Channel Estimation for Downlink LTE System Based on LAGRANGE Polynomial Interpolation

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Abstract—**Long Term Evolution (LTE) uses different techniques to achieve high throughput required, such as the HARQ techniques, Multiple-Input Multiple-Output (MIMO) transmission schemes and estimation techniques. In the present paper, research focuses on Downlink LTE channel estimation which is based on the interpolation to estimate channel coefficients. Thus, we propose an adaptive method for polynomial interpolation based on Lagrange polynomial. We perform the Downlink LTE system for Single-Input Single-Output (SISO) and MIMO transmission then compare the obtained results with linear and Sinus Cardinal Interpolations. The simulation results show that the Lagrange method outperforms the linear interpolation in term of Block Error Rate (BLER) and throughput vs. Signal to Noise Ratio (SNR).**

Keywords-LTE; MIMO; SISO; Interpolation; Lagrange

I. INTRODUCTION

In modern world, requirement of high data rate communication has become inevitable. Applications such as streaming transmission, video images, and World Wide Web browsing require high speed data transmission with mobility. In order to fulfill these data requirements, the 3rd Generation Partnership Project (3GPP) [1][2] introduced Long Term Evolution (LTE), to provide high speed data rate for mobile communication. The LTE system affords an important effective bit rate and allows increasing system capacity in terms of numbers of simultaneous calls per cell. In addition, it has a low latency compared to 3G/3G + networks. It offers a theoretical speed of 100 Mbits/ s in the Downlink and 50Mbits/s in the Uplink transmission. The LTE uses Orthogonal Frequency Division Modulation (OFDM) and Orthogonal Frequency Division Modulation multiple access technique (OFDMA) in the downlink transmission [3]. The OFDM provides the signal transmitted robustness against the multipath effect and can improve the spectral efficiency of the system [4][5]. On the other hand, the implementation of MIMO system increases channel capacity and decreases the signal fading by sending the same information at the same time through multiple antennas[5]. The combination of these two powerful technologies (MIMO-OFDM) in the LTE system improving thus the spectral efficiency and throughput offered without increasing resources for base bands and power output. To best exploit the power of MIMO-OFDM technology, it is imperative to manage at best the estimation of the channel coefficients; this operation is ensured by the interpolation of pilots.

In this paper, we represent a polynomial interpolation algorithm using the method of Lagrange [10] which greatly reduces the complexity of the transceiver. The simulation is made on a 'Vehicular A' (Veh A) [11][12] channel through SISO and MIMO system using Least Square equalizer (LS). Section II of this paper give an over view of MIMO-OFDM transmission. In Section III, we present Lagrange interpolation algorithm. Finally, Section IV provides the numerical results.

II. MIMO-OFDM TRANSMISSION

A. MIMO OFDM transmissions schemes

In this section, we are going to describe the MIMO OFDM transmission scheme. A modulation block is used to modulate the original binary data symbol using the complex constellation QPSK, 16 QAM or 64 QAM according to the LTE standard [6][7]. Pilot insertion is generated according to the LTE standards, followed by Inverse Fast Fourier Transform operation (IFFT); at the end, a cyclical prefix is inserted to remedy the phenomenon of the Inter Symbol Interference (ISI) and the Inter Sub carriers Interference. Transmission is made through a multipath Fast Fading channel over a multiple antenna system. Multiple antennas can be used in the transmitter and the receiver; consequently, MIMO encoders are needed to increase the spatial diversity or the channel capacity. Applying MIMO allows us to get a diversity gain to remove signal fading or getting a gain in terms of capacity. Generally, there are three types of MIMO receivers, as presented in [1]. At the reception, the cyclical prefix is firstly removed, followed by the Fast Fourier Transform operation (FFT); after the extraction of pilots, parameters of channel is estimated through the block interpolation followed by equalization. The method of interpolation chosen is essential to make the estimation more efficient and to reduce the equalizer complexity.

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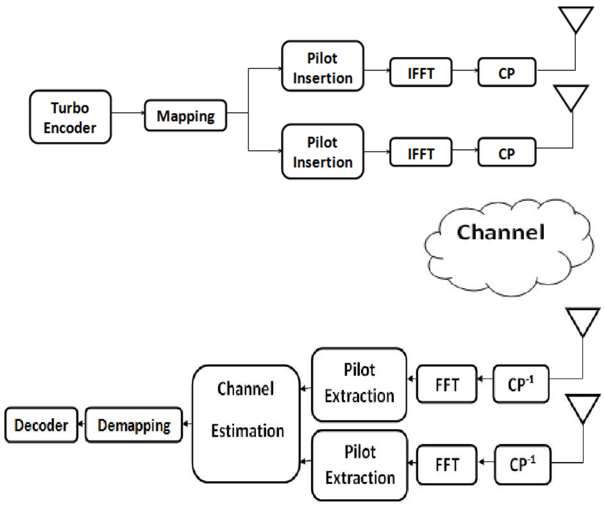

Fig. 1. MIMO-OFDM transmission.

