

# Carrier Frequency Recovery for Oversampled Perfect Reconstruction Filter Bank Transceivers

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**Abstract**—In this paper, we consider the problem of data-aided carrier frequency offset (CFO) estimation for filter bank multicarrier systems, with emphasis on oversampled perfect reconstruction filter banks. By exploiting statistical properties of the transmitted pilots in such systems, the maximum likelihood (ML) estimator of the CFO is derived and its performance is investigated numerically for different channel scenarios. The Cramer Rao bound (CRB) on CFO estimator variance for the additive white Gaussian noise (AWGN) channel is also derived as a performance benchmark. Simulation results show that the proposed ML estimator reaches the provided CRB over AWGN channel, while it also exhibits a robust performance in the case of frequency selective channels.

**Keywords**—Data-aided estimation, carrier frequency offset estimation, maximum likelihood, filter bank multicarrier systems.

## I. INTRODUCTION

Due to its desirable characteristics, multicarrier modulation (MCM) is currently the main choice for high speed wireless communications. For instance, one specific form of MCM, orthogonal frequency division multiplexing (OFDM), has been used in many standards, including WiMAX and LTE-Advanced. Recently, to overcome certain limitations of OFDM, alternative forms of MCM have been proposed, which fall into the general category of filter bank multicarrier systems (FBMC). Filtered multitone (FMT) [1] and oversampled perfect reconstruction filter banks (OPRFB) [2], [3] are examples of such systems. However, while FMT and OPRFB have been shown to be less sensitive than OFDM to carrier frequency offset (CFO) [4], some counter measure techniques should be also applied to fully exploit the benefits of MCM in this case.

To mitigate this sensitivity and remove the CFO

effect, limited number of frequency estimation or synchronization algorithms for FBMC systems have been considered. In [5], a *non data-aided* (i.e., blind) CFO estimator for FMT systems is obtained based on the maximum likelihood (ML) principle. Another blind CFO estimator based on the best linear unbiased estimation (BLUE) principle, under the assumption of additive white Gaussian noise (AWGN) channel, is proposed in [6]. Alternatively, a *data-aided* joint symbol timing and frequency synchronization scheme for FMT systems is presented in [7], where the synchronization metric is derived by calculating the time-domain correlation between the received signal and a known pseudo-random training sequence. In [8], a synchronization scheme for data-aided symbol timing and frequency offset recovery is developed by employing the least-squares (LS) approach and exploiting the known structure of a special training sequence whose estimation error does not reach the provided lower bound. Authors in [9], [10] propose CFO estimation schemes based on the ML criterion that are specifically tailored for FBMC systems. While [9] uses a sequence of consecutive pilots, the approach in [10] employs scattered pilots instead, which helps estimate and track channel variations during multicarrier burst transmissions. This method exhibits an improved estimation accuracy when compared to the blind ones, while requiring a moderate pilot overhead and a low complexity.

Although these data-aided methods perform well for FMT systems with root raised cosine prototype filters, they do not demonstrate the same level of accuracy for OPRFB systems. In particular, since OPRFB transceiver employs longer prototype filters, some of the essential assumptions of these methods, e.g., constant CFO effect during the filter support in [9], [10], are not valid in prac-

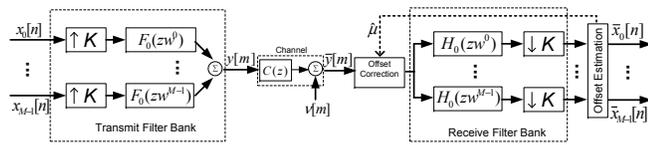


Fig. 1. DFT modulated OPRFB transceiver

tice, which in turn significantly degrades the estimation accuracy as we have been able to observe and therefore, they can not be applied to OPRFB systems that employ longer filters. Consequently, finding a synchronization method well suited for OPRFB systems is of particular interest.

