Review of Shape-based Similarity Algorithms and Design Retrieval Methods for Computer-aided Design and Manufacturing

Leila Zehtaban and Dieter Roller
Institute of Computer Aided Product Development Systems
Universität Stuttgart, Stuttgart, Germany
Email: zehtaban@informatik.uni-stuttgart.de, roller@informatik.uni-stuttgart.de

Abstract— **Reusing engineering data has opened a new opportunity to improve product quality, shorten design lead-time and reduce costs using existing know-how within the design process. Geometrical aspects or 3D shape information of a product is an essential data which can be reused in CAD software. In order to compare and retrieve the existing 3D models, having a precise computational representation of a shape, so-called shape index or shape signature, is a main challenge. The shape signature is often used for the shape similarity comparison. There are several specifications for a shape signature like quick to compute, easy to index, invariant under transformation, independent of 3D representations, tessellation, genus or topology. The algorithms or the methods which decompose a shape into a signature can be classified into seven main classes. This paper aims to focus on the discussion of the first three methods, i.e., Invariant-based methods, Harmonics-based methods, and Graph-based methods, and provide the related literature review on their underlying approaches with highlighting methodologies, advantages and disadvantages.**

Keywords-shape signature; shape similarity comparison; 3D shape retrieval; reused design.

I. INTRODUCTION

New technological progress has enabled Computer-aided Design (CAD) software to incorporate engineering know-how into the design process in order to improve product quality, shorten design lead-time and reduce costs. Any manufacturer has an accumulated amount of know-how related to design, production and performance of existing or previously manufactured products. Accessibility and the possibility of reusing this accumulated knowledge is a key factor ***for*** optimizing design and performance of a new product. Using a capable procedure for identifying similarity between a new possible product with items listed in the existing product data-bases enables a design engineer to find a professional base to design a new product. The new design can be well optimized using existing know-how of the existing product in design, production and performance.

The similarity comparison between two objects could include diverse similarity aspects like similarity in shape (structure), design intent (functionality), production specifications, etc. However, the shape similarity comparison is one of the most important bases for any comprehensive similarity comparison in product design. The complexity of shape similarity comparison arises from the challenge of finding a computational representation (signature) for a shape which can be applied for the shape similarity comparison. The current shape similarity methods can be classified as follows [1]: Invariant-based methods, Harmonics-based methods, Graph-based methods, Statistics/probability-based methods, 3D object recognition-based methods, Feature recognition-based methods and Group Technology-based methods.

In this paper, we highlight the first three classes by having a literature review on their different underlying methods. Although the methods which are classified under the same classification, originally apply an identical concept to decompose a shape into a signature, nevertheless there are still differences regarding the utilized techniques. In the following, Section II describes the invariant-based methods, Section III describes the harmonics-based methods, Section IV describes the graph-based methods, and finally, Section V summaries the paper.

II. FIRST CLASSIFICATION: INVARIANT-BASED METHODS

These approaches use invariants or descriptors of the 3D shape such as volume, surface area, aspect ratio, higher order moments or moment invariants as signatures [1]. At the following four methods which belong to the category of invariant-based will be briefly discussed. These methods include: RTS-invariants, Moments and relevance feedback, Non-dimensional and scale-independent features, and Elementary-shape-based features and active learning will be explained.

A. *RTS-invariants*

In the method from Cybenko et al. [2], solid objects given in a standard digital representation like the IGES file format are converted into a surface triangular mesh representation. Afterwards, the triangular mesh representation is converted into a voxel model representation using a flood filling method. For the shapes represented by voxel model, geometrical moments are calculated and used to normalize the object into a canonical form. Shape features are computed by calculating variant volumetric invariants. They are called RTS-invariants because these features are invariant against rotation, translation and scaling. The following RTS-invariants can be calculated: second-order

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3D moments invariants, spherical-kernel moments invariants, axis aligned bounding box and centroid of the object, and the surface area of the objects. In the first step of similarity measurement, feature vectors are used to compute a set of best candidate objects. On this set of the best candidates a voxel-by-voxel comparison is performed as the second step of similarity measurement. This step allows a detailed comparison between voxel model representation of objects and is based on template matching.

