

# Cooperative Communications Using Multilevel Bit-Interleaved Coded Modulations

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**Abstract**—This paper deals with cooperative communications for *non-ergodic* block fading multiple-input multiple-output (MIMO) channels. A 2-level bit-interleaved coded modulation (BICM) with 16-QAM constellation is used in order to obtain a flexible scheme where one or two users can share the resources of the relay node in order to achieve better performance. The block fading channel is assumed to be perfectly known at the receiver but not at the transmitter. The channel between the user and the relay is assumed to be error free. Iterative multistage decoding is done. The performance of the system is derived using Monte Carlo simulation.

**Keywords**—Cooperative communications; MIMO channel; bit-interleaved coded modulations; multilevel coded modulations.

## I. INTRODUCTION

Cooperative communications for wireless networks have recently gained a lot of interest. The idea is to allow the users of the network to cooperate and share resources in order to provide more diversity and more resources like bandwidth and antennas, in order to achieve better capacity, performance, system coverage, etc. Sendonaris *et al.* [1] [2] proposed the concept of cooperative diversity, with decode and forward strategy applied by the cooperative node. T.E. Hunter *et al.* [3] and B. Zhao *et al.* [4] studied the principles of coded cooperation and distributed coding.

Several coded cooperation schemes based on multilevel coded modulations (MLC) [5] or bit-interleaved coded modulations (BICM) [6] were introduced [7][8][9].

In this paper, a multilevel bit-interleaved coded 16-QAM modulation [10] with  $n_t$  transmit antennas is applied by the users or nodes in order to send information to destination points with  $n_r$  receive antennas.

In the presence of a relay node, a part or the entire data sent by the user is retransmitted (decode and forward) by the relay in order to achieve higher diversity. This depends on whether or not the relay is shared by two users or is transmitting its own data.

In the absence of a relay node, and in order to achieve better performance, the user can reduce the data rate by sending information over one of the two levels of the modulation.

The paper is organized as follows. The channel model is described in Section III. In Section IV, we introduce the 2-level BICM combined to a 16-QAM constellation. This is the modulation used by the users and the relays in order to transmit

data. In Section V, the capacity of the proposed scheme is derived. The iterative multistage decoder (MSD) is studied in Section VI. The different transmission and cooperation strategies are introduced in section VII. In Section VIII, the performance of the different cooperation and non-cooperation schemes are determined using Monte Carlo simulation. Results are compared to the performance of a 1-level BICM and to the outage probability of a *non-ergodic* block fading MIMO channel.

## II. RELATED WORKS

In [7], a multilevel cooperation scheme, where each source or relay node uses a level of the modulation, was introduced. However, a frequency selective channel, with a single transmit and a single receive antenna was considered. Moreover, there's no use of bit-interleaved coding and iterative decoding.

In [8], a one level BICM is applied by each user, and iterative decoding is done between users or between the decoder and the constellation. Flexible spectral efficiency is reached by controlling the portion of information data repeated by the relay.

In [9], a cooperative communications scheme, combining BICM and orthogonal frequency division multiplexing (OFDM), is proposed.

In this paper, the main originality is the combination of MLC and BICM into a multilevel coding scheme with a bit-interleaved coded modulation at each level. Moreover, a MIMO channel is considered. Notice that each source or relay uses one level of the modulation in order to transmit its own data, and the other levels for cooperation as in [7]. On the other hand, iterative multistage decoding is applied. This is similar to the second decoding strategy introduced in [8].

## III. CHANNEL MODEL

Let us consider a flat fading MIMO channel with  $n_t$  transmit and  $n_r$  receive antennas. The received and the transmitted signals are related by  $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$  where  $\mathbf{y} = (y_1, y_2, \dots, y_{n_r})^t$  denotes the vector of complex received signals during any given channel use,  $\mathbf{x} = (x_1, x_2, \dots, x_j, \dots, x_{n_t})^t$  denotes the vector of complex transmitted symbols. The superscript  $t$  stands for transpose. The superscript  $h$ , that will be used later, stands for transpose conjugate. The symbols  $x_j$  belong to a Quadrature Amplitude Modulation (QAM) constellation of size  $M = 2^m$ . The

channel matrix  $\mathbf{H} = [h_{i,j}]$  is assumed to be perfectly known at the receiver but not at the transmitter. The fading coefficients  $h_{i,j}$  are complex, Gaussian, circular, mutually independent and satisfy  $E[|h_{i,j}|^2] = 1$ .

