Evaluative Study of Detect-Split-Forward Scheme over MIMO Relays

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***Abstract*—Virtual multi-input multi-output (vMIMO) schemes in wireless communication systems improve coverage, throughput, capacity, and quality of service. In this paper, we propose two uplink vMIMO relaying schemes based on detect-split-and-forward (DSF). In addition, we investigate the effect of several physical parameters on the performance of the relaying systems, such as distance, modulation type and number of relays. In addition, we provide analytical tools to evaluate the performance of the proposed vMIMO relaying schemes.**

Keywords–Virtual MIMO, distributed antenna, relaying, detect-split-and-forward, STBC, V-BLAST.

I. INTRODUCTION

Since the introduction of the multiple-input-multiple-output (MIMO) technology, there have been great advancements in data rate speeds and wireless network efficiency. The main purpose of MIMO implementation is to boost the transmission rate by exploiting the randomness of parallel channels. Using MIMO technology, the capacity of a propagation environment decreases with increasing the correlation of the channel coefficients. Practically, for none-line-of-sight (NLOS) and omni-directional wireless mobile communications, there are restrictions on handset manufacturing caused by wavelegths. Hence, the designers should select applicable wavelengths to realize the full potential of MIMO receivers. Obviously, large antenna arrays of different sizes are not always practical for handsets mobile. Furthermore, the developers of the next generation wireless systems are investing in virtual MIMO (vMIMO) [1]. Virtual MIMO (vMIMO) is a recent model adapted from the broadcasting model of wireless channels where all communication nodes (relays) support each other. The goal of vMIMO is to provide better quality-of-service (QoS) at higher data rates, especially for users who are at the cell edge. This technique acts in a way similar to multi-user (MU-MIMO) technique in the uplink side, also called network MIMO [1]. vMIMO is based on the concept of relaying over virtual antenna arrays. with results in enhancing the end-to-end link performance, offering good QoS and extending coverage range in NLOS environment. vMIMO systems execute the communication process in a distributed manner to take advantages of the MIMO system while reducing consumption of battery, improving capacity and expanding network lifetime [2].

Relay structures have evolved by the introduction of virtual antenna arrays and MIMO relays [3] and [4]. The relaying technique, as introduced by Van der Meulen [5], has transpired through the years as the most well-known approach to improve the reliability and performance of wireless networks. It makes use of node cooperation and it allows a network to extend its coverage without exhausting its power resources. The two well-known relaying protocols, are: amplify-and forward (AF) and decode-and forward (DF) [6]. AF is the simplest as it only amplifies the received signal then forwards it. However, its drawback is that it amplifies the noise in addition to the signal. This technique, as described in the IEEE802.16j [7] standard, does not require the mobile station (MS) to be aware of intermediate relays. On the other hand, DF is a protocol that uses error detection and correction as it decodes data once received and confirms its correctness then forwards the data. This technique is generally used with hybrid automatic repeat request (HARQ) to ensure that correct data was decoded and intact [6].

The authors of [8] and [9] extended relaying concepts to MIMO by considering several relay transmission and topology schemes (e.g., parallel, serial and hybrid) and taking into consideration practical MIMO systems. The authors of [10] and [11] investigated the performance of space-time block coding (STBC) with MIMO relaying using AF as an effective way to introduce spatial diversity. Virtual spatial multiplexing with AF relaying was investigated in [12] and closed-form expressions were derived at high-SNRs. In [13], IEEE802.16e described the uplink virtual MIMO (UL-vMIMO) as follows: each user is equipped with a single antenna and shares the same channel resources with other users. By utilizing simultaneous transmissions over a common burst, vMIMO increases the peak transmission rate and improves the system performance. In addition, DF virtual relaying scheme for MIMO systems was analyzed in [14].

In [15], an AF virtual spatial multiplexing scheme is proposed in which each transmitter is equipped with a single antenna. The transmitters form a virtual antenna array and send identical signals to relays that amplify-and-forward different portions of the signal at a reduced data rate to the destination. The receiver is equipped with multiple antennas in order to null and cancel the interference from the different relays and detect the original signal transmitted from the source. Another approach proposed by [16] is to let the relays detect a sub-stream from the original stream. Then, all relays forward their low rate sub-streams simultaneously over the same physical channel. This scheme has the advantage of controlling noise, as in digital systems, so that it is not amplified. Another advantage is that the vMIMO relay can send data with lower modulation rates which improves the bit error rate (BER). Because of the practical difficulty associated with antenna coupling, another alternative technique is proposed in [16]. In this paper we refer to the scheme of [16] as detect-split-forward (DSF) using vertical-bell laboratories layered space-time (VBLAST).