In this paper, we propose a data-aided, ML-based CFO estimation method that is well suited to OPRFB-based MCM systems with longer prototype filters, and investigates its performance numerically for different wireless channel scenarios. By applying judicious simplifications to the log-likelihood function and ignoring the negligible terms, we are able to significantly reduce the implementation complexity of the proposed ML estimator. The Cramer Rao bound (CRB) on CFO estimator variance for the AWGN channel is also derived as a performance benchmark. Results of simulation experiments show that the proposed ML estimator reaches the provided CRB over AWGN channel, while it exhibits a robust performance in the case of frequency selective channels, despite the fact that no channel state information (CSI) is employed.

The paper organization is as follows. The OPRFB system model is outlined in Section II along with a discussion of CFO in such systems. The proposed ML estimator for CFO and the CRB are presented in Section III. The estimator performance is assessed in Section IV. Finally, conclusions are drawn in Section V.

**Notations:** Bold-faced letters indicate vectors and matrices. Superscript  $(\cdot)^*$  denotes the complex conjugation,  $\text{Re}[\cdot]$  the real part and  $|\cdot|$  the absolute value of a complex number.  $\mathbf{I}_K$  denotes the  $K \times K$  identity matrix. Moreover,  $(\cdot)^T$  represent transpose,  $(\cdot)^H$  Hermitian,  $(\cdot)^{-1}$  the inverse of a matrix, while  $E[\cdot]$  stands for the statistical expectation.

## II. PROBLEM FORMULATION

### A. OPRFB System Model

We consider a DFT modulated OPRFB transceiver system, as depicted in Fig. 1, where parameters  $M$  and

$K$  represent the number of subbands and the upsampling/downsampling factor, respectively, and  $K > M$  is assumed. Here,  $x_i[n]$  denotes the complex-valued data sequence transmitted on the  $i$ th subband at discrete-time  $nT_s$ , where  $i \in \{0, \dots, M-1\}$ ,  $n \in \mathbb{Z}$ ,  $T_s = F_s^{-1}$  and  $F_s$  is the input sampling rate. In DFT modulated FBMC systems, the transmit and receive subband filters can be derived from common prototypes with finite impulse responses (FIR) of length  $D$  and respective system functions  $F_0(z) = \sum_{n=0}^{D-1} f_0[n]z^{-n}$  and  $H_0(z) = \sum_{n=0}^{D-1} h_0[n]z^n$ . For convenience in analysis,  $H_i(z)$  is assumed non-causal although in practice, causality can be restored simply by introducing an appropriate delay in the receiver. Defining  $w = e^{-j2\pi/M}$ , the transmit and receive filters for the  $i$ th subband are respectively obtained as

$$F_i(z) = F_0(zw^i), \quad H_i(z) = H_0(zw^i). \quad (1)$$

In this work, the filter length  $D$  is restricted to be a multiple of  $M$  and  $K$ , i.e.,  $D = d_P P$ , where  $P$  denotes the least common multiple of  $M$  and  $K$  and  $d_P$  is a positive integer. As proposed in [2], [3], to enforce the perfect reconstruction (PR) property, the paraconjugates of the transmit filters are employed as receive filters, i.e.,  $h_i[n] = f_i^*[n]$ . Therefore, PR can be expressed as

$$\sum_{q=-\infty}^{\infty} f_j[q - pK] f_i^*[q - nK] = \delta_{ij} \delta_{np}, \quad (2)$$

where  $\delta_{ij}$  denotes the Kronecker delta function. As shown in Fig. 1, the transmitter output signal at discrete-time  $mT_s/K$  is given by

$$y[m] = \sum_{i=0}^{M-1} \sum_q x_i[q] f_i[m - qK], \quad (3)$$

where the range of the summation over  $q$  is delimited by the finite support of the subband FIR filter,  $f_i[m]$ .

