Application of the Conditional Gradient Method to Optimal Allocation of Total Network Resources

Igor Konnov
Department of System Analysis and Information Technologies, Kazan Federal University, Kazan 420008, Russia.
Email: konn-igor@ya.ru

Aleksey Kashuba
LLC ”AST Povolzhye” Kazan, 420029, Russia
Email: leksser@rambler.ru

Erkki Laitinen
Department of Mathematical Sciences University of Oulu Oulu, Finland
Email: erkki.laitinen@oulu.fi

***Abstract*—We propose a new two-level iterative method for solution to a general problem of optimal allocation of a homogeneous resource (bandwidth) in a wireless communication network, which is divided into zones (clusters). In order to satisfy changing network users requirements, the network manager can buy additional volumes of this resource. We apply a dual Lagrangian method where the upper level problem is single- dimensional but calculation of the cost function value requires a solution to a convex optimization problem. This optimization problem is suggested to be solved with conditional gradient method with linear search. We give some results of numerical experiments on the proposed method which confirm its preference over the previous ones.**

Keywords–Resource allocation; wireless networks; bandwidth; zonal network partition; dual Lagrange method; linear search; conditional gradient method.

I. INTRODUCTION

The necessity of efficient allocation of limited resources in wireless communication networks arises from increasing demand of services and its variability, which leads to serious congestion effects, whereas significant network resources (say, bandwidth and batteries capacity) may be utilized inefficiently. This situation forces one to develop more flexible allocation mechanisms; see, e.g., [1]–[4]. Moreover, experience of dealing with these very complicated systems usually shows that a proper decomposition/clustering approach, which can be based on zonal, time, frequency and other attributes of nodes/units, might be very efficient here [5][6]. In [7] and [8], several optimal resource allocation problems in telecommunication networks and proper decomposition based methods were suggested. In this paper, we consider a further development of these models, where a system manager can utilize additional external resources for satisfying current users requirements. This manager strategy is rather typical for contemporary wireless communication networks, where WiFi or femtocell communication services are utilized in addition to the usual network resources; see, e.g., [9].

This approach leads to a two-level optimization problem. In [10], we considered embedded procedures within a unique iterative scheme that correspond to a sequential application of the dual decomposition method at each level of the problem. In this paper, we consider some other approach for solving this optimization problem. We apply a dual Lagrangian method where the upper level problem is single- dimensional but the calculation of its cost function value requires a solution of a convex optimization problem. This optimization problem is suggested to be solved with conditional gradient method with linear search.

We present some results of computational experiments on test problems. The results of the new method are essentially better than those of the method described in [10] and confirm its usefulness.

II. PROBLEM DESCRIPTION

Let us consider a network with nodes (attributed to users), which is divided into n zones (clusters) within some fixed time period. For the k-th zone ($k = 1, \ldots, n$), I_k denotes the index set of nodes (currently) located in this zone, b_k is the maximal fixed resource value. A manager of the network can allocate both the inner network resource x_k and external resource z_k, which brings maintenance expenses $f_k(x_k)$ and side payments $h_k(z_k)$, respectively, for each $k = 1, \ldots, n$. We suppose that there exists the upper bound c_k for the additional amount of the external resource in the k-th zone, and the upper bound B for the total inner amount of the resource. Next, if the i-th user receives the resource amount y_i with the upper bound a_i, then he/she pays the charge $\varphi_i(y_i)$. The problem of the network manager is to find an optimal allocation of the resource among the zones for maximization of the network profit subject to the above constraints. It is written as follows:

$$\max_{(x,y,z)\in W, \sum\limits_{k=1}^{n} x_k \leq B} \to \mu(x,y,z) \tag{1}$$

where

$$\mu(x,y,z) = \sum_{k=1}^{n} \left[\sum_{i\in I_k} \varphi_i(y_i) - f_k(x_k) - h_k(z_k) \right] \tag{2}$$

and

$$W = \left\{ (x,y,z) \;\middle|\; \begin{array}{l} \sum_{i\in I_k} y_i = x_k + z_k, \\ 0 \leq y_i \leq a_i, \ i \in I_k, \\ 0 \leq x_k \leq b_k, \ 0 \leq z_k \leq c_k, \\ k = 1, \ldots, n \end{array} \right\}. \tag{3}$$

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III. SOLUTION METHOD

In what follows, we suppose that all the functions $-\varphi_i(y_i)$, $f_k(x_k)$, and $h_k(z_k)$ are convex and differentiable.