B. Moments and relevance feedback

Elad et al. [4][5], used moments as shape features of 3D models and relevance feedback as an iterative and interactive method to improve the performance retrieval. For the models given in VRML file format (Virtual Reality Modeling Language), the geometrical moments are calculated and approximated up to the third-order and used to normalize the object in a canonical form. For the normalized objects moments are approximated again (up to forth-to-seventh-order is sufficient) and used as a feature vector of objects.

In the first step of similarity measurement a set of the best candidates is presented to the user by computing the Euclidean distance between feature vectors of the primary object and objects from the database. After that the user has the ability to influence the future search results by applying the method of relevance feedback. The user can mark a subset of presented results as relevant or as irrelevant. Based on these markings, which capture the user-perceived similarity between objects, the distance measure can be adapted and a new search results calculated. The adaption of the distance measure is based on Support Vector Machine (SVM) learning algorithms and can be repeated until the user is satisfied with the search results.

C. Non-dimensional and scale-independent features

Rea et al. [6] and Corney et al. [7], used various non-dimensional and scale-independent features as signature for 3D CAD models in an internet search engine. Most of these features are computed using object characteristics such as volume, surface area and convex hull of objects.

For example, the features like crinkliness, compacness, hull crumbliness, etc. are calculated as following [7]: Crinkliness is defined as the surface area of the model divided by the surface area of the sphere having the same volume as model. Compactness is defined as a ratio of the volume squared over the cube of the surface area and used as a non-dimensional feature. Hull crumbliness is defined as a ratio of objects surface area to the surface area of its convex hull. Hull packaging is defined as the percent of the convex hull volume not occupied by the original object. Hull compactness is defined as ratio of the convex hull's surface area cubed over the volume of the convex hull squared.

Further features being used are: ratio of the longest edge to the shortest edge of the bounding box, number of the holes of the object and number of the facets of the object. User can specify combination of features which are used in the similarity search and tolerance values for these features.

D. Elementary-shape-based features and active learning

The method from Zhang et al. [8][9], describes that features such as volume, surface area, moments and Fourier transform coefficients can be well extracted from a mesh representation and be considered as the signature of an object. The inspiration of this method is to compute features for elementary shapes such as triangles and tetrahedrons in advance and sum up the feature values of the elementary shapes in order to get the feature value of the whole object. Annotation of the object was used as a method to improve the performance retrieval. The hidden annotation has to be performed as a learning stage before a database can be used for the similarity search. By using an active learning method the system determines the sample objects to the annotator. The sample objects are selected so that annotation of the object can provide the maximum information or knowledge gain to the system.

Using this method reduces the number of training samples by selecting the most informative ones to the annotator.

E. Evaluation of invariant-based methods

All invariant-based methods have the advantage of being robust to small changes in shape. The disadvantage of these methods refers to the improbable partial matching.

The method from Cybenko et al., suffers from the requirement of a huge storage requirement for every object and its different models. In addition, the voxelization of models is a time and memory consuming process. [2]

In the method from Elad et al., with using the relevance feedback, not only the geometrical similarity is being computed between the objects, but also the user-perceived similarity can be incorporated in the similarity search process. Hence, the retrieval performance is being improved by retrieving more objects than user has in mind. [4][5]

The method from Corney et al. and Rea et al., can be useful as a coarse filter in huge databases. However, to perform a finer comparison between objects when the sets of retrieved objects are large, combination of this method with further methods might be necessary. [6][7]

The disadvantage of the method from Zhang et al., lies on the requirement of an explicit routine to compute a feature value for elementary shape. Nevertheless, it is difficult to develop explicit routine to compute the high order moments for triangles and tetrahedrons. Zhang et al., used the method of hidden annotation and active learning to improve the retrieval performance of the system. In practice, an annotation of large databases can hardly be performed because of the manual effort. Besides, with applying partial annotation, it is difficult to decide how much annotation is sufficient for specific database. [8][9]

III. SECOND CLASSIFICATION: HARMONIC-BASED METHODS

These methods use a set of harmonic functions of a shape as signature. Spherical or Fourier functions are usually used to decompose a discrete 3D model into an approximate sum of its (first n) harmonis components [1]. The four methods of this category will be discussed in the following sections:

Ray-based SH-descriptor, Rotation-invariant SH-descriptor, Layered depth sphere-based SH-descriptor; and Concrete radialized spherical projection descriptor.