We assume that during a time interval equal to the processing delay of the transceiver system (i.e.,  $2DT_s/K$ ), the transmission channel can be modelled as a linear time-invariant system with FIR  $c[l]$  of length  $Q$  and corresponding system function  $C(z) = \sum_{l=0}^{Q-1} c[l]z^{-l}$ . The channel output is corrupted by an AWGN sequence  $\nu[m]$ , with zero-mean and variance  $E[|\nu[m]|^2] = \sigma_\nu^2$ , assumed to be statistically independent from the input data. The input-output relationship of the noisy channel can therefore be expressed as

$$\bar{y}[m] = \sum_{l=0}^{Q-1} c[l]y[m-l] + \nu[m], \quad (4)$$

where  $\bar{y}[m]$  denotes the received baseband discrete-time signal. On the receiver side,  $\bar{y}[m]$  is passed through a bank of analysis filters and downsampled by  $K$ . Accordingly, for each subband, the reconstructed signal  $\bar{x}_i[n]$  can be written as

$$\bar{x}_i[n] = \sum_q \bar{y}[q] f_i^*[q - nK]. \quad (5)$$

### B. Effect of Carrier Frequency Offset

In practice, there often exists a mismatch between the carrier frequency in the receiver and the transmitter, denoted as CFO. In this case, the received signal  $\bar{y}[m]$  can be modelled as [6]–[8]

$$\bar{y}[m] = e^{j2\pi\mu m} \sum_{l=0}^{Q-1} c[l] y[m-l] + \nu[m], \quad (6)$$

where  $\mu$  is a normalized CFO with respect to the subband spacing  $F_s K/M$ . Upon substitution of (3) and (6) into (5), the reconstructed signal for the  $i$ th subband,  $\bar{x}_i[n]$ , can be written in terms of the input signals  $x_j[n]$ , for  $j \in \{0, \dots, M-1\}$ , as

$$\bar{x}_i[n] = \sum_p \sum_{j=0}^{M-1} \Gamma_{i,j}^{n,p}(\mu) x_j[p] + \nu_i[n], \quad (7)$$

where  $\Gamma_{i,j}^{n,p}(\mu)$  and  $\nu_i[n]$  are defined as

$$\Gamma_{i,j}^{n,p}(\mu) = \sum_{l=0}^{Q-1} \sum_q e^{j2\pi\mu q} c[l] f_j[q-l-pK] f_i^*[q-nK], \quad (8)$$

$$\nu_i[n] = \sum_q \nu[q] f_i^*[q-nK]. \quad (9)$$

Here, the complex factor  $\Gamma_{i,j}^{n,p}(\mu)$  (8) characterizes the interference level of the  $p$ th input sample from the  $j$ th subband on the  $n$ th output sample of the  $i$ th subband, in the presence of CFO with magnitude  $\mu$ . We note that for  $|n-p| > (D+Q)/K$ , due to the finite support of  $f_i[n]$ ,  $\Gamma_{i,j}^{n,p}(\mu) = 0$ ; accordingly, the range of the sum over  $p$  in (7) is finite. The term  $\nu_i[n]$  (9) represents the additive noise passed through the  $i$ th subband of the receive filter bank. This term has zero-mean and, due to the PR property imposed on  $f_i[n]$ , its covariance is given by

$$E[\nu_i[q] \nu_j^*[p]] = \delta_{ij} \delta_{qp} \sigma_\nu^2. \quad (10)$$

Considering the reconstructed signal  $\bar{x}_i[n]$  in (7), it appears that even if the channel could be perfectly equalized, which is equivalent to  $c[0] = 1$  and  $c[l] = 0$  for

$l \neq 0$ , the presence of the CFO term  $e^{j2\pi\mu q}$  in the interference factors  $\Gamma_{i,j}^{n,p}(\mu)$  (8) would render the transceiver system non-PR. That is, the terms  $\Gamma_{i,j}^{n,p}(\mu) x_j[p]$  would be non-zero for  $j \neq i$  or  $p \neq n$ , and this in turn would result in a loss of performance in the data transmission process. It is worth to mention that in previous work [9], [10], it is assumed that the CFO factor  $e^{j2\pi\mu q}$  can be taken out of the summation in (8) and consequently the interference terms  $\Gamma_{i,j}^{n,p}(\mu) x_j[p]$  when  $j \neq i$  or  $p \neq n$  are negligible, which does not hold for the OPRFB systems. Our interest therefore lies in the development of a suitable, data-aided ML-based approach for the estimation of the CFO parameter  $\mu$ .

