. Only the S-DF with source re-transmission (S-DF/RT), where the source re-transmits the packet to the destination on behave of a relay if it fails to decode [2], is treated explicitly here; the S-DF with source idle in which source stays idle in the case of relay decoding failure can be viewed as a special case of S-DF/RT. A closed-form BER expression is derived, and simulation results are given to confirm the accuracy of the proposed analysis.

This paper is organized as follows. Section II describes the system models. BER performance is analyzed in Section III with numerical results presented in Section IV. Finally, conclusions are given in Section V.

## II. SYSTEM MODELS

We consider the cooperative relaying network with one source,  $R$  relays and one destination which are indexed by  $0, 1, \dots, R$  and  $R+1$ , respectively. Each node is equipped with one antenna, and relays operate in the half-duplexing manner implying that they cannot transmit and receive simultaneously. In the S-DF/RT relaying [2], transmission of a packet is divided into two phases; at phase-I, the source broadcasts a packet to relays and the destination, and, at phase-II, relays forward the received packet over orthogonal channels to the destination if it is decoded correctly (with a CRC). In the case of decoding failure at a relay, on the other hand, the relay keeps silent, and the source retransmits the

packet on the relay's behave over the orthogonal channel allotted to that relay. For convenience, the orthogonal channel allotted to relay  $j$  at phase-II will be denoted as orthogonal-channel  $j$ .

#### A. Channel Model

A frequency-nonselctive fading channel is considered. Define  $h_{0,j}(k)$ ,  $j=1,\dots,R+1$  the channel gain between the source and node  $j$  at the  $k$ -th channel use,  $h_{i,R+1}(k)$   $i=1,\dots,R$  the channel gain between relay  $i$  and the destination, and  $h_{0,R+1}^{(j)}(k)$ ,  $j=1,\dots,R$  the channel gain between the source and the destination at phase-II that uses orthogonal-channel  $j$  for retransmission. The channels are assumed to be mutually independent, and under the assumption of a symbol inter-leaver with a depth larger than the channel coherent time, the channel gains of a channel are independent and identically distributed (i.i.d.) over different time index  $k$ .

The general Nakagami- $m$  fading model is adopted in this study, with the probability density function (pdf)

$$p(h) = \frac{2m^m h^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(-\frac{mh^2}{\Omega}\right) \quad (1)$$

to characterize the channel gain  $h$  [12], where  $m$  is the shaping factor (assumed to be a positive integer),  $\Omega$  is the average power of the channel, and  $\Gamma(\cdot)$  is the Gamma function. Perfect channel-state-information (CSI) will be assumed available at all receiving nodes.

#### B. Bit-interleaved Coded Modulation

BICM is employed at all nodes. At the source, an information bit sequence  $\mathbf{b}$  of length  $K$  is encoded into a coded sequence  $\mathbf{c}$  of length  $N$ . After interleaving, the sequence is partitioned successively into groups of  $l$  bits, called the labels, which are then mapped to a sequence of complex symbols  $\{x(k) \in \mathcal{X}\}$  for transmission according to a signal mapper  $\mu$  and a signal constellation  $\mathcal{X}$ .

At phase-I, the received signals at relays and the destination at time  $k$  are given by

$$y_{0,j}(k) = h_{0,j}(k)\sqrt{P_0}x(k) + \omega_{0,j}(k), \quad j=1,2,\dots,R+1, \quad (2)$$

where  $P_0$  is the source transmit power, and  $\omega_{0,j}(k)$  is the AWGN (additive white Gaussian noise) at node  $j$ . All noises are modeled as i.i.d. zero-mean, circularly-symmetric complex Gaussian random variables with variance  $N_0/2$  per dimension. Upon receiving  $y_{0,j}(k)$ , relay  $j$  calculates the maximum log-likelihood ratio (LLR) for the  $i$ -th bit of the  $k$ -th symbol according to