Let us define the Lagrange function of problem (1)–(3) as follows:

$$L(x,u,z,\lambda) = \mu(x,y,z) - \lambda\left(\sum_{k=1}^{n} x_k - B\right).$$

That is, we utilize the Lagrangian multiplier λ only for the total resource bound. We can now replace problem (1)–(3) with its one-dimensional dual:

$$\min_{\lambda \geq 0} \to \psi(\lambda), \tag{4}$$

where

$$\psi(\lambda) = \max_{(x,y,z)\, \in\, W} L(x,y,z,\lambda) = \lambda B$$
$$+ \max_{(x,y,z)\, \in\, W} \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - \lambda x_k - h_k(z_k)\right]$$

Its solution can be found by one of well-known single-dimensional optimization problem.

In order to calculate the value of $\psi(\lambda)$ in (4) we have to solve the inner problem:

$$\max \to \sum_{k=1}^{n}\left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - \lambda x_k - h_k(z_k)\right]$$

subject to

$$\sum_{i\in I_k} y_i = x_k + z_k,\ 0 \leq y_i \leq a_i,\ i \in I_k,$$
$$0 \leq x_k \leq b_k,\ 0 \leq z_k \leq c_k,\ k = 1,\dots,n.$$

Obviously, this problem decomposes into n independent zonal convex optimization problems

$$\max \to \left[\sum_{i\in I_k}\varphi_i(y_i) - f_k(x_k) - \lambda x_k - h_k(z_k)\right], \tag{5}$$

subject to

$$\sum_{i\in I_k} y_i = x_k + z_k,\ 0 \leq y_i \leq a_i,\ i \in I_k, \tag{6}$$
$$0 \leq x_k \leq b_k,\ 0 \leq z_k \leq c_k, \tag{7}$$

for each $k = 1,\dots,n$. Note that the cost function in (5) is differentiable. The constraints in (6)–(7) give a polyhedral set, which is independent of λ. In what follows we also suppose each this set is nonempty and bounded. Then, we can apply the well known conditional gradient method [11], [12], with an inexact linear search procedure.

Let us describe this method to a convex optimization problem of form

$$\min_{v\in V} \to \eta(v), \tag{8}$$

where V is a convex polyhedron and η is a convex and differentiable function.

Conditional Gradient Method (CGM): Take an arbitrary initial point $v^0 \in V$ and numbers $\alpha \in (0,1)$ and $\gamma \in (0,1)$. At the k-th iteration, $k = 0,1,\dots$, we have a point $v^k \in V$ and calculate $u^k \in V$ as a solution of the linear programming problem

$$\min_{u\in V} \to \langle \eta'(v^k), u\rangle.$$

Then we set $p^k = u^k - v^k$. If $\|p^k\| \leq \delta$, stop, we have an approximate solution. Otherwise we find m as the minimal non-negative integer such that

$$\eta(v^k + \gamma^m p^k) \leq \eta(v^k) + \alpha\gamma^m\langle\eta'(v^k), p^k\rangle,$$

set $\sigma_k = \gamma^m$, $v^{k+1} = v^k + \sigma_k p^k$ and go to the next iteration.

It is known that the conditional gradient method generates a sequence $\{v^k\}$ which converges to a solution of problem (5) under the assumptions above. Therefore, the two-level method based on the solution of the dual problem (4) and the sequential solution of problems (5)–(7) with the conditional gradient method is well-defined.

IV. NUMERICAL EXPERIMENTS

In order to evaluate the performance of the new method denoted as CGDM (Conditional Gradient Dual Method) and to compare it with that from [10] denoted as DML (Dual Multi Level method) we made a number of computational experiments.