A. *Ray-based SH-descriptor (Spherical Harmonics)*

In the method from Vranic [10][11], 3D models represented by polygon meshes are normalized to achieve invariance against rotation, translation and scaling. For that purpose a Principal Component Analysis (PCA) which can be applied to a discrete set of points, as well as the union of all polygons of the mesh with infinitely number of points. After normalization, the 3D models are characterized by defining a function on a sphere which measures the extension of an object in different directions.

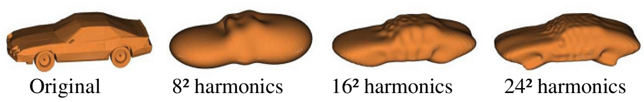

Figure 1. Multi-resolution representation used to derive feature vectors from Fourier coefficients for spherical harmonics [10]

For each direction a ray is casted from the center of mass in order to compute the last point of the intersection with the polygonal mesh which is used as a sample of the function. After sampling the function Fast Fourier Transformation (FFT) is performed to obtain the Fourier coefficients to be applied as feature vector. Figure 1 represents reconstruction of the different levels of a primary object when using three different spherical harmonics coefficients.

B. *Rotation-invariant SH-descriptor*

Kazhdan et al. [3] claim that the methods using PCA are unstable referring to the multiplicity of eignevalues and its sensitivity to outliers.

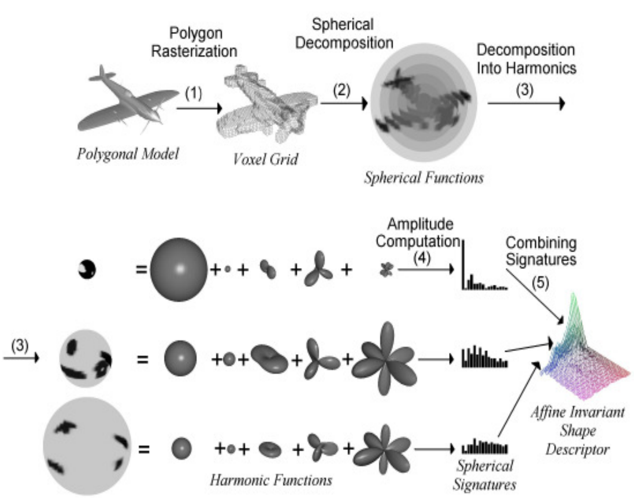

Figure 2. Computing the Harmonic Shape Representation [3]

As a solution a new method [12][13], to compute the harmonic shape representation is proposed. In this method, the model polygon is rasterized into a $64 \times 64 \times 64$ voxel grid. The voxel grid is decomposed into 32 functions on concentric spheres by restricting the voxel grid to spheres with radii 1 to 32. By decomposing each of these functions as a sum of its first 16 harmonic components, analogous to a Fourier decomposition into different frequencies and define the signature of each spherical function as a list of these 16 norms and combining the different signatures, a 32×16 signature for 3D model is obtained. In order to compare two harmonic presentations, the Euclidean distance between them should be computed. An example of the explained method is shown in Figure 2.

C. *Layered Depth Spheres (LDS)-based SH-descriptor*

Vranic [14][15], described a further harmonics-based method which captures information about internal structure of objects. The shape descriptor is extracted from a triangle mesh representation of the objects. Invariance against translation and scaling is achieved using Continues PCA (CPCA). 3D model is decomposed into a family of function on the sphere restricting function values by lower and upper bounds which describe a bounded area of the model. Using ray cast method for rays emanating from the origin in many directions all points of intersection with the polygonal mesh are computed.

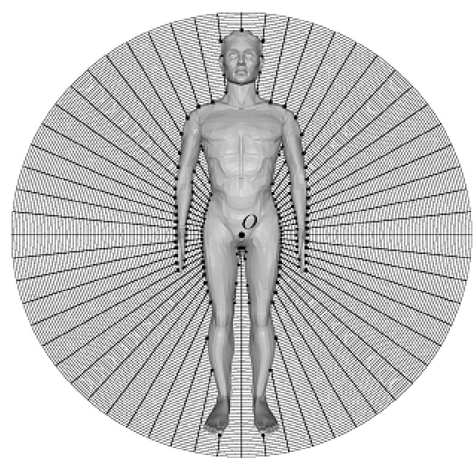

Figure 3. Concept of Layered depth Sphere with an example in 2D [15]

For intersection points the closest sphere and a set of corresponding value of the function on that sphere is determined. If two intersection points of the same ray belong to the same sphere then the larger distance determines the function value. On each sampled function on the sphere Fast Fourier Transformation (FFT) is performed to obtain a set of coefficients. The PCA method can be performed during the normalization step or the properties of spherical harmonics can be used to achieve rotation invariance.