$$\min_{x \in \mathcal{X}_b^i} \frac{|y_{0,j}(k) - h_{0,j}(k)\sqrt{P_0}x|^2}{N_0} - \min_{x \in \mathcal{X}_b^i} \frac{|y_{0,j}(k) - h_{0,j}(k)\sqrt{P_0}x|^2}{N_0}, \quad (3)$$

where  $\mathcal{X}_b^i$  is the subset of signal points in  $\mathcal{X}$  with the binary value  $b$  at the  $i$ -th position of the label. The LLRs of the coded sequence are then de-interleaved and decoded. The Max-log MAP (maximum a posteriori probability) decoder [16] is employed at all receiving nodes

In the S-DF/RT relaying, relay  $j$  forwards the received packet to the destination if  $\mathbf{b}$  is decoded correctly. Otherwise, it notifies the source to re-transmit the packet over orthogonal-channel  $j$ . Define  $\Theta \subseteq \{1,\dots,R\}$  as the set of active relays which have decoded successfully at phase-I and  $\bar{\Theta}$  as its complement set. Then, at phase-II, the signals received at the destination can be expressed by

$$y_{j,R+1}(k) = h_{j,R+1}(k)\sqrt{P_j}x(k) + \omega_{j,R+1}(k), \quad j \in \Theta, \quad (4)$$

and

$$y_{0,R+1}^{(j)}(k) = h_{0,R+1}^{(j)}(k)\sqrt{P_j}x(k) + \omega_{0,R+1}^{(j)}(k), \quad j \in \bar{\Theta}, \quad (5)$$

where  $P_j$  is the transmit power over orthogonal-channel  $j$ , and  $\omega_{j,R+1}(k)$  and  $\omega_{0,R+1}^{(j)}(k)$  are the corresponding AWGNs. For  $j \in \bar{\Theta}$ ,  $P_j$  may assume different values depending on whether the orthogonal channels are implemented in the time, frequency or code domain. The received signals at phase-I and phase-II are combined and decoded jointly at the destination.

The max-log likelihood ratio for the  $i$ -th bit of the  $k$ -th symbol at the destination is evaluated by

$$\min_{x \in \mathcal{X}_b^i} \sum_{j=0}^R \frac{|\tilde{y}_{j,R+1}(k) - \tilde{h}_{j,R+1}(k)\sqrt{P_j}x|^2}{N_0} - \min_{x \in \mathcal{X}_b^i} \sum_{j=0}^R \frac{|\tilde{y}_{j,R+1}(k) - \tilde{h}_{j,R+1}(k)\sqrt{P_j}x|^2}{N_0}, \quad (6)$$

where  $\tilde{y}_{j,R+1}(k) \doteq \begin{cases} y_{j,R+1}(k), & \text{if } j=0 \text{ or } j \in \Theta \\ y_{0,R+1}^{(j)}(k), & \text{if } j \in \bar{\Theta} \end{cases}$ , and

$$\tilde{h}_{j,R+1}(k) \doteq \begin{cases} h_{j,R+1}(k), & \text{if } j=0 \text{ or } j \in \Theta \\ h_{0,R+1}^{(j)}(k), & \text{if } j \in \bar{\Theta} \end{cases}. \quad \text{The max-log}$$

likelihood ratios of the coded sequence are then de-interleaved and passed to the decoder. For notation simplicity,  $\Theta' \doteq \Theta \cup \{0\}$  is used in the rest of the paper.

### III. BER ANALYSIS

Let  $p_{b,R+1}^{\text{RT}}$  denote the BER at the destination using the S-DF/RT relaying. Under the packet-based forwarding, the BER is given by

$$p_{b,R+1}^{\text{RT}} = \sum_{\Theta \subseteq \{1,2,\dots,R\}} p_{b,R+1}^{\text{RT}}(\Theta) \prod_{j \in \Theta} (1 - p_{f,j}) \prod_{j \in \bar{\Theta}} p_{f,j}, \quad (7)$$

where  $p_{b,R+1}^{\text{RT}}(\Theta)$  is the BER at the destination given  $\Theta$ , and  $p_{f,j}$  is the packet-error-rate (PER) at relay  $j$ . In what follows,  $p_{b,R+1}^{\text{RT}}(\Theta)$  is analyzed first, followed by the

analysis of  $p_{f,j}$ . Recall that in [6]-[10] SER at the destination was analyzed for the un-coded relaying system under an impractical symbol-based forwarding.