As seen from Fig. 1, once a suitable estimate of  $\mu$  is available, say  $\hat{\mu}$ , it can be used to compensate the CFO at the receiver front-end and thereby avoid its deleterious effects. In this paper, we focus on a simplified model of the noisy channel, i.e., AWGN for which the above condition on the channel coefficients  $c[l]$  is satisfied, but extensions of our proposed approach to more complex time dispersive channels with joint equalization and CFO recovery is possible.

## III. FREQUENCY OFFSET ESTIMATION

In this section, we first derive a novel CFO estimator based on the ML principle, which employs known transmitted pilots. We then propose a number of practical simplifications in the calculation of the associated log-likelihood function (LLF) that result in a lower implementation complexity for this estimator. Finally, the CRB on the variance of unbiased CFO estimators is derived as a performance benchmark.

### A. ML Estimator

As indicated above, we consider a simplified AWGN channel model (i.e.,  $C(z) = 1$ ) in the derivation of the proposed ML-based CFO estimator; consequently, the resulting approach will not require the use of a priori CSI. In this special case, (8) reduces to

$$\Gamma_{i,j}^{n,p}(\mu) = \sum_q e^{j2\pi\mu q} f_j[q-pK] f_i^*[q-nK]. \quad (11)$$

In this work, we define a data frame as the set of  $M$  subband inputs  $x_i[n]$  ( $i \in \{0, 1, \dots, M-1\}$ ) entering the transmit filter bank at time  $n$ . We assume that within a burst of  $N$  consecutive frames, say from  $n = 0$  to  $N-1$ , a total of  $L$  frames, with time indices  $0 \leq t_0 < t_1 < \dots < t_{L-1} \leq N-1$ , are selected for

the transmission of pilot tones. At any given time  $t_n$ , a subset of  $S$  subbands, with indices  $0 \leq s_0 < s_1 < \dots < s_{M-1} \leq M-1$ , are dedicated to the transmission of a unit-energy pilot symbol  $p_{s_i}[t_n]$ . We therefore consider a rectangular lattice of  $N_P = LS$  pilot tones distributed over the time-frequency plane. However, our approach can be applied to other distributions of pilot symbols. Without loss in generality (since the pilot symbols are known to the receiver), we set  $p_{s_i}[t_n] = 1$  for all pair  $(s_i, t_n)$ . Let  $z_{s_i}[t_n]$  denote the reconstructed signal corresponding to the transmitted pilot  $p_{s_i}[t_n]$ . From (7), it follows that

$$z_{s_i}[t_n] = \gamma_{s_i}^{t_n}(\mu) + \nu_{s_i}[t_n], \quad (12)$$

where we defined

$$\gamma_{s_i}^{t_n}(\mu) = \sum_p \sum_{j=0}^{M-1} \Gamma_{s_i, j}^{t_n, p}(\mu) \quad (13)$$

In order to express (12) in compact vector form, we introduce:

$$\mathbf{z}_{s_i} = [z_{s_i}[t_0], z_{s_i}[t_1], \dots, z_{s_i}[t_{L-1}]]^T \quad (14)$$

$$\boldsymbol{\lambda}_{s_i}(\mu) = [\gamma_{s_i}^0(\mu), \gamma_{s_i}^1(\mu), \dots, \gamma_{s_i}^{L-1}(\mu)]^T \quad (15)$$

$$\boldsymbol{\nu}_{s_i} = [\nu_{s_i}[t_0], \nu_{s_i}[t_1], \dots, \nu_{s_i}[t_{L-1}]]^T \quad (16)$$

Therefore, we can write (12) as

$$\mathbf{z}_{s_i} = \boldsymbol{\lambda}_{s_i}(\mu) + \boldsymbol{\nu}_{s_i}. \quad (17)$$