For all the one-dimensional optimization problems we used the golden section method. The program was implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

The initial intervals for finding λ (and the additional dual variables in DML) were taken as $[0,1000]$. The initial intervals for choosing the zonal allocation shares u_k in DML were taken as $[0,R]$ with $R = B + \sum_{k=1}^{n} c_k$, B was chosen to be 1000.

The coefficients b_k and c_k were generated by trigonometric functions with values in $[1,11]$, a_i were generated by trigonometric functions with values in $[1,3]$. We took the trikonometric functions ($\sin$, $\cos$) in order to verify different methods on the same sets of data.

Values γ and α in CGDM was chosen to be 0.33. The number of zones was varied from 5 to 105, the number of users was varied from 210 to 1010. Users were distributed in zones either uniformly or according to the normal distribution. The processor time and number of iterations, which were necessary to find an approximate solution of problem (4) within the same accuracy, were not significantly different for these two cases of distributions.

Further we report the results of tests, which include the time and number of iterations needed to find a solution of problem (4) within some accuracies. Let ε and δ denote the desired accuracy of finding a solution to problem (4) and solutions of auxiliary inner problems in DML. Let J denote the total number of users, N_ε the number of upper iterations in λ, T_ε the total processor time in seconds. For the same accuracy, both methods gave the same numbers of upper iterations, so that the main difference was in the processor time. The results of computations are given in Tables I–VI. We inserted also the results for DML with adaptive strategy of choosing the inner accuracies. We named DMLA (Dual Multi Level Adaptive method) this version of the method. We named CGDM0 the version of CGDM where the zero initial point was taken in

TABLE I. RESULTS OF TESTING WITH $J = 510$, $n = 70$, $\delta = 10^{-2}$

ε_λ	N_ε	T_ε DML	T_ε CGDM0	T_ε CGDMB
10^{-1}	20	11.3023	0.0260	0.1563
10^{-2}	24	12.8280	0.0311	0.2083
10^{-3}	29	15.9376	0.0470	0.2500
10^{-4}	34	17.7663	0.0630	0.3076

TABLE II. RESULTS OF TESTING WITH $n = 70$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

J	N_ϵ	T_ε DML	T_ε DMLA	T_ε CGDM0	T_ε CGDMB
210	24	5.4117	3.9327	0.0103	0.0727
310	24	7.9010	5.7400	0.0157	0.1193
410	24	10.3387	7.5053	0.0317	0.1610
510	24	12.8280	9.2973	0.0311	0.2083
610	24	15.2397	11.0733	0.0363	0.2447
710	24	17.6827	12.8903	0.0467	0.2813
810	24	20.1670	14.6303	0.0470	0.3230
910	24	22.5993	16.3910	0.0627	0.3700
1010	24	25.0993	18.1983	0.0677	0.4170

each conditional gradient method and CGDMB the version of CGDM where a boundary initial point was taken in each conditional gradient method. In Tables I and IV, we vary the accuracy ε, in Tables II and V we vary the total number of users, and in Tables III and VI, we vary the number of zones.

In all the computational experiments, we took the quadratic functions $f_k(x_k)$ and $h_k(z_k)$:

$$\begin{aligned} f_k(x_k) = \quad & \alpha''_k x_k^2 + \alpha'_k x_k + \alpha_k, \alpha''_k > 0, \alpha'_k \geq 0, \alpha_k \geq 0, \\ & k = 1, \dots, n; \\ h_k(z_k) = \quad & \beta''_k z_k^2 + \beta'_k z_k + \beta_k, \beta''_k > 0, \beta'_k \geq 0, \beta_k \geq 0, \\ & k = 1, \dots, n. \end{aligned}$$

The charge functions $\varphi_i(y_i)$ were chosen different. In Tables I–III, we give the results of computations with the logarithmic functions, i.e.,

$$\begin{aligned} \varphi_i(y_u) = \quad & \log(\gamma'_k x_k + \gamma_k), \gamma'_k > 0, \gamma_k \geq 1, i \in I_k, \\ & k = 1, \dots, n \end{aligned}$$

In Tables IV–VI, we give the results of computations with the concave quadratic functions, i.e.,

$$\begin{aligned} \varphi_i(y_u) = \quad & \gamma''_i y_i^2 + \gamma'_i y_i + \gamma_i, \gamma''_k < 0, \gamma'_k \geq 0, \gamma_k \geq 0, \\ & i \in I_k, k = 1, \dots, n. \end{aligned}$$

From the results, we can conclude that the new method CGDM has the significant preference over those in [10]. Moreover, they clearly enable us to apply CGDM for online solution of such resource allocation problems.