Using the assumptions of ideal interleaving and symmetrization in [12],  $p_{b,R+1}^{\text{RT}}(\Theta)$  can be estimated by

$$p_{b,R+1}^{\text{RT}}(\Theta) \approx \sum_{d_h=d_f}^N w_l(d_h) f_{\text{ex}}^{\text{RT}}(d_h, \Theta), \quad (8)$$

where  $w_l(d_h)$  is the total information bits of the error events with Hamming weight  $d_h$  divided by  $K$ ,  $d_f$  is the free distance of the code, and  $f_{\text{ex}}^{\text{RT}}(d_h, \Theta)$  is the expurgated upper bound of the pair-wise error probability (PEP) between two coded sequences with Hamming distance  $d_h$ .

As is shown in [14],  $f_{\text{ex}}^{\text{RT}}(d_h, \Theta)$  can be expressed as

$$f_{\text{ex}}^{\text{RT}}(d_h, \Theta) = \frac{1}{2\pi j} \int_{s_0-j\infty}^{s_0+j\infty} \prod_{k=1}^{d_h} \left[ \frac{1}{l2^l} \sum_{i=1}^l \sum_{b=0}^1 \sum_{x(k) \in \mathcal{X}_b^i} \prod_{j=0}^R \Phi_{\Delta_j(x(k), \hat{z}(k))}(s) \right] ds \quad (9)$$

where  $j = \sqrt{-1}$ ,

$$\Phi_{\Delta_j(x(k), \hat{z}(k))}(s) = E_{\tilde{h}_{j,R+1}(k)} \left[ \exp \left[ (-s + s^2) \frac{P_j}{N_0} \tilde{h}_{j,R+1}^2(k) |x(k) - \hat{z}(k)|^2 \right] \right] \quad (10)$$

is the moment generating function of the metric difference

$$\Delta_j(x(k), \hat{z}(k)) \doteq E_{\tilde{h}_{j,R+1}(k)} \left[ \begin{array}{l} \log p(\tilde{y}_{j,R+1}(k) | x(k), \tilde{h}_{j,R+1}(k)) \\ -\log p(\tilde{y}_{j,R+1}(k) | z(k), \tilde{h}_{j,R+1}(k)) \end{array} \right], \quad (11)$$

and  $\hat{z}(k)$  is the nearest neighbor of  $x(k)$  in  $\mathcal{X}_b^i$ .

To evaluate  $f_{\text{ex}}^{\text{RT}}(d_h, \Theta)$ , firstly,  $\Phi_{\Delta_j(x(k), \hat{z}(k))}(s)$  can be derived as [14]

$$\Phi_{\Delta_j(x(k), \hat{z}(k))}(s) = \left( 1 - (-s + s^2) \frac{\tilde{\Omega}_{j,R+1} P_j |x(k) - \hat{z}(k)|^2}{\tilde{m}_{j,R+1} N_0} \right)^{-\tilde{m}_{j,R+1}} \quad (12)$$

with the region of convergence (ROC)

$$\frac{1}{2} - \sqrt{\frac{\tilde{m}_{j,R+1} N_0}{\tilde{\Omega}_{j,R+1} P_j |x(k) - \hat{z}(k)|^2} + \frac{1}{4}} < \text{Re}\{s\} < \frac{1}{2} + \sqrt{\frac{\tilde{m}_{j,R+1} N_0}{\tilde{\Omega}_{j,R+1} P_j |x(k) - \hat{z}(k)|^2} + \frac{1}{4}}, \quad (13)$$