Moreover, by arranging, we can write

$$\mathbf{Z} = \boldsymbol{\Lambda}(\mu) + \mathbf{W} \quad (18)$$

where

$$\mathbf{Z} = [\mathbf{z}_{s_0}^T, \mathbf{z}_{s_1}^T, \dots, \mathbf{z}_{s_{S-1}}^T]^T, \quad (19)$$

$$\boldsymbol{\Lambda}(\mu) = [\boldsymbol{\lambda}_{s_0}(\mu)^T, \boldsymbol{\lambda}_{s_1}(\mu)^T, \dots, \boldsymbol{\lambda}_{s_{S-1}}(\mu)^T]^T, \quad (20)$$

$$\mathbf{W} = [\boldsymbol{\nu}_{s_0}^T, \boldsymbol{\nu}_{s_1}^T, \dots, \boldsymbol{\nu}_{s_{S-1}}^T]^T. \quad (21)$$

As a consequence of the AWGN model assumption, it follows that  $\mathbf{W}$  is a zero-mean Gaussian random vector with diagonal covariance matrix  $\mathbf{C}_W = E[\mathbf{W}\mathbf{W}^*] = \sigma_\nu^2 \mathbf{I}$ , where  $\mathbf{I}$  is the identity matrix. Accordingly, for a given value of the unknown CFO parameter  $\mu$ , the observation vector  $\mathbf{Z}$  in (18) is also Gaussian with mean  $\boldsymbol{\Lambda}(\mu)$  and covariance  $\sigma_\nu^2 \mathbf{I}$ . The probability density function (PDF) of  $\mathbf{Z}$ , say  $f(\mathbf{Z}; \mu)$  can therefore be formulated and subsequently maximized to produce the desired estimate

of  $\mu$ . Take the natural logarithm of this PDF, the LLF [11] can be expressed (up to a constant term) in the form

$$\begin{aligned} \log(f(\mathbf{Z}; \mu)) &= -\frac{1}{\sigma^2} [\mathbf{Z} - \boldsymbol{\Lambda}(\mu)]^H [\mathbf{Z} - \boldsymbol{\Lambda}(\mu)] \\ &= -\frac{1}{\sigma^2} \sum_{i=0}^{S-1} \sum_{n=0}^{L-1} |z_{s_i}[t_n] - \gamma_{s_i}^{t_n}(\mu)|^2 \end{aligned} \quad (22)$$

Finally the ML estimator of the CFO can be written as:

$$\hat{\mu} = \arg \max_{\mu \in \mathcal{M}} \{\log(f(\mathbf{Z}; \mu))\}, \quad (23)$$

where  $\mathcal{M}$  is the search range for  $\mu$ . According to (22), maximization of the LLF attempts to find the CFO  $\mu$ , such that the skewed pilots by this hypothetical  $\mu$  best match (in the LS sense) the reconstructed pilot data at the output of the receive filter bank.

### B. Simplifications of $\Gamma_{i,j}^{n,p}(\mu)$

Here, we propose two simplifications of  $\Gamma_{i,j}^{n,p}(\mu)$  to speed up the calculation of the (22). First consider (11), which includes a summation over the length  $D$  (often large) of the prototype filter  $f_0[q]$ . Recall that  $f_i[q] = f_0[q]w^{-iq}$ , therefore, (11) can be first simplified as

$$\Gamma_{i,j}^{n,p}(\mu) = w^{K(pj-in)} \varphi_{i-j}^{n,p}(\mu), \quad (24)$$

where

$$\varphi_{\Delta}^{n,p}(\mu) = \sum_q e^{j2\pi\mu q} f_0[q - pK] f_0^*[q - nK] w^{q\Delta}. \quad (25)$$

By this implementation, instead of calculating  $\Gamma_{i,j}^{n,p}(\mu)$  for all the  $M^2$  possible pairs  $(i, j)$ , it is sufficient to compute  $\varphi_{\Delta}^{n,p}(\mu)$  for  $2M-1$  possible different values of  $i-j = \Delta \in \{-M+1, \dots, M-1\}$  and find the corresponding  $\Gamma_{i,j}^{n,p}(\mu)$  by a multiplication as in (24). Therefore, we can roughly reduce the number of operations needed to compute the terms  $\Gamma_{i,j}^{n,p}(\mu)$  by a factor of  $M/2$ .