Up to the authors knowledge, there is no analysis for the error probability of DSF schemes over vMIMO channels. Thus, in this paper we provide analytical tools to evaluate the performance of DSF-vMIMO schemes, for both VBLAST and STBC. Both simulations and analysis are conducted to evaluate the system performance in terms of several physical parameters such as distance, modulation type, and number of relays.

The remainder of this paper is organized as follows. Section II introduces the system and channel models. The analysis of system performance and the average capacity are conducted in Section III and Section IV. In Section V, some simulation and numerical results are presented and discussed. Finally, conclusions are given in Section VI.

II. System Model

We consider a $1 \times N_R \times N_D$ uplink system, where 1 indicates a source with a single antenna, N_R is number of relays, each equipped with a single antenna. N_D is number of receiving antennas at the destination. The source modulates a block of k information bits and transmits an 2^k-ary modulated symbol x, which is received by all relays. Then, each relay detects the information bits and splits them into N_R blocks of length m bits, where $mN_R = k$. At each relay, m-bit block is modulated using lower level modulation schemes (2^m-ary symbol) and will be transmitted through N_R relays, which creates a vMIMO system. At the relays, two vMIMO schemes are considered for relaying the N_R symbols. The first scheme uses spatial multiplexing based on VBLAST and the second scheme uses STBC.

A. DSF-vMIMO Schemes

To illustrate DSF vMIMO schemes, consider a $1 \times 2 \times 2$ system with 2 bps/Hz efficiency. The relays receive the following signals from the source:

$$Y_{R_1} = h_{SR_1} x + n_{R_1} \tag{1}$$

$$Y_{R_2} = h_{SR_2} x + n_{R_2}, \tag{2}$$

where x is the transmitted symbol, Y_{R_i} is the received signal at relay i, and h_{SR_i} is the complex Gaussian fading coefficient, with zero mean and variance of one, from the source to relay i. In addition, n_{R_i} is the additive white Gaussian noise at relay i with zero-mean and variance N_o, where N_o is the noise power spectral density. Each relay detects the transmitted symbol x. The detection, splitting and forwarding of each vMIMO scheme is explained next.

1) DSF-VBLAST Scheme: The DSF-VBLAST scheme detects the 2^k-ary symbol, splits it into N_R parallel symbols, where each symbol carries m bits and then forwards the signals simultaneously to the destination as shown in Figure 1. As an example, for a 2 bps/Hz efficiency, the source modulates 4 bits using 16-QAM and sends it to the relays. Each relay detects the 16-QAM symbol and demodulates the 4 bits. Then, these 4 bits will be splitted into two blocks, each consisting of two bits. These two bits will be modulated using QPSK and then spatially multiplexed and forwarded to the destination. Since 4 bits have been transmitted using two hops, the system efficiency is 2 bps/Hz.

The destination receives the following vMIMO signals:

$$\begin{pmatrix} Y_{D_1} \\ Y_{D_2} \end{pmatrix} = \begin{pmatrix} h_{R_1D_1} & h_{R_2D_1} \\ h_{R_2D_2} & h_{R_1D_2} \end{pmatrix} \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} + \begin{pmatrix} n_{D_1} \\ n_{D_2} \end{pmatrix}, \tag{3}$$

where Y_{D_j} is the received signal at antenna j at the destination, $\hat{x}_i$ is the transmitted signal from relay i, and $h_{R_iD_j}$ is the complex Gaussian fading coefficient, with zero mean and unit variance, between relay i and antenna j at the destination. In addition, n_{D_j} is the additive white Gaussian noise at antenna j at the destination.

The destination applies the VBLAST detection algorithm with successive interference cancellation (SIC) and it performs nulling, based on zero-forcing (ZF), cancelation and ordering.