We consider the case where the channel matrix  $\mathbf{H}$  is fixed in time. This is the so called *non-ergodic* block fading channel. Notice that the *ergodic* fading channel is the one where the channel matrix varies at each channel use.  $\mathbf{n}$  denotes the vector of additive white complex Gaussian noise with zero mean and variance  $2N_0$ .

#### IV. THE 2-LEVEL BIT-INTERLEAVED CODED MODULATION

The 2-level bit-interleaved coded modulation with  $n_t$  transmit antennas is represented in Figure 1. This is a Multilevel Coded (MLC) modulation with a Bit-Interleaved Coded Modulation at each level.

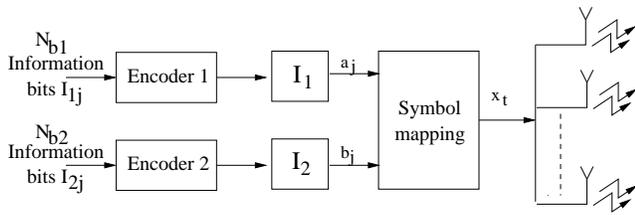


Fig. 1. 2-level BICM transmitter.

At each level, the information bits are encoded into  $N_c/2$  coded bits using a non-recursive non-systematic convolutional encoder, then randomly interleaved.

Afterwards, the bits  $a_j$  at level 1 and the bits  $b_j$  at level 2 are mapped into  $N_s = N_c/4$  16-QAM symbols  $x_t$ . Gray labeling of coded bits is applied as in Figure 2.

10	x	x	x	x
11	x	x	x	x
01	x	x	x	x
ab=00	x	x	x	x
ab=00	01	11	10	

Fig. 2. Gray labeled 16-QAM constellation. a  $\rightarrow$  level 1. b  $\rightarrow$  level 2.

Finally, the block of  $N_s = N_c/4$  symbols is divided into signal vectors of length  $n_t$  symbols transmitted at each channel use (the signal vector is function of  $L = 4.n_t$  coded bits).

The total rate of the system in bits per channel use is equal to  $R = 4 \times (R_1 + R_2)$  where  $R_1$  and  $R_2$  are the rates of encoders 1 and 2 respectively. Note that  $R_1$  and  $R_2$  are chosen in a way to achieve the best performance. For large code length, this is done using the capacity design rule [5], which states that the rate  $R_i$  at level  $i$  is equal to the capacity  $C_i$  of the equivalent channel that represents the transmission of the bits of level  $i$  (cf. section V).

#### V. CAPACITY

Let us consider an *ergodic* Rayleigh flat fading MIMO channel. Since  $\mathbf{H}$  is known to the receiver and not to the transmitter, it can be considered as part of the output. The capacity is therefore obtained by maximizing the mutual information  $I(\mathbf{X}; (\mathbf{Y}, \mathbf{H}))$ .

It can be shown after some simple manipulations that this quantity is equal to  $E_{\mathbf{H}}\{I(\mathbf{X}; \mathbf{Y}|\mathbf{H} = H)\}$  where  $H$  denotes a realization of  $\mathbf{H}$ . The mutual information in this last expression is maximized when  $\mathbf{X}$  is a zero mean complex circularly symmetric Gaussian vector. This leads to the following equation [11]:

$$C_{MIMO} = E_{\mathbf{H}}\{\log_2(\det(\mathbf{I}_{n_r} + \frac{\rho}{n_t}\mathbf{H}\mathbf{H}^h))\} \quad (1)$$

where  $\rho = \frac{E_s}{N_0} = \frac{E\{\mathbf{x}^h\mathbf{x}\}}{2N_0}$ , and  $E_s$  represents the total transmitted energy per channel use. This is also the total energy at each receiver antenna since all elements  $h_{i,j}$  are independent and satisfy  $E\{|h_{i,j}|^2\} = 1$ .

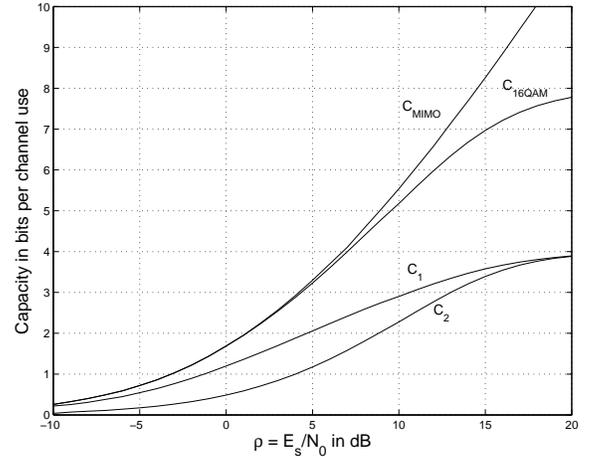


Fig. 3. Capacity of a MIMO channel. 16-QAM constellation.  $n_t = n_r = 2$ .