Due to the excellent spectral containment of the prototype filters, we can assume that the main source of the CFO-induced interference on each target subband is due to the first few neighbouring subbands, and that the interference from more distant subbands is negligible. Therefore, as the second proposed simplification, to derive the total interference from the other subbands on the subband with pilot index  $s_i$ , it is sufficient to only factor in the contribution from the two neighbouring subbands on each side of the  $s_i$ th one. As a result, (13) is approximated as

$$\gamma_{s_i}^{t_n}(\mu) \approx \sum_p \sum_{j=s_i-2}^{s_i+2} \Gamma_{s_i, j}^{t_n, p}(\mu) \quad (26)$$

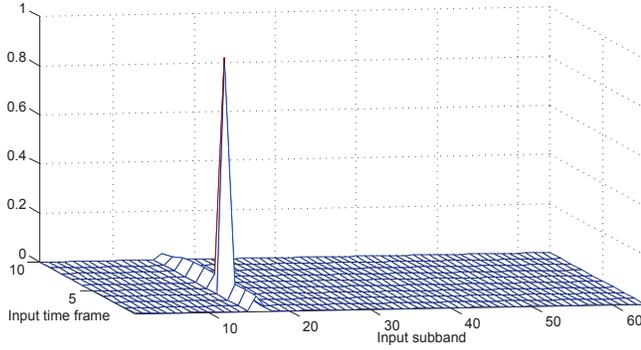


Fig. 2. Interference level  $\Gamma_{i,j}^{n,p}(\mu)$  of  $p$ th input sample from  $j$ th subband on the  $n$ th output sample of the  $i$ th subband ( $p \in \{0, 1, \dots, 10\}$ ,  $j \in \{0, 1, \dots, 63\}$ ,  $n = 4$ ,  $i = 16$  and  $\mu = 0.08$ )

### C. Cramer Rao Bound

Next, we derive a compact expression for the CRB on the variance of an unbiased data-aided CFO estimator obtained over the AWGN channel. Considering  $\partial \mathbf{C}_W / \partial \mu = \mathbf{0}$ , the Fisher  $\mathcal{I}(\mu)$  [11] is

$$\begin{aligned} \mathcal{I}(\mu) &= 2\text{Re} \left[ \frac{\partial \mathbf{\Lambda}(\mu)^H}{\partial \mu} \mathbf{C}_Z^{-1} \frac{\partial \mathbf{\Lambda}(\mu)}{\partial \mu} \right] \\ &= \frac{2}{\sigma^2} \sum_{i=0}^{S-1} \sum_{n=0}^{L-1} \left| \frac{\partial \gamma_{s_i}^{t_n}(\mu)}{\partial \mu} \right|^2, \end{aligned} \quad (27)$$

where  $\text{Re}[\cdot]$  represents the real part of its argument and

$$\frac{\partial \gamma_{s_i}^{t_n}(\mu)}{\partial \mu} = j2\pi \sum_p \sum_{j=0}^{M-1} \sum_q q e^{j2\pi\mu q} f_j[q-l-pK] f_{s_i}^*[q-t_nK]. \quad (28)$$

Therefore, we can obtain the CRB on the variance of an unbiased CFO estimator  $\hat{\mu}$  as

$$\text{Var}(\hat{\mu}) \geq \frac{1}{\mathcal{I}(\mu)} = \left( \frac{2}{\sigma^2} \sum_{i=0}^{S-1} \sum_{n=0}^{L-1} \left| \frac{\partial \gamma_{s_i}^{t_n}(\mu)}{\partial \mu} \right|^2 \right)^{-1}. \quad (29)$$

It can be seen that this CRB is inversely proportional to the signal-to-noise ratio (SNR), or proportional to the noise variance  $\sigma^2$ . Moreover, it generally depends on the number of the observed pilots  $N_p = LS$ . Specifically, it is a decreasing function of both  $L$  and  $S$ .