D. *Concrete Radialized Spherical Projection Descriptor (CRSP)*

In the method from Papadakis et al. [16], a shape descriptor is extracted from a triangle mesh representation of 3D models. In this method, scaling and axial flipping invariance is achieved referring the properties of spherical harmonics.

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Rotation and translation invariance is achieved by applying CPCA and Principal Component Analysis on the model's Normal (NPCA). This algorithm, results in two versions of an object and, therefore, two descriptors for an object. For each version of the object a set of functions on spheres is defined, which are sampled by casting rays from the origin of the object.

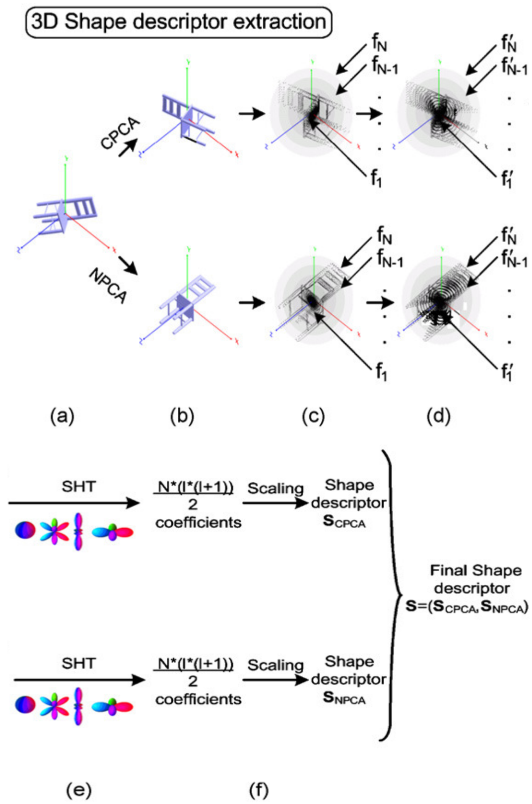


Figure 4. The stage of the shape matching using CPCA and NPCA [16]

A function on a sphere represents intersection points of the models surface with rays and also all points in the direction of each ray that are closer to the origin than the furthest intersection point. For every function Short-time Fourier Transform (SFT) is performed to obtain the Fourier coefficients. Scaling invariance of the descriptor is achieved using properties of spherical harmonics. Figure 4 illustrates the stages of the shape decomposition and matching as well as obtaining the shape descriptor/signature respectively.

E. *Evaluation of Harmonics-based methods*

All harmonics-based methods have an advantage which feature extraction and similarity measurements are efficiently performed. Drawbacks of these methods are as following: first; specific details of shape can not be captured, and second; partial matching is not possible in these methods. [1]

Kazhdan used a coarse voxel grid to achieve robustness against small changes of shape. However, coarse voxel grid causes loss of many details. [11] Voxelization also affects efficiency of feature extraction.

The ray-based method allows an embedded multi-resolution representation of the descriptor. This means that a descriptor contains all descriptors having lower dimension. [11]

Unlike the ray-based method, LDS-based method captures information about the internal structure of objects by defining several functions on spheres instead of only one.

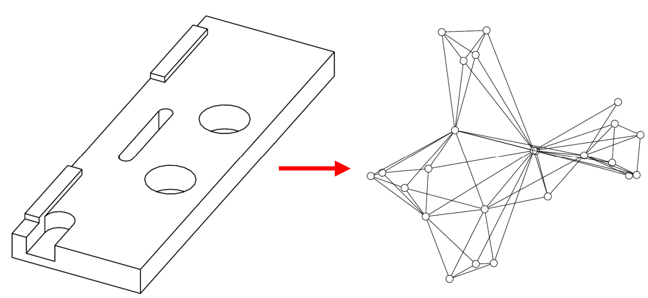
The CRPS method improves the invariance properties of the descriptor by applying two normalization methods, CPCA and NPCA. Thus, the retrieval performance of the descriptor is improved. Although this process increases the complexity of descriptor, since for each object two descriptors are extracted. [16]

IV. THIRD CLASSIFICATION: GRAPH-BASED METHODS

In Graph-based approaches sub-graph isomorphism is used in order to match B-Rep graphs, or to match eigenvalues of a model signature graph which is constructed from the B-Rep graph. Five different methods belonging to the graph category are briefly explained in the following [1]. These five methods include: Model signature graphs, Attributed graphs, Reeb graphs and Skeletal graph with parameter-controlled thinning.