TABLE III. RESULTS OF TESTING WITH $J = 510$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

n	N_ϵ	T_ε DML	T_ε DMLA	T_ε CGDM0	T_ε CGDMB
5	24	12.5940	9.1250	0.0157	0.1873
15	24	12.5053	9.1407	0.0677	0.2343
25	24	12.6150	9.1667	0.0517	0.1977
35	24	12.6927	9.1873	0.0520	0.2033
45	24	12.6617	9.2243	0.0310	0.1873
55	24	12.8543	9.2603	0.0310	0.2033
65	24	12.8340	9.2917	0.0310	0.1873
75	24	12.8957	9.3440	0.0210	0.1930
85	24	12.9483	9.3803	0.0257	0.1663
95	24	12.9427	9.3960	0.0263	0.1873
105	24	12.9743	9.4327	0.0313	0.1923

TABLE IV. RESULTS OF TESTING WITH $J = 510$, $n = 70$, $\delta = 10^{-2}$

ε_λ	N_ε	T_ε DML	T_ε CGDM0	T_ε CGDMB
10^{-1}	20	5.3753	0.0523	0.2083
10^{-2}	24	6.3180	0.0883	0.2657
10^{-3}	29	7.6197	0.1303	0.3700
10^{-4}	34	8.7447	0.1820	0.5263

TABLE V. RESULTS OF TESTING WITH $n = 70$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

J	N_ϵ	T_ε DML	T_ε DMLA	T_ε CGDM0	T_ε CGDMB
210	24	2.7500	2.0160	0.0153	0.0990
310	24	3.9950	3.0517	0.0260	0.1560
410	24	5.2030	3.8437	0.0577	0.2083
510	24	6.3180	4.6353	0.0883	0.2657
610	24	7.4950	5.4633	0.1043	0.3387
710	24	8.6410	6.3073	0.1407	0.3593
810	24	9.8647	7.1773	0.1930	0.4323
910	24	11.0107	8.0203	0.2033	0.5573
1010	24	12.1823	8.8807	0.2500	0.6200

V. CONCLUSION

We considered a general resource allocation problem in telecommunication networks, where a system manager can buy additional external resources for satisfying current users requirements, which is rather typical for contemporary wireless communication networks. We suggested a new approach for solving this problem, which consists in solving the single-dimensional dual Lagrangian such that the calculation of its cost function decomposes into a set of independent convex optimization problems. They are solved with a conditional gradient method. The results of computational experiments on test problems showed rather rapid convergence of the method and its essential preference over the previous iterative schemes.

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TABLE VI. RESULTS OF TESTING WITH $J = 510$, $\varepsilon = 10^{-2}$, $\delta = 10^{-2}$

n	N_ϵ	T_ε DML	T_ε DMLA	T_ε CGDM0	T_ε CGDMB
5	24	6.0110	4.3647	0.0570	0.1563
15	24	5.9740	4.3857	0.2760	0.4530
25	24	6.0837	4.4323	0.1300	0.3597
35	24	6.1150	4.4740	0.1407	0.2813
45	24	6.1563	4.5207	0.1093	0.2707
55	24	6.2550	4.5573	0.0310	0.2550
65	24	6.3333	4.5937	0.0417	0.3020
75	24	6.3440	4.6613	0.0260	0.2657
85	24	6.4167	4.7863	0.0260	0.2553
95	24	6.5363	4.8490	0.0470	0.2550
105	24	6.4737	4.9690	0.0417	0.2713

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