where

$$(\tilde{m}_{j,R+1}, \tilde{\Omega}_{j,R+1}) = \begin{cases} (m_{j,R+1}, \Omega_{j,R+1}), & j \in \Theta' \\ (m_{0,R+1}, \Omega_{0,R+1}), & j \in \bar{\Theta} \end{cases}, \quad (14)$$

and  $m_{i,j}$  and  $\Omega_{i,j}$  are the shaping factor and the average power of the channel between node  $i$  and  $j$ , respectively. Since the saddle point 0.5 always lies in the ROC, the

integration in (9) can be evaluated efficiently along with the vertical line of  $s = 0.5 + jt$  [15]. Using this, (9) becomes

$$f_{\text{ex}}^{\text{RT}}(d_h, \Theta) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \left[ \frac{1}{l2^l} \sum_{i=1}^l \sum_{b=0}^1 \sum_{x \in \mathcal{X}_b^i} \prod_{j=0}^R \left( 1 + \left( t^2 + \frac{1}{4} \right) \frac{\tilde{\Omega}_{j,R+1} P_j |x - \hat{z}|^2}{\tilde{m}_{j,R+1} N_0} \right)^{-\tilde{m}_{j,R+1}} \right]^{d_h} \left( t^2 + \frac{1}{4} \right)^{-1} dt, \quad (15)$$

where the time index  $k$  has been dropped because  $\tilde{m}_{j,R+1}$  and  $\tilde{\Omega}_{j,R+1}$  are the same for all  $k$ . In addition, (15) only contains the real part because the imaginary part of the integral in (9) is an odd function of  $t$ .

Secondly, some of  $(x, \hat{z})$  pairs in (15) have the same squared Euclidean distance  $|x - \hat{z}|^2$  and can be grouped together. By doing so, (15) is rewritten as

$$f_{\text{ex}}^{\text{RT}}(d_h, \Theta) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \left[ \sum_{i=1}^M C_i \prod_{j=0}^R \left( 1 + \left( t^2 + \frac{1}{4} \right) \frac{\tilde{\Omega}_{j,R+1} P_j D_i}{\tilde{m}_{j,R+1} N_0} \right)^{-\tilde{m}_{j,R+1}} \right]^{d_h} \left( t^2 + \frac{1}{4} \right)^{-1} dt, \quad (16)$$

where  $D_i = |x - \hat{z}|^2$  is a squared Euclidean distance,  $C_i$  is the number of  $(x, \hat{z})$  pairs with  $|x - \hat{z}|^2 = D_i$  over  $l2^l$ , and  $M$  is the number of distinct  $D_i$ 's. the examples,  $M$ ,  $C_i$  and  $D_i$  for QPSK, 16-QAM and 64-QAM with Gray mapping, can be found in [14].

Lastly, from [14], a close-form expression of (16) is obtained as

$$f_{\text{ex}}^{\text{RT}}(d_h, \Theta) = \frac{1}{4} \sum_{\substack{u_1, u_2, \dots, u_M \\ d_h = u_1 + \dots + u_M}} \frac{d_h!}{u_1! u_2! \dots u_M!} \prod_{i=1}^M \left[ C_i \prod_{j=0}^R \left( A_{i,j} - \frac{1}{4} \right)^{B_{i,j}} \right] \cdot \left( \sum_{i=1}^M \sum_{j=0}^R \sum_{k=1}^{B_{i,j}} E_{i,j,k} \frac{(2k-3)!!}{(2k-2)!!} A_{i,j}^{-\frac{2k-1}{2}} + E' \right) \quad (17)$$

$$\text{where } A_{i,j} = \frac{1}{4} + \frac{\tilde{m}_{j,R+1} N_0}{\tilde{\Omega}_{j,R+1} P_j D_i}, \quad B_{i,j} = u_i \tilde{m}_{j,R+1},$$