A. *Model signature graphs*

McWherter et al. [17][18][19][20], developed Model Signature Graph (MSG) for similarity measurement between 3D CAD models. MSGs are labeled, undirected graphs, which are generated using the Boundary Representation (B-Rep) on CAD models. The boundary representation consists of a set of edges and a set of faces. For the definition of MSGs every face of the model is represented as a graph vertex and every edge in the B-Rep is represented as a graph edge. Labels of edges and vertices contain attributes of faces and edges in B-Rep, such as topological identifier, underlying geometrical representation, etc. For every MSG the eigenvalue spectrum and Invariant Topology Vector (ITV) are extracted and used to perform similarity comparison. ITV contains graph invariants, such as vertex and edge counts, maximum, minimum, mode degrees graph diameter, etc.



(a) Solid Model (b) Model Signature Graph

Figure 5. A model and its transformation into a Model Signature Graph [19]

The spectrum of a graph is sorted as the eigenvalues of its adjacency matrix, which holds information related to the graph structure. In addition, the eigenvalues of the graphs can be used to partition the graph into two or more sub-graphs in order to compare substructure of the graphs.

B. *Attributed graphs*

In the approach of El-Mehalawi und Miller [21][22], attributed graphs are used as signature of 3D CAD models, which are extracted from STEP files of these models. Attributed graphs are quite similar to MSGs, hence, graph nodes describe the faces of CAD components and graph edges describe the edges of CAD components. In addition, the node attribute correspond to the surface attribute, such as type of surface and direction of the normal. Edge attributes correspond to the edge attributes in B-Rep, such as type of the edge, direction of the normal and length of the edge.

For the retrieval process, abstract information is extracted from attributed graphs and used in the first step of the retrieval process. The abstract data is the total number of nodes, number of nodes representing plan, cylindrical and conical surfaces. These data are used as an index, where the set of graphs candidate similar to the query graph can be calculated very quickly. In addition, a more accurate comparison is applied for the set of candidate graphs. To finalize the method and to complete the retrieval procedure, inexact graph matching algorithm based on an integer programming model is applied. This algorithm has a polynomial computational complexity.

C. *Skeletal graphs with thinning*

Sundar et al. [23][24], used skeletal graphs as signature of 3D models for similarity measurement as figure 6 present it.

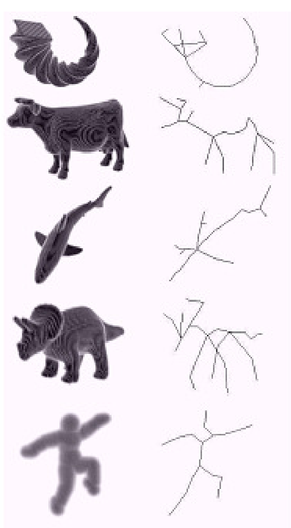


Figure 6. Skeletal graphs based on the thinning algorithm [23]

To extract the skeletal graph of an object, the belonging 3D model ought to be converted into a voxel model. In the next step, for the skeletonization process, a parameter-controlled thinning algorithm is used to calculate a subset of voxels. In this thinning method the thickness of the skeleton is determined by the parameter given the user. Hence, a family of the different voxel sets can be calculated, each one is thinner than its parent. The thinness parameter classifies the importance of the voxels for the boundary coverage by comparing the distance transform of the voxel with its 26 neighbors. After skeletonization the Minimum Spanning Tree (MSN) algorithm is applied in order to generate an undirected acyclic graph out of unconnected skeletal points. For every node in the graph Topological Signature Vector (TSV) is defined which holds information related to the node underlying sub-graphs structure.