Now, in the case of a discrete 16-QAM input, the maximum of the mutual information is achieved by a uniform distribution. The capacity  $C_{16QAM} = E_{\mathbf{H}}\{I(\mathbf{X}; \mathbf{Y}|\mathbf{H} = H)\}$  is calculated using the following equation

$$C_{16QAM} = E_{\mathbf{H}}\left(\frac{1}{L} \sum_{i=1}^L \int_y p_H(y|x_i) \log\left(\frac{p_H(y|x_i)}{\frac{1}{L} \sum_j p_H(y|x_j)}\right) dy\right) \quad (2)$$

where, for a fixed realization  $\mathbf{H} = H$ , the output of the channel  $\mathbf{Y}$  is a vector of Gaussian random variables.

$$p_H(\mathbf{y}|\mathbf{x}_i) \propto \exp\left(-\frac{\|\mathbf{y} - H\mathbf{x}_i\|^2}{2N_0}\right) \quad (3)$$

In the case of the 2-level BICM, the random variable  $\mathbf{X}$  representing the input of the channel can be written as  $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2)$  where  $\mathbf{X}_1$  and  $\mathbf{X}_2$  are the discrete random variables representing the transmission of the coded bits of level 1 and

2 respectively. Applying the chain rule of mutual information [12]:

$$I(\mathbf{X}; \mathbf{Y}) = I(\mathbf{X}_1; \mathbf{Y}) + I(\mathbf{X}_2; \mathbf{Y}|\mathbf{X}_1) \quad (4)$$

we get that the total capacity of the MIMO channel with 16-QAM input is equal to

$$C_{16QAM} = C_1 + C_2 \quad (5)$$

where the quantities  $C_1 = E_{\mathbf{H}}\{I(\mathbf{X}_1; \mathbf{Y}|\mathbf{H} = H)\}$  and  $C_2 = E_{\mathbf{H}}\{I(\mathbf{X}_2; \mathbf{Y}|\mathbf{X}_1, \mathbf{H} = H)\}$  are the capacities of the equivalent channels representing the transmission of the bits at level 1 and 2 respectively.

These capacities, as well as the capacity of the MIMO channel with continuous input, are represented in Figure 3 as function of the signal-to-noise ratio  $\rho = \frac{E_s}{N_0}$ .

## VI. THE ITERATIVE MULTISTAGE DECODER

The receiver, represented in Figure 4, is divided into two parts. The first part is non iterative and computes the received signal conditional probabilities  $p(y_r|c_1, \dots, c_{4.n_t})$  at every antenna  $r$  using

$$p(y_r|c_1, \dots, c_{4.n_t}) = \frac{e^{-\frac{\|y_r - \sum_{t=1}^{n_t} h_{t,r} x_t\|^2}{2\sigma^2}}}{2\pi\sigma^2} \quad (6)$$

The second part of the receiver is an iterative multistage decoder (MSD) [5]. MSD consists in decoding the first level of bits  $a_j$  then the second level of bits  $b_j$  knowing the first level. This is an implementation of the chain rule equation 5.

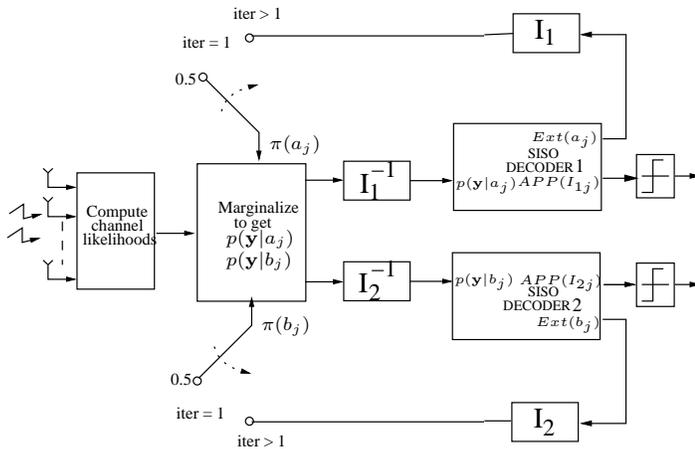


Fig. 4. Two stage iterative MSD decoder.