## IV. RESULTS AND DISCUSSION

In this section, the performance of the proposed ML estimator of the CFO is assessed and compared with the CRB (29). In addition to the AWGN channel (where  $Q = 1$  and  $c[0] = 1$ ), we consider a frequency selective channel consisting of  $Q = 5$  independent Rayleigh-fading taps with an exponentially decaying power delay profile, i.e.,  $E[|c[l]|^2] = \beta e^{-\alpha l}$  for  $l \in \{0, \dots, Q-1\}$ ,  $\alpha = 0.5$ , and  $\beta$  is a constant such that  $\sum_{l=0}^{Q-1} E[|c[l]|^2] =$

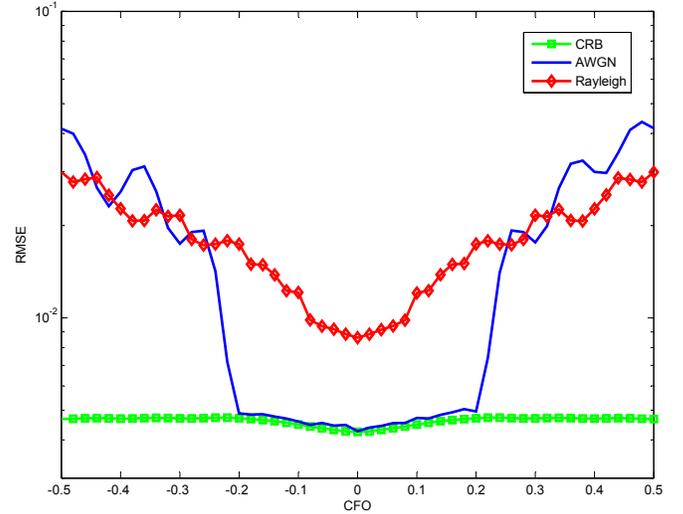


Fig. 3. RMSE of CFO estimator  $\hat{\mu}$  versus  $\mu$  (SNR=10dB,  $L = 10$ )

1. The results are reported for an OPRFB system with  $M = 64$  subbands, up/downsampling factor  $K = 72$  and real prototype filter of length  $D = 1728$ , designed based on the method [2], [3]. Results are presented for different values of the SNR, defined as  $E_s/N_0$ , where  $E_s = E[|x_i[n]|^2]$  is the input symbol energy and  $N_0 = \sigma^2$  is the variance of the channel induced Gaussian noise. Moreover,  $L$  known pilots are inserted at the start of the transmitted burst on all the available subbands ( $S = M$ ).

Figure 2 justifies the assumptions made in Section III-B about the cross-channel interference. In this figure, the level of interference  $\Gamma_{i,j}^{n,p}(\mu)$  from the  $p$ th input sample of the  $j$ th subband on the  $n$ th output sample of the  $i$ th subband is plotted for  $p \in \{0, 1, \dots, 10\}$ ,  $j \in \{0, 1, \dots, 63\}$ ,  $n = 4$ ,  $i = 16$  and  $\mu = 0.08$ . It is evident that only a few subbands surrounding the target subband are contributing as interference sources.

The root mean square error (RMSE) of the proposed estimator, i.e.,  $\sqrt{E[|\hat{\mu} - \mu|^2]}$ , is shown in Figure 3 versus the true CFO  $\mu$  for SNR=10dB and  $L = 10$  pilot frames. Here, the AWGN acquisition range (i.e., the CFO values where the algorithm's RMSE coincides with the CRB) is observed to be  $|\mu| < 0.2$  (i.e., 20% of subband spacing). Clearly, in this interval, the RMSE of the proposed estimator over the AWGN channel is almost independent of the CFO. It is worth mentioning that the results reported in [10] show an acquisition range  $|\mu| < 0.1$  when implemented for FMT transceivers, whereas in the OPRFB context, it leads to unsatisfactory results (i.e., RMSE level of the order of 10%) due to its underlying assumptions. Moreover, the proposed method also