2) DSF-STBC Scheme: To improve the diversity order and provide more link reliability, STBC is used in the second hop. At the relays, the 2^k-ary symbol is detected and split into N_R symbols, each carries k/N_R bits. After that, the splitted symbols are mapped to an STBC as shown in Figure 2. As an example, for a 2 bps/Hz, the source sends 6 bits using a 64-QAM symbol to the two relays. Then, each relay detects the 64-QAM symbol and demodulates the 6 bits and split them into two blocks, each consists of 3 bits. Each block of 3 bits is modulated using 8PSK and then mapped to Alamouti STBC code. The STBC codes will be forwarded to the destination over two time slots. Since 6 bits have been transmitted during three time slots, the system efficiency is 2 bps/Hz. The signals received by the destination could be expressed as:

$$\begin{pmatrix} Y^2_{D_1} \\ Y^2_{D_2} \\ Y^{3*}_{D_1} \\ Y^{3*}_{D_2} \end{pmatrix} = \begin{pmatrix} h^2_{R_1D_1} & h^2_{R_1D_2} \\ h^2_{R_2D_1} & h^2_{R_2D_2} \\ h^{3*}_{R_1D_1} & -h^{3*}_{R_1D_2} \\ h^{3*}_{R_2D_1} & -h^{3*}_{R_2D_2} \end{pmatrix} \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} + \begin{pmatrix} n^2_{D_1} \\ n^2_{D_1} \\ n^{3*}_{D_1} \\ n^{3*}_{D_1} \end{pmatrix}, \tag{4}$$

where $Y^t_{D_j}$ is the received signal at time slot t and antenna j, $h^t_{R_iD_j}$ is the channel from relay j to antenna j at the destination at time slot t. The MIMO channel from the relay to the destination is assumed to be quasi-static complex Gaussian channel, where each cofficient has a zero mean and unit variance.

III. Performance Analysis

A. DSF-VBLAST Scheme

To calculate the block error rate (BLER) for the DSF V-BLAST scheme, we analyze the detection process at each hop. Assume that P_{B,H_1} is the BLER of the first hop and P_{B,H_2} is the BLER of the second hop. Then the total BLER at the destination will be:

$$P_B = 1 - (1 - P_{B,H_1})(1 - P_{B,H_2}), \tag{5}$$

Since in the first hop, the source transmit the modulated symbol to N_R single antenna relays, the BLER at the first hop is

$$P_{B,H_1} = 1 - (1 - P_{e,R})^{N_R}. \quad (6)$$

Where $P_{e,R}$ is the symbol error rate (SER) at each relay for M-ary modulation over fading channels [17], taking into account the appropriate signal set levels and energy after splitting. Thus SER at each relay is:

$$P_{e,R} = \left(\frac{M-1}{M}\right)\left(1 - \sqrt{\frac{1.5\gamma_R}{M^2 - 1 - 3\gamma_R}}\right), \quad (7)$$

where $M = 2^m$ is the new cardinality of the signal set after splitting and γ_R denotes the average received SNR at each relay.

Since VBLAST detection is used in the second hop, each layer of the V-BLAST scheme has a different error probability depending on its diversity order [18].

Assume that $P_{e,i}$ is the SER for layer i over Rayleigh fading channels, then the BLER of the second hop is:

$$P_{B,H_2} = 1 - \prod_{i=1}^{N_R}(1 - P_{e,i}), \quad (8)$$

For M-QAM signals, $P_{e,i}$ is [19]:

$$\begin{aligned} P_{e,i} = {} & 4(1 - \tfrac{1}{\sqrt{M}})(\tfrac{1-\zeta_i}{2})^{D_i} \sum_{j=0}^{D_i-1} \binom{D_i - 1 + j}{j} (\tfrac{1+\zeta_i}{2})^j \\ & -4(1 - \tfrac{1}{\sqrt{M}})^2 \{\tfrac{1}{4} - \tfrac{\zeta_i}{\Pi}\{(\tfrac{\Pi}{2} - \tan^{-1}\zeta_i) \sum_{j=0}^{D_i-1} \binom{2j}{j} (\tfrac{1-\zeta_j}{4})^j \\ & + \sin(\tan^{-1}\zeta_i) \sum_{j=1}^{D_i-1} \sum_{r=1}^{j} \tfrac{Jrj}{(1+\beta_i)^j} [\cos(\tan^{-1})\zeta_i]^{2(j-r)+1}\}\}, \end{aligned} \quad (9)$$

where the diversity order of layer i is:

$$D_i = N_D - N_R + i, \quad (10)$$

and N_D is the total number of receiver antenna at the destination. Let d be the distance between the relays and the source, and v be the path loss, then the parameters in (9) are defined as

$$\zeta_i = \frac{\beta_i}{1 + \beta_i} \quad (11)$$

$$\beta_i = \frac{3d^{-v}\gamma_D}{2N_R(M-1)} \quad (12)$$

$$J_{rj} = \frac{\binom{2j}{j}}{\binom{2(j-r)}{(j-r)} 4^i (2(j-r)+1)}. \quad (13)$$

where γ_D denotes the average received SNR at each receive antenna at the destination.