$$(2k+1)!! = 1 \cdot 3 \cdot 5 \cdots (2k+1), \quad (2k)!! = 2 \cdot 4 \cdot 6 \cdots (2k),$$

$$E_{i,j,k} = \frac{1}{(B_{i,j} - k)! ds^{B_{i,j} - k}} \left( (A_{i,j} + s)^{B_{i,j}} G(s) \right) \Big|_{s=A_{i,j}}, \quad (18)$$

$$E' = \left( \frac{1}{4} + s \right) G(s) \Big|_{s=-\frac{1}{4}}, \quad (19)$$

and  $G(s) = \prod_{i=1}^M \prod_{j=0}^R (A_{i,j} + s)^{-B_{i,j}} \left( s + \frac{1}{4} \right)^{-1}$ . With (17), the

BER  $p_{b,R+1}^{\text{RT}}(\Theta)$  is now estimated by

$$\hat{p}_{b,R+1}^{\text{RT}}(\Theta) = \sum_{d_h=d_f}^N w_l(d_h) f_{\text{ex}}^{\text{RT}}(d_h, \Theta). \quad (20)$$

On the other hand, the PER at relay  $j$ ,  $p_{f,j}$ , can be approximated by

$$\hat{p}_{f,j} = \begin{cases} 1, & \hat{p}_{b,j} \geq 1 \\ 1 - (1 - \hat{p}_{b,j})^K, & \text{otherwise} \end{cases}, \quad (21)$$

where  $\hat{p}_{b,j}$  is an estimation of the BER  $p_{b,j}$  at relay  $j$ . Following the same steps as in (8)-(20),  $\hat{p}_{b,j}$  is given by

$$\hat{p}_{b,j} = \sum_{d_h=d_f}^N w_i(d_h) \frac{1}{4} \cdot \sum_{u_1, u_2, \dots, u_M} \frac{d_h!}{u_1! u_2! \dots u_M!} \prod_{i=1}^M \left[ C_i^{u_i} \left( \tilde{A}_{i,j} - \frac{1}{4} \right)^{\tilde{B}_{i,j}} \right], \quad (22)$$

$$\cdot \left( \sum_{i=1}^M \sum_{k=1}^{\tilde{B}_{i,j}} \tilde{E}_{i,j,k} \frac{(2k-3)!!}{(2k-2)!!} \tilde{A}_{i,j}^{-\frac{2k-1}{2}} + \tilde{E}' \right)$$

where  $\tilde{A}_{i,j} = \frac{1}{4} + \frac{m_{0,j} N_0}{\Omega_{0,j} P_0 D_i}$ ,  $\tilde{B}_{i,j} = u_i m_{0,j}$ , and  $\tilde{E}_{i,j,k}$  and  $\tilde{E}'$  are obtained as in (18) and (19) with  $A_{i,j}$ ,  $B_{i,j}$  and  $G(s)$  replaced by  $\tilde{A}_{i,j}$ ,  $\tilde{B}_{i,j}$ , and  $\tilde{G}_j(s) \triangleq \prod_{i=1}^M (\tilde{A}_{i,j} + s)^{-\tilde{B}_{i,j}} \left( s + \frac{1}{4} \right)^{-1}$ , respectively. Using  $\hat{p}_{b,R+1}^{\text{RT}}(\Theta)$  and  $\hat{p}_{f,j}$ ,  $p_{b,R+1}^{\text{RT}}$  now can be estimated by

$$\hat{p}_{b,R+1}^{\text{RT}} \doteq \sum_{\Theta \in \{1,2,\dots,R\}} \hat{p}_{b,R+1}^{\text{RT}}(\Theta) \prod_{j \in \Theta} (1 - \hat{p}_{f,j}) \prod_{j \in \bar{\Theta}} \hat{p}_{f,j}. \quad (23)$$

#### IV. SIMULATION RESULTS

In this section, simulation results are given to verify the accuracy of the proposed BER analysis. In all the simulations, the half-rate convolutional code with the generator matrix  $(1+D+D^2+D^3+D^6, 1+D^2+D^3+D^5+D^6)$  is employed, an S-random interleaver with depth 20 is used to break to correlation between bits in a label, and the Gray mapping is used for all QAM constellations. In addition, we assume  $P_0 = P_1 = \dots = P_R = P$  for simplicity, where  $P = E_b \cdot R_c \cdot l$  with the bit energy  $E_b$  and the channel code rate  $R_c = 0.5$ . The considered network configurations are summarized in Table I.