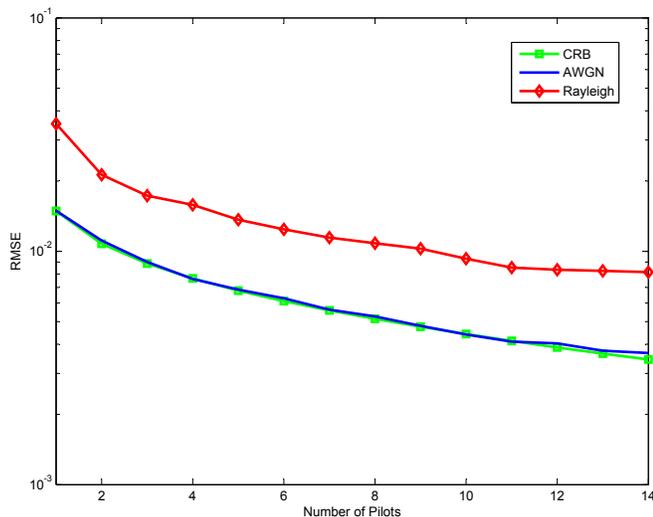


Fig. 4. RMSE of CFO estimator  $\hat{\mu}$  versus  $L$  (SNR=10dB,  $\mu = 0.08$ )

exhibits a robust performance over frequency selective Rayleigh channel, with an RMSE of about 1% within the range  $|\mu| < 0.1$ .

Sensitivity of the proposed estimator to  $L$ , the number of the pilots being used, is depicted in Figure 4 with SNR= 10dB and CFO  $\mu = 0.08$ . As expected, by increasing  $L$ , a better estimation accuracy can be achieved. In addition, over AWGN channel, the CRB is always attainable. Figure 5 exhibits the RMSE performance of the proposed estimator as a function of SNR with true CFO  $\mu = 0.08$  and  $L = 10$ . Interestingly, in contrast to the result for the CFO synchronization method reported in [8], the proposed ML estimator can achieve a performance very close to the CRB over a wide range of SNR values..

## V. CONCLUSION

In this paper, we considered the problem of data-aided CFO estimation and recovery for FBMC systems with emphasis on OPRFBs. By exploiting statistical properties of inserted pilots transmitted by such systems over an AWGN channel, the ML estimator for the CFO was derived. The complexity of the proposed estimator is considerably reduced by identification of the insignificant LLF terms and consequently neglecting them in the estimation. This method was tested over AWGN and frequency selective channels with various CFOs, SNRs and number of pilots. The results show that over the AWGN channel, the proposed estimator exhibits a performance close to the CRB with wider acquisition range compared to other methods, whereas its performance remains satisfactory over frequency selective channels.

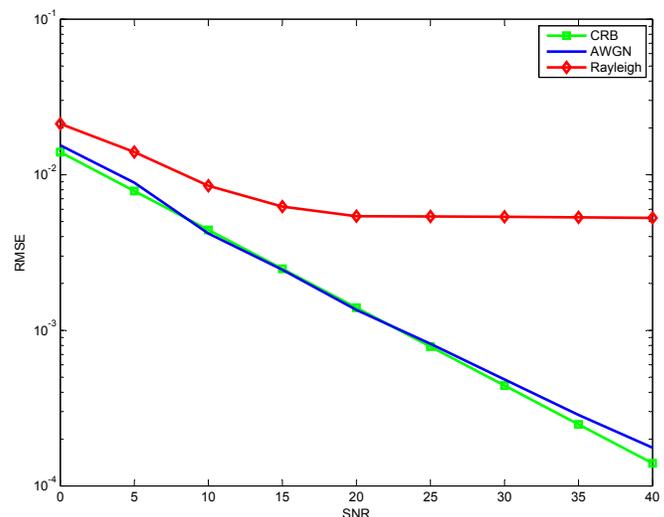


Fig. 5. RMSE of CFO estimator  $\hat{\mu}$  versus SNR (CFO=0.08,  $L = 10$ )

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