For M-PSK case, $P_{e,i}$ is [19]:

$$\begin{aligned} P_{e,i} = {} & \frac{M-1}{M} - \frac{\mu_t}{\sqrt{\mu_t^2+1}}(\frac{1}{2} + \frac{\omega_t}{\Pi}) \sum_{\tau=0}^{D_t-1} \binom{2\tau}{\tau} [4(\mu_t^2+1)]^{-\tau} \\ & - \frac{\mu_t}{\sqrt{\mu_t^2+1}} \frac{1}{\Pi} \sin(\omega_t) \sum_{\tau=1}^{D_t-1} \sum_{i=1}^{\tau} \frac{Ji\tau}{(\mu_t^2+1)^\tau} [\cos(\omega_t)]^{2(\tau-i)+1}, \end{aligned} \quad (14)$$

where

$$\mu_t = \sqrt{\rho_t}\sin(\frac{\Pi}{M}) \quad (15)$$

$$\rho_t = d^{-v} \cdot \gamma_D \quad (16)$$

$$\omega_t = \tan^{-1}(\frac{\sqrt{\rho_t}\cos(\frac{\Pi}{M})}{\sqrt{\mu_t^2+1}}). \quad (17)$$

B. DSF-STBC Scheme

For M-QAM STBC, the SEP at the second hop could be calculated using (9) with a diversity order $N_D \cdot N_R$ and with one layer ($L = 1$). Therefore, the second hop BLER is $P_{B,H_2} = P_{e,1}$ with a diversity $D = N_D \cdot N_R$. Then the total BLER P_B is calculated using (5).

IV. Capacity Analysis

The DSF-vMIMO capacity is presented in this section. The analysis of the capacity of DSF-vMIMO for V-BLAST and STBC is based on the fact that the instantaneous capacity of the two hop relay system is determined by the weakest link. The first hop consists of two SISO channels and the second hop consists of a MIMO channel. The second MIMO hop can be either V-BLAST or STBC. Therefore, the instantaneous capacity of DSF-VBLAST can be calculated as:

$$C_{DSF-VBLAST} = \frac{\min\{C_{SISO1}, C_{SISO2}, C_{VBLAST}^{ZF}\}}{N_H}, \quad (18)$$

and the capacity of DSF STBC is:

$$C_{DSF-STBC} = \frac{\min\{C_{SISO1}, C_{SISO2}, C_{STBC}\}}{N_H}, \quad (19)$$

where N_H is the total number of hops.

In the above equations, C_{SISO} is the instantaneous capacity of single-input single-output flat Rayleigh fading channels [18] at the first hop, which is expressed as:

$$C_{SISO} = \log_2(1 + \gamma_R|h|^2) \ bps/Hz, \quad (20)$$

where the channel coefficient h is a complex Gaussian random variable with zero mean and unit variance, and γ_R is the average SNR at each relay. For a given h, there is only one way to increase the capacity of the SISO channel and that is by increasing SNR. Also, the capacity increases logarithmically with increasing SNR.

The second hop is either a VBLAST or an STBC. For VBLAST, the instantaneous capacity with N_R relays and with ZF interference nulling and serial cancelation is given by [20]:

$$C_{VBLAST}^{ZF} = N_R.min_{j=1,2,\ldots,R_N}\{\log_2(1 + \frac{\gamma_D}{N_R\|W_{ZF,j}\|_F^2})\}, \quad (21)$$

where $W_{ZF,j}$ is the ZF projection vector of the j^{th} relay, γ_D is the SNR per receive antenna at the destination, and $\|(\cdot)\|_F^2$ is the squared Frobenius norm.