B. Aanalysis of standard LTE pilot scattering

In the LTE standards, pilots are placed in a well-defined ways to cover up the frequency and time domain. The location of pilots for 2x2 MIMO transmissions scheme in LTE system is shown in the following figures.

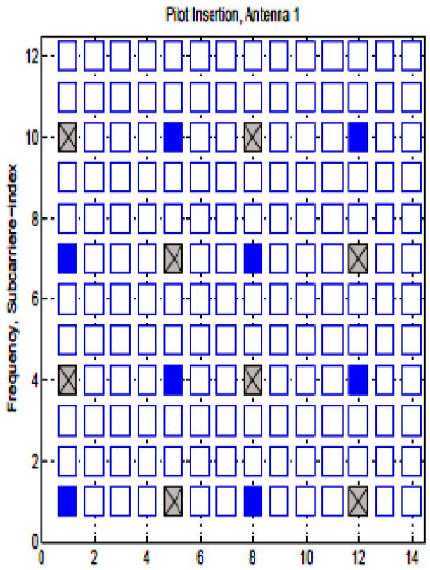

Fig. 2. Pilot structure of Transmit

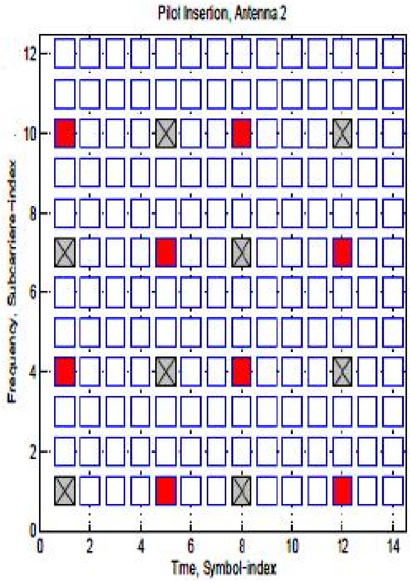

Fig. 3. Pilot structure of Transmitter 2.

It can be seen that, through the first antenna, pilots are disposed in OFDM symbols numbers 1, 5, 8 and 12 while for the second antenna, they are placed in the same OFDM symbols, but in different subcarriers index.

Those positions allow a better coverage of the frequency and time and reduce the risk of interference in reception [3].

III. Description of the Interpolation Algorithm

A. Linear Interpolation

In linear polynomial interpolation, the channel coefficients are estimated using the linear relationship between two successive pilots.

Linear interpolation is given by the following expression:

$$H_{(k)}^{(i)} = i/d * H_{(k)(p)}^{(i)} + (1 - i/d) * H_{(k)(p+1)}^{(i)} \qquad (1)$$

where $H_{(k)}^{(i)}$ is the channel coefficient to estimate, $H_{(k)(p)}^{(i)}$ and $H_{(k)(p+1)}^{(i)}$ two successive pilots, $\boldsymbol{i}$ is the subcarriers index, $\boldsymbol{k}$ is the OFDM symbol index, $\boldsymbol{p}$ is the pilot index and $\boldsymbol{d}$ is the distance between two pilots [8].

B. Sinus Cardinal Interpolation

Sinus Cardinal (SinC) interpolation is given by the following expression [7]:

$$S\ (x) = \sum_{i=0}^{n} S\ (k)\mathrm{SINC}(x-k)(2)$$

where $S\ (k)$ is the pilots, $\boldsymbol{k}$ is the position of $\boldsymbol{y}$,$S\ (x)$is the SinC interpolation function.

In this work, we use 2 pilots to estimate channel coefficients using SinC interpolation.