Fig. 1 shows the analytical and simulation results of  $p_{b,2}^{\text{RT}}(\{1\})$ ,  $p_{b,2}^{\text{RT}}(\emptyset)$ ,  $p_{b,1}$  and  $p_{f,1}$  for Network-1 (3-node), which is used to illustrate the accuracy of the analyses given

TABLE I. THE CONSIDERED NETWORK CONFIGURATIONS

Networks <sup>o</sup>	$R^o$	S-D link <sup>o</sup>	S-R link(s) <sup>o</sup>	R-D link(s) <sup>o</sup>
Network-1	1 <sup>o</sup>	$m_{0,2} = \Omega_{0,2} = 1^o$	$m_{0,1} = \Omega_{0,1} = 4^o$	$m_{1,2} = \Omega_{1,2} = 2^o$
Network-2 <sup>o</sup>	1 <sup>o</sup>	$m_{0,2} = \Omega_{0,2} = 1^o$	$m_{0,1} = \Omega_{0,1} = n^o$	$m_{1,2} = \Omega_{1,2} = 2^o$
Network-3 <sup>o</sup>	2 <sup>o</sup>	$m_{0,2} = \Omega_{0,2} = 1^o$	$m_{0,1} = \Omega_{0,1} = 4,^o$ $m_{0,2} = \Omega_{0,2} = 3^o$	$m_{1,3} = \Omega_{1,3} = 1,^o$ $m_{2,3} = \Omega_{2,3} = 2^o$

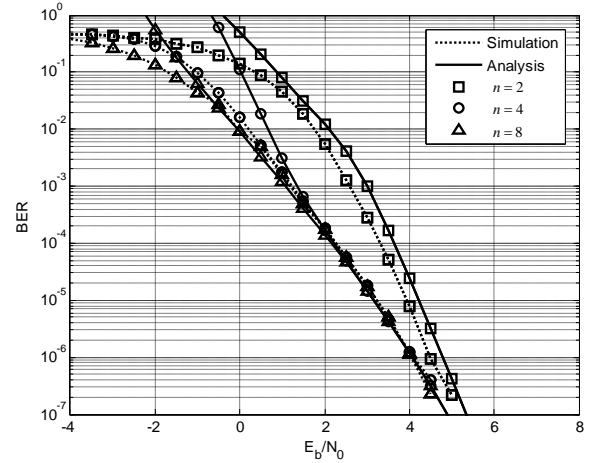


Figure 1. Simulation and analytical results of  $p_{b,2}^{\text{RT}}(\{1\})$ ,  $p_{b,2}^{\text{RT}}(\emptyset)$ ,

$p_{b,1}$  and  $p_{f,1}$ .

in (20), (21) and (22). Recall that in the 3-node network, the BER at the destination is given by (see (7))

$$p_{b,2}^{\text{RT}} = p_{b,2}^{\text{RT}}(\{1\})(1 - p_{f,1}) + p_{b,2}^{\text{RT}}(\emptyset)p_{f,1}. \quad (24)$$

As can be seen, the analytical results obtained in (20) and (22) predict the BERs very accurately at performance of interest, say  $\text{BER} = 10^{-5}$ . The PER prediction obtained in (21), on the other hand, is a bit mismatch with the true one, but the difference is only around 0.5 dB at  $\text{PER} = 10^{-3}$ .