For STBC, the instantaneous capacity rate r_c code and N_R relays is [21]

$$C_{STBC} = r_c\{\log_2(1 + \frac{\gamma_D}{N_R}\|\mathbf{H}\|_F^2)\}. \qquad (22)$$

V. Numerical Results

In this section, the performance of DSF V-BLAST and DSF-STBC schemes are evaluated using the analytical results and verified by Monte-Carlo simulations. The channel models used in the simulation are Rayleigh flat-fading channels characterized by complex Gaussian random variables with zero-mean and 0.5 variance per dimension. In addition additive white Gaussian noise is added at each receive antenna. Both BLER performance and average capacity are evaluated The spectral efficiency is computed as the total number of bits received at the destination divided by the total number of transmission time slots.

The BLER performance of $1 \times 2 \times 2$ DSF-VBLAST and DSF-STBC are shown in Fig. 3 and 4, respectively. The performance is examined at several spectral efficiencies as shown in Tables I and II. The relays are placed at a normalized distance of 0.3 from the source. The results in this figure illustrate the accuracy of our analysis.

BLER comparison between DSF-VBLAST and DSF-STBC schemes is shown in Fig. 5 The result shows the effect of relay location on the performance of both systems and the inherent tradeoffs. The performance is examined at low, medium, and high SNRs and the normalized distance is changed from zero to one. The result shows that when the relays are closer to the source, the DSF-STBC performs better than DSF-V-BLAST. However, when the relays are placed further than 0.4, the performance of DSF-VBLAST becomes better. From this result, we propose to design a hybrid system where the relays use adaptive techniques to determine the best scheme to be used based on the distance from the source.

The average capacities of DSF-VBALST and DSF-STBC schemes are shown in Fig. 6. At a certain distance and since both systems consist of SISO and MIMO hops, the SISO channel will dominate the overall channel capacity. The result in Fig. 6 shows that at distances greater than $d = 0.5$, the DSF-VBLAST channel capacity will perform the same as the DSF-STBC since both systems are dominated by the weakest channel, which is the SISO channel at the first hop.

VI. Conclusion

In this paper, we analyzed detect-split-forward uplink virtual MIMO relying schemes based on VBLAST and STBC. The relays split the incoming source signal into lower modulation levels. This will enhance the energy efficiency of the relaying system. Block error rate expressions and outage capacity rates were presented. The analysis presented in this paper matched the simulation results and it showed the effect of several physical parameters such as distance, modulation type and number of relays. For future work, there exists a tradeoff between VBLAST and STBC which leads to designing an adaptive relaying system that can adapt the MIMO scheme based on the location of the relay to satisfy a certain quality of service.

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TABLE I. $1 \times 2 \times 2$ DSF-VBLAST

Time Slots	1^{st} Hop	2^{nd} Hop	bps/Hz
2	QPSK	BPSK	1
2	16-QAM	QPSK	2
2	64-QAM	8PSK	3
2	256-QAM	16-QAM	4

TABLE II. $1 \times 2 \times 2$ DSF-STBC

Time Slots	1^{st} Hop	2^{nd} Hop	bps/Hz
3	QPSK	BPSK	0.66
3	16-QAM	QPSK	1.33
3	64-QAM	8PSK	2
3	256-QAM	16-QAM	2.66

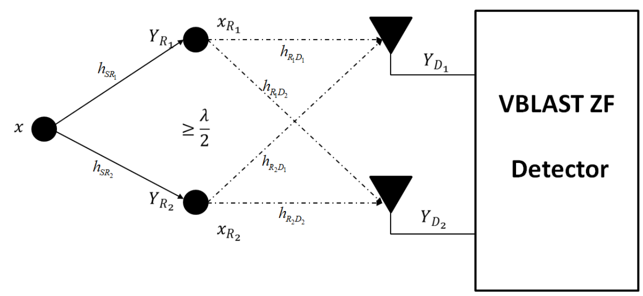

Figure 1. System model of $1 \times 2 \times 2$ DSF-VBLAST relaying scheme.

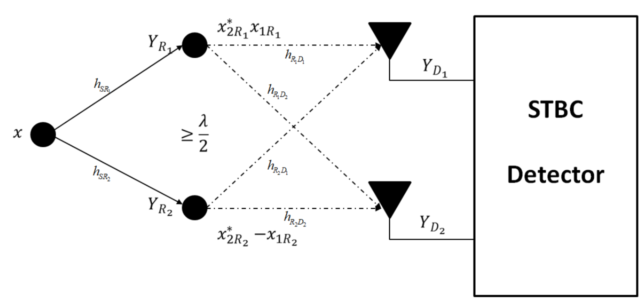

Figure 2. System model of $1 \times 2 \times 2$ DSF-STBC relaying scheme.