The interpolation is represented as follow:

1-Extract received $y_{(k)(P)}^{(i)}$ pilots from received signal $y_{(k)}^{(i)}$

2-Calculate the channel coefficients of pilots symbols with Least Square estimator

$$H_{(k)(p)}^{(i)} = y_{(k)(p)}^{(i)} / x_{(k)(p)}^{(i)} \quad (3)$$

3- Estimate $H_{(k)}^{(i)}$ with SinC interpolation:

$$H_{(k)}^{(i)} = \text{sinc}(x - x_{p_0}) * H_{(k)(p0)}^{(i)} + \text{sinc}(x - x_{p_1}) \quad (4)$$

C. Lagrange polynomial Interpolation

Lagrange polynomials allow interpolating a set of points by a polynomial which goes exactly through these points. The Lagrange polynomial is given by the following expression [10]

$$P(x) = \sum_{i=0}^{n} y_i L_i(x) \quad (5)$$

$$L(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} (x - x_i)/(x_i - x_j) \quad (6)$$

Where $\boldsymbol{y_i}$ the pilots, $\boldsymbol{x}$ is the position of $\boldsymbol{y}$, $\boldsymbol{L}$ is the coefficients of Lagrange and $\boldsymbol{n}$ is the Lagrange polynomial order.

D. Algorithm description

The received signal for MIMO system model consisting of N_T transmits antennas and N_R receives antennas can be represented by the following Equation:

$$Y_{(k)}^{(i)} = X_{(k)}^{(i)} H_{(k)}^{(i)} + N \quad (7)$$

where $Y_{(k)}^{(i)} = [y_0^0 \dots\dots\dots y_0^{Nsc} \dots\dots\dots y_{N\,OFDM\,SYM\,N_R}^{Nsc}]$ is the received vector, $H_{(k)}^{(i)}$ is the channel coefficient matrix of the dimensions $N_T x N_R$ express the channel gain and $N= [n1, n2 \dots\dots nN_R]^T$ is the noise vector.

The matrix $H_{(k)}^{(i)}$ is written as follow [11]:

$$H_{(k)}^{(i)} = \begin{pmatrix} H_{(k)}^{(i)} & h_{(k)1,2}^{(i)} & \cdots & h_{(k)1,N_T}^{(i)} \\ h_{(k)2,1}^{(i)} & h_{(k)2,2}^{(i)} & & h_{(k)2,N_T}^{(i)} \\ \vdots & & \ddots & \vdots \\ h_{(k)N_R,1}^{(i)} & h_{(k)N_R,2}^{(i)} & \cdots & h_{(k)N_R,N_T}^{(i)} \end{pmatrix} \quad (8)$$

For each reception antennas, after eliminating Cyclical Prefix and Fast Fourier Transform operation, pilots are extracted and then interpolation block is attacked to estimate the parameter $H_{(k)}^{(i)}$ of the channel. The interpolation operation is necessary for both frequency and time domain.

In the present work, we use a Lagrange polynomial interpolation for frequency domain and linear interpolation for temporary. The interpolation algorithm is represented as follow:

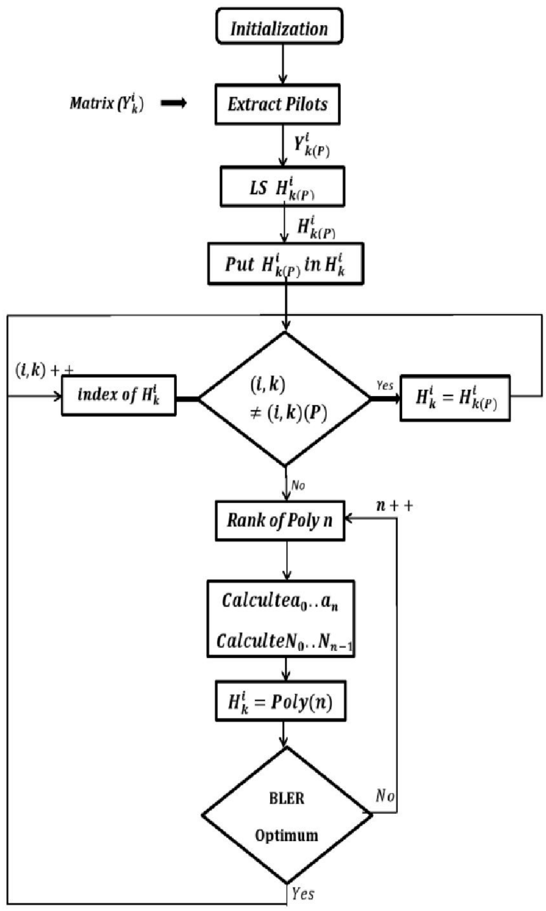

Fig. 4. Algorithm of Interpolation.