Fig. 2 shows the simulation and analytical results of  $p_{b,2}^{\text{RT}}$  for Network-2, which is a 3-node network as well but with a varying channel condition on the S-R channel. Clearly, the analysis in (23) predicts the BER performance at the destination very accurately at  $\text{BER} = 10^{-5}$  for all the cases of  $n = 2, 4$  and 8. The slight difference between the analytical and simulation results for  $n = 2$  is due to that in this case  $p_{b,2}^{\text{RT}}(\emptyset)p_{f,1}$  in (24) plays a more prominent role in determining  $p_{b,2}^{\text{RT}}$ , and there is a bit error in predicting  $p_{f,1}$  as is shown in Fig. 2. In the typical scenarios of applying relay stations, the S-R channel often has a good channel condition; for example there is light-of-sight between the source and relay, and that results in a better BER prediction by the proposed analysis because in this case  $p_{b,2}^{\text{RT}}$  is well approximated by  $p_{b,2}^{\text{RT}}(\{1\})$  and which can be predicted accurately. (see the cases of  $n = 4$  and 8 in Fig. 2.)

Fig. 3 shows the simulation and analytical results for Network-3 (4 nodes) for the S-DF/RT with different constellations. As is shown, the predictions are quite accurate; a less than 0.3 dB error at  $\text{BER}$  of  $10^{-5}$  is observed.

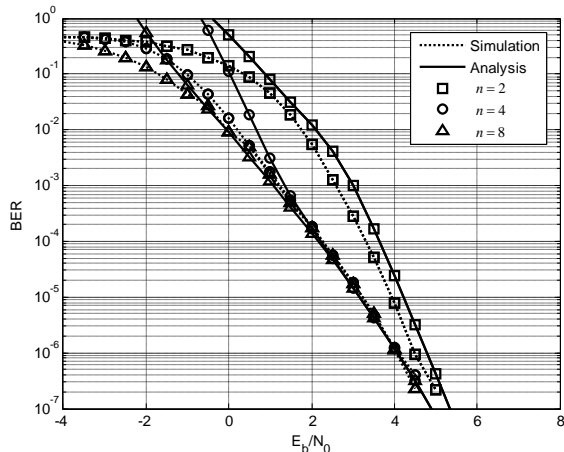


Figure 2. Simulation and analytical results of  $p_{b,2}^{RT}$  for Network-2 with  $n = 2, 4, \text{ and } 8$ .

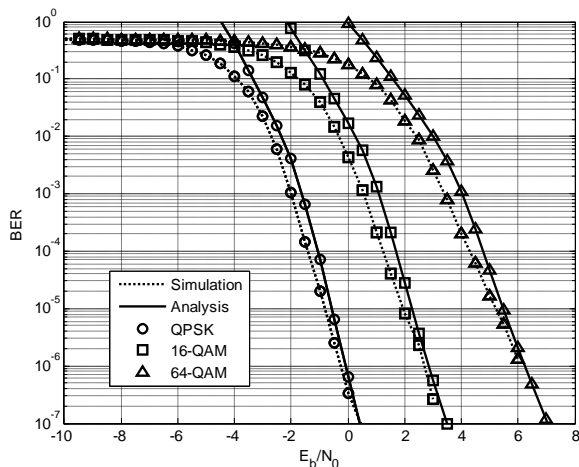


Figure 3. Simulation and analytical results of  $p_{b,3}^{RT}$  for Network-3.

### V. CONCLUSIONS

This paper investigates the performance of BICM-coded cooperative networks with the S-DF/RT relaying over the Nakagami- $m$  fading channels. In particular, the BER performance is analyzed with a packet-based forwarding which can be implemented easily with CRC. In the literature, performance has been reported only for the un-coded system that uses an impractical symbol-based forwarding. A closed-form expression is derived and verified by computer simulations. The numerical results show that the prediction error is less than 0.5 dB at  $\text{BER} = 10^{-5}$ .

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