The decoding at each level is done using a Soft-Input Soft-Output (SISO) decoder that calculates the *a posteriori probability* (APP) associated to a coded bit

$$APP(c_j) \propto Ext(c_j) \times obs(c_j) \quad (7)$$

where  $Ext(c_j)$  is the *extrinsic information*, and  $obs(c_j)$  is the observation. Notice that iterative decoding is run between the SISO decoder and the "Margenalize" block as in the second method of decoding used in [8]. This is based on the fact that the observation  $obs(c_j) = p(y|c_j)$  of a coded bit at level 1 ( $c_j = a_j$ ) or 2 ( $c_j = b_j$ ) depends also on the a priori probabilities  $\pi(c_j)$  according to the following equation

$$p(y|c_j) = \sum_{c_i, i \neq j} \left( \prod_{r=1}^{n_r} p(y_r|c_1, \dots, c_{4.n_t}) \prod_{l \neq j} \pi(c_l) \right) \quad (8)$$

## VII. TRANSMISSION AND COOPERATION STRATEGIES

Let us consider a user node (U1) that would like to transmit information to a certain destination. The transmission strategy of (U1) depends on the presence or not of a second user (U2) capable to work as a relay node (RN).

In the absence of a relay node (no cooperation), the user can transmit information at a full rate, of  $R$  bits per channel use, by sending data over the two levels of the modulation as previously detailed in section IV. However, in the presence of a severe degradation in signal-to-noise ratio, the user can reduce the data rate by sending information over the level 1 only. In this case, the coded bits of the level 2 are assumed to be equal to  $b_j = 0$ . Analyzing the gray labeling of the 16-QAM, we find that symbol mapping will lead to symbols of a sub QPSK constellation. Therefore, the modulator scheme is now equivalent to a 1-level BICM scheme combined to a QPSK constellation with a reduced rate  $R_s = 4 \times R_1$  bits per channel use.

In the presence of a relay node (RN), we only studied the case where the user is transmitting data at a full rate using the two levels of the modulation. Half-duplex relay channel with decode and forward relaying protocol is considered, where the channel between the user and the relay is assumed to be error free. Two cooperation strategies are analyzed.

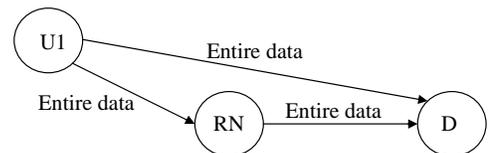


Fig. 5. First cooperation strategy.

In the first strategy, the entire data of the user is retransmitted by the relay using the two levels of the relay (see Figure 5). The cooperation scheme operates as a repetition code where each coded symbols is sent twice over the channel. This is equivalent to a non cooperation scheme with twice the number of received antennas.

Now, in the presence of another user (U3) who would like to cooperate with the same relay (RN) or in case the relay needs to send its own data, half of the coded symbols of user (U1) can be retransmitted by the relay using the level 2 of the relay (RN). The other level is left to the relay (RN) or the user (U3). This is the second cooperation strategy (see Figure 6). Notice that in this case, half of the coded bits  $a_j$  of level 1

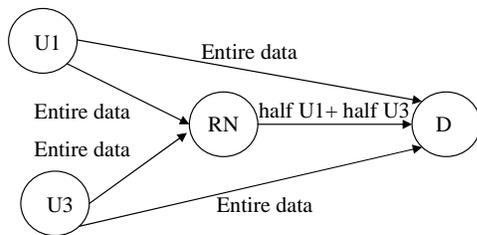


Fig. 6. Second cooperation strategy.

and half the coded bits  $b_j$  of level 2 are retransmitted by the relay.

### VIII. SIMULATION RESULTS

A *non-ergodic* block fading channel with  $n_t = n_r = 2$  transmit and receive antennas is considered. For the 16-QAM constellation, the total rate is equal to  $R = 4$  bits per channel use. From Figure 3, we get that the optimal code rates of the encoders at levels 1 and 2, for  $R = 4$ , are respectively equal to  $R_1 = 0.6$  and  $R_2 = 0.4$ . The length of the coded symbols is  $N_s = 100$  symbols ( $N_b = 200$  information bits). Encoder 1 is a punctured 16-state non-recursive non-systematic convolutional encoder with octal generators equal to  $g = (23, 35)$ . Encoder 2 is a rate-2/5 16-state non-recursive non-systematic convolutional encoder with octal generators equal to  $g = (27, 71, 52, 65, 57)$ . The frame error rate, as function of the signal-to-noise ratio  $\frac{E_s}{N_0}$  in dB, is determined using Monte Carlo simulation.