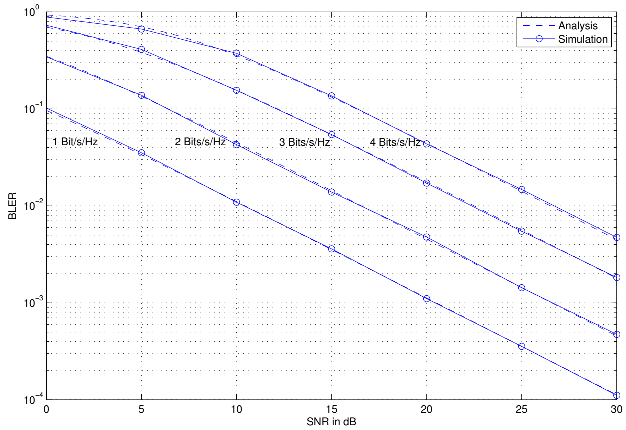

Figure 3. BLER performance comparison of DSF-VBLAST relaying schemes for $1 \times 2 \times 2$ vMIMO system.

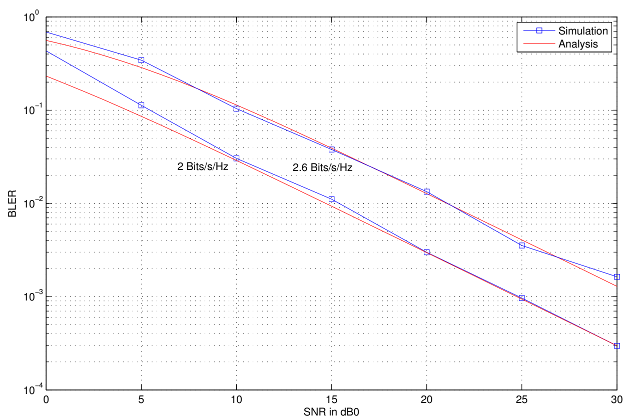

Figure 4. BLER performance of DSF-STBC relaying schemes at 0.3 normalized distance from the source.

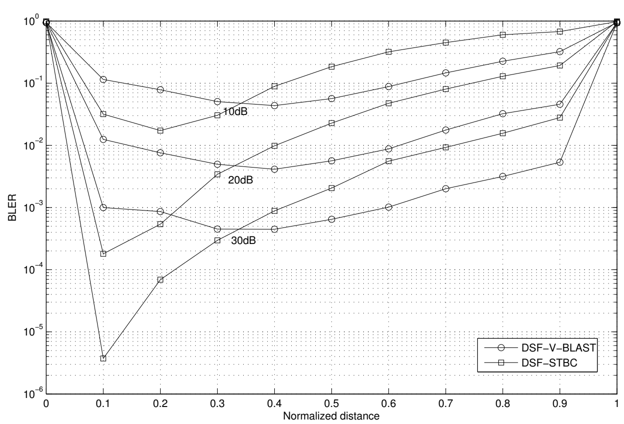

Figure 5. Effect of the source-relay distance on the BLER performance of DSF-vMIMO relaying schemes.

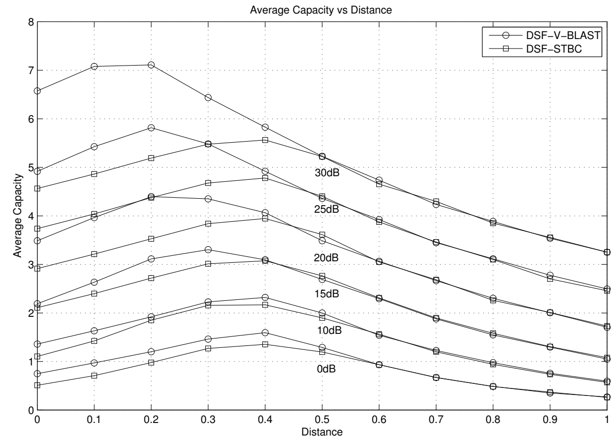

Figure 6. Average capacity of DSF-vMIMO relaying schemes at different source-relay distances.

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