1-Extract $y_{(k)(P)}^{(i)}$ pilots from received signal $y_{(k)}^{(i)}$

2-Calculate the channel coefficients of pilots symbols with Least Square estimator

$$H_{(k)(p)}^{(i)} = y_{(k)(p)}^{(i)} / x_{(k)(p)}^{(i)} \quad (9)$$

3-Calculate L_0……………..L_n Coefficients of Lagrange with $\boldsymbol{n}$ order of Lagrange polynomial and $\boldsymbol{p}$ index of pilots, we start with $\boldsymbol{n=2}$. For example for 12 first coefficients to estimate we use 3 first pilots placed respectively at x_{p_0}= 0, x_{p_1}= 6 and x_{p_2}=12 frequency index

$$L_0 = ((x_i - x_{p_1})*(x_i - x_{p_2}))/((x_{p_0} - x_{p_1})*(x_{p_0} - x_{p_2})) \quad (10)$$

$$L_1 = ((x_i - x_{p_0})*(x_i - x_{p_2}))/((x_{p_1} - x_{p_0})*(x_{p_1} - x_{p_2})) \quad (11)$$

$$L_2 = ((x_i - x_{p_0})*(x_i - x_{p_1}))/((x_{p_2} - x_{p_0})*(x_{p_2} - x_{p_1})) \quad (12)$$

where L_0 , L_1 and L_2 are the coefficients of Lagrange polynomial of order ***n=3***,$x_i$is the frequency index of $H_{(k)}^{(i)}$ to estimate and x_{p_0},.x_{p_1},$x_{p_2}$are the frequency index of first tree pilots.

4-Estimate $H_{(k)}^{(i)}$ with Lagrange polynomial:

$$H_{(k)}^{(i)} = L_0 * H_{(k)(p)}^{(i)} + L_1 * H_{(k)(p+1)}^{(i)} + L_2 * H_{(k)(p+2)}^{(i)} \quad (13)$$

where $H_{(k)(p)}^{(i)}$, $H_{(k)(p+1)}^{(i)}$ and $H_{(k)(p+2)}^{(i)}$are three successive pilots.

5-Testing the estimation operation performance by incrementing the polynomial of order ***n*** until having optimal performance. For our simulation, the optimal performance was noticed at a value of ***n=5***.

IV. SIMULATIONS RESULTS

Our simulation was performed for LTE downlink transmission through a channel which uses the profile of ITU-Veh A for SISO and MIMO system with use of 16 QAM (CQI=7) constellation. We show simulation results for known channel, Lagrange polynomial interpolation algorithm, SinC interpolation and linear interpolation. All simulations are used over a Least Square equalizer. Simulation results are compared in term of Block Error Rate (BLER) and Throughput vs. SNR. Both Systems are simulated using the parameters shown in TABLE I [11][12].

TABLE I. PARAMETERS SIMULATION

Transmission Bandwidth	1.4 MHz
Carrier Frequency	2.1 GHz
Data Modulation	16QAM (CQI 7)
Channel	ITU-Veh A
Interpolation	Polynomial interpolation OF LAGRANGE

A. Simulation results and discussion

To observe the effect of the Lagrange polynomial interpolation compared with linear and SinC interpolation techniques, we simulate and trace the performance of LTE Downlink system in SISO and MIMO transmission over multipath channel (ITU-Veh A) using an LS equalizer . The simulations have been carried out for the 16-QAM (CQI=7). The Block Error Rate (BLER) and throughput vs. SNR results were study. Figures 5 and 6 show Block Error Rate vs. SNR for known channel, Lagrange polynomial, Sinus Cardinal and linear interpolations for both SISO and MIMO transmission.

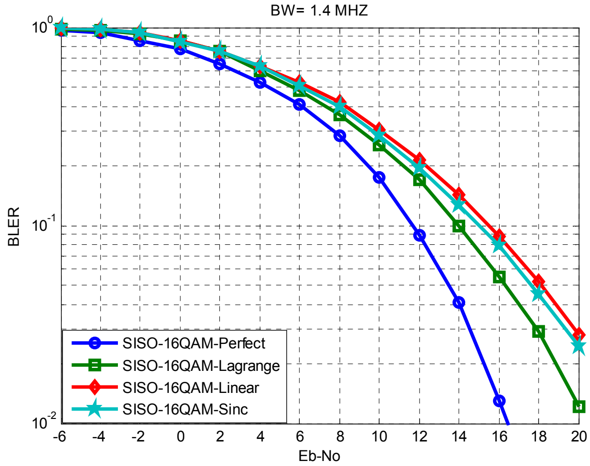

Fig. 5. BLER vs. SNR for SISO Transmission over Veh-A channel, CQI=7.