In Figure 7, we validate the good performance of the 2-level BICM by comparing it to the one obtained using a 1-level BICM based on a rate-1/2, 16-state, non-recursive non-systematic convolutional code. We also represented the outage probability  $P_{out} = Prob(C_H < R)$  of the non-ergodic block fading channel with continuous and discrete 16-QAM inputs.  $P_{out}$  is the probability that the code rate is not supported by the channel and  $C_H$  is the capacity for a fixed channel use  $\mathbf{H} = H$ . This is the optimal performance that can be achieved over the MIMO channel.

The frame error rate of the two cooperation strategies are sketched in Figure 8. We also represented the performance in case of no cooperation for both, full rate ( $R = 4$ ) using a 16-QAM constellation, and reduced rate ( $R_s = 2.4$ ) using the sub QPSK constellation.

In the absence of a relay node, results in Figure 8 show a gain of 2 dB obtained by reducing the code rate to  $R_s = 2.4$  bits per channel use. In the presence of a relay node retransmitting the entire data (first cooperation strategy), Figure 8 shows a gain close to 8 dB. However, taking in consideration the energy transmitted by the relay, the total energy transmitted per channel use will be  $E_s^t = 2 \times E_s$ . Therefore, the real gain is equal to  $8 - 10 \times \log_{10}(2) \approx 5$  dB. Notice that the performance is within 2 dB of the optimal performance achieved over the channel with a relay network retransmitting the entire data sent by the user.

Finally, when the relay node retransmit half of the coded symbols (second cooperation strategy), the total energy trans-

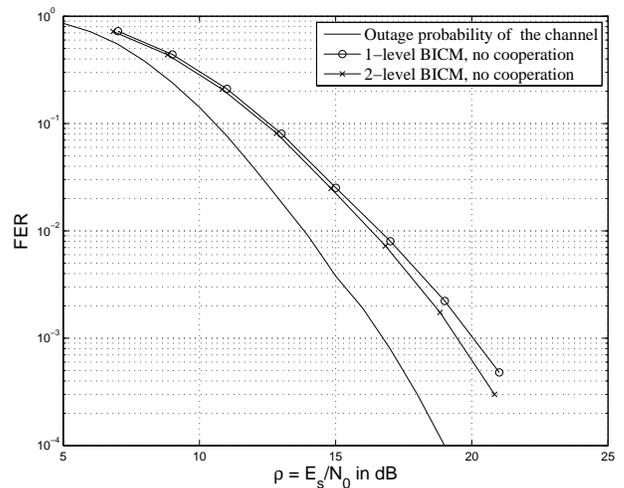


Fig. 7. Frame error rate of the 2-level bit-interleaved coded modulation.

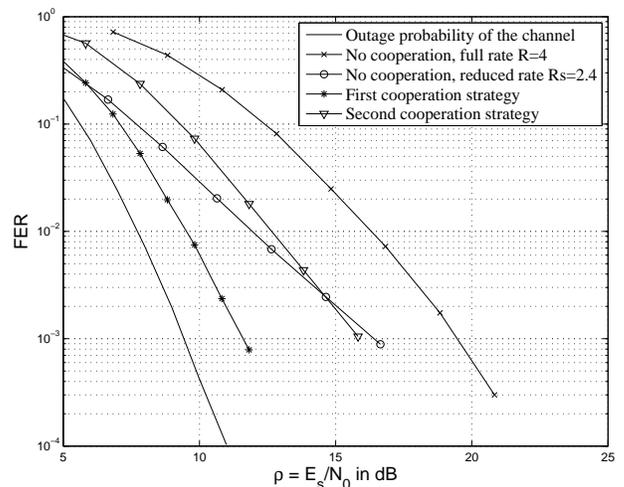


Fig. 8. Frame error rate of the two cooperation strategies compared to non cooperation.

mitted per channel use will be  $E_s^t = 1.5 \times E_s$ . The real gain is equal to  $3.5 - 10 \times \log_{10}(1.5) \approx 1.75$  dB.

### IX. CONCLUSION AND FUTURE WORK

In this paper, cooperative communications is done using a 2-level bit-interleaved coded 16-QAM modulation transmitted over a *non-ergodic* block fading channel with  $n_t = 2$  transmit and  $n_r = 2$  receive antennas. The scheme is flexible allowing cooperation to be done at half and full rate. In both cases, a significant improvement in performance is reached.

Further work could be on studying the cooperation between more than two sources using an L-level BICM ( $L > 2$ ). Analyzing the performance over a frequency selective channel is another future research item.

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