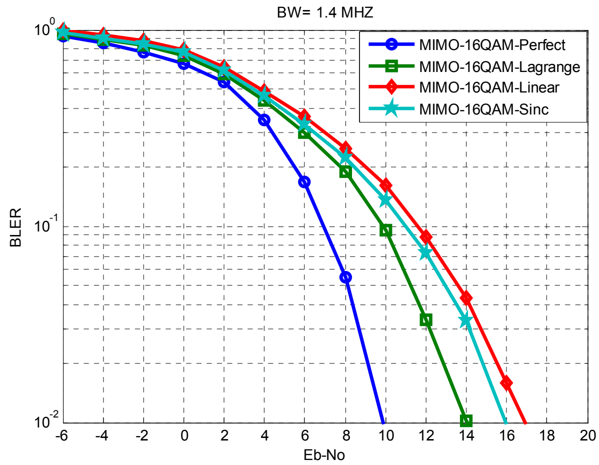

Fig. 6. BLERvs. SNR for MIMO Transmission over Veh-A channel, CQI=7.

The BLER results of LTE downlink transmission for SISO and MIMO transmission scheme is shown in Fig. 5 and Fig. 6. We can see that the Lagrange polynomial interpolation enhances the performance of downlink LTE system by almost than 2 dB for BLER $=10^{-1}$.

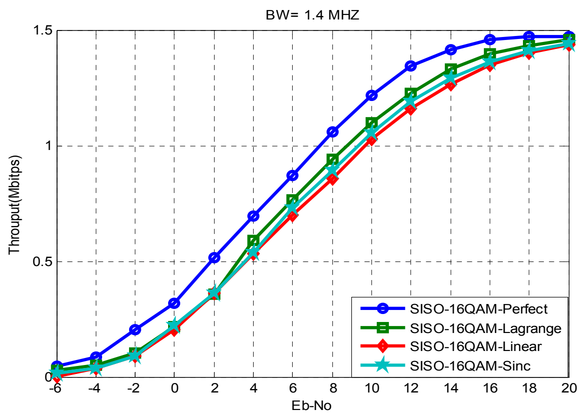

Fig 7. Throughput vs. SNR for SISO Transmission over Veh-A channel, CQI=7.

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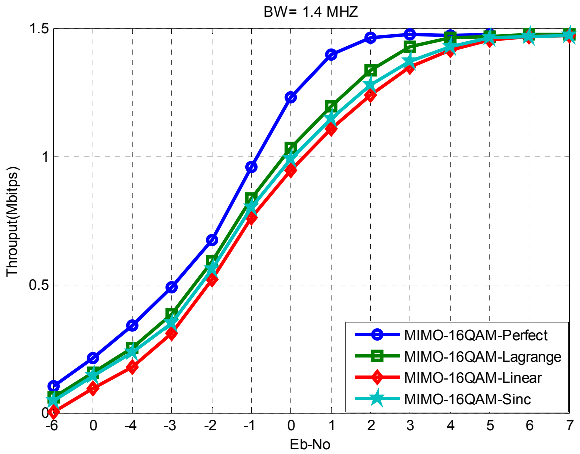

Fig 8 .Throughputvs. SNR for MIMO Transmission over Veh-A channel, CQI=7.

As shown in Figures 7 and 8 the throughput of SISO and MIMO transmission for CQI=7 over Vehicular A channel can be compared. We can note that the suggested algorithm of interpolation improve throughput compared to the linear and Sinus Cardinal interpolation. For example, with throughput=1 MHz we have a gain almost than 1dB for SISO and MIMO systems.

After studying performance, we find that the Lagrange polynomial interpolation offers a significant improvement compared to the linear and Sinus Cardinal Interpolation, as a result of the precision given by using ***n*** pilots to estimate each parameter channel. In fact, the use of ***n*** pilots in estimation of the channel coefficients takes into account the correlation between the pilot subcarriers; which makes this calculation more accurate and thus enhances the system efficiency.

It is obvious that this polynomial interpolation algorithm is more complex than the linear and Sinus Cardinal Interpolation, however it significantly improves system performance especially in the case of a fast fading channel (our case).

On the other hand, this algorithm has the advantage of having an adaptable order of polynomial interpolation ***n*** according to transmission condition. For example, in our case, we use a channel ITU-Veh A in the Bandwidth of 1.4 MHz where we have ***n = 5*** for same Bandwidth but for ITU-PEDISTRIAN-B channel ***n = 4***.

V. CONCLUSION

In the present work , adaptive polynomial interpolation algorithm was described in relation with the method of Lagrange for Downlink LTE system. Simulation is achieved through an ITU-Veh A channel with CQI = 7 and for SISO and MIMO systems .We conclude that, despite the complexity of this algorithm (compared to the linear and Sinus Cardinal Interpolation), it offers a considerable improvement of the performance of Downlink LTE system. In effect, using a maximum number of pilots to estimate the parameters of the channel (5 in our case) against two for a linear interpolation optimize considerably the estimation of these parameters.

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