TSV contains the eigenvalues of the sub-graph's adjacency matrix and is used as an index to fast determination of a set of best candidate graphs. On the set of candidate graphs a graph matching algorithm is performed by reformulating the problem of largest isomorphic sub-graph as the problem of finding the "maximum cardinality, minimum weight matching" in a bipartite graph. To preserve the hierarchical structure of the graph a greedy form of the above bipartite formulation is combined with a recursive depth search.

D. *Skeletal graphs with parameter-based thinning*

In the method from Iyer et al. [25][26][27], skeletal graphs and feature vectors jointly present the signature of 3D models. 3D models are normalized into a canonical form and converted into voxel model. In the skeletonization process, iterative thinning algorithm is applied by deleting border points satisfying conditions of topology preservation. On the generated skeleton, the skeleton-marching algorithm is performed to identify the basic entities and construct the skeletal graph. Basic skeletal entities are vertex, edge and loop.

For the definition of feature vectors the following shape descriptors are extracted from the voxel model: moments, geometry parameters such as volume and surface area, voxelization parameters such as voxel size, and graph parameters such as number of loops edges and nodes. For the similarity measurements the Euclidean distance of the feature vectors, as well as the distance between skeletal graphs are calculated. For the graph matching a decision-tree based algorithm developed by Messmer et al. [28] is applied.

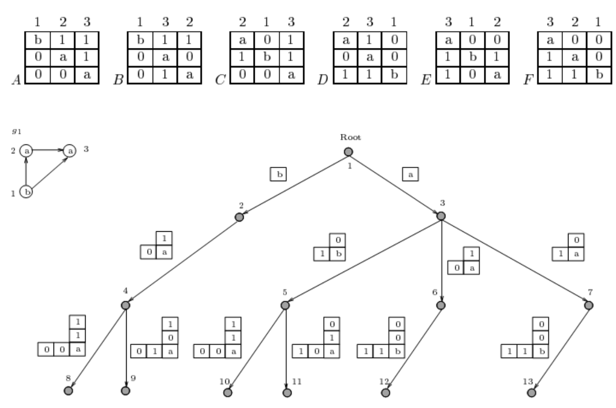


Figure 7. Decision tree for the related (above) adjacency matrix [28]

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In this algorithm all graphs in a database are indexed in the form of a decision tree using the different permutations of the adjacency matrix as described in Figure 7. The space requirements are exponential, but the search requirement is sub-polynomial in the number of query graph nodes.

E. *Reeb Graphs*

Hilaga et al. [29] developed Multiresolutional Reeb Graphs (MRGs) for similarity measurements between 3D models. MRG describes the skeletal and topological structure of a 3D model. A reeb graph is generated using a continuous scalar function on the 3D model. In this method, geodetic distance is used because of the translation and rotation invariance of this function. Because the reeb graph might contain many nodes, a MRG is constructed as a row of reeb graphs at several levels of detail. 3D object is divided into a number of ranges using values of the scalar function. A graph node represents a connected component in a particular range, and graph edge represent connected components of the adjacent ranges that contact each other.

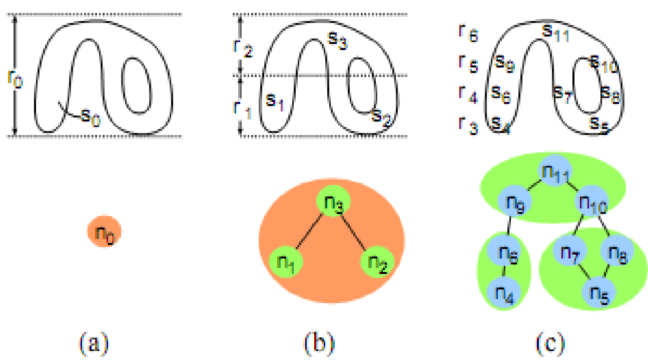

Figure 8. Multiresolutional Reeb graph [29]

The construction starts with the generation of a reeb graph having the finest resolution. Construction of the MRGs with coarser resolution is followed by merging adjacent ranges and unifying connected nodes form this ranges into one. For every node attributes are calculated and used to estimate similarity between nodes in the similarity measurements process. The calculation of node attributes is based on shape features such as area of triangles, area of whole object and certain values of continuous function. The similarity measurement is performed using a coarse-to-fine strategy, while preserving the consistency of the graph structure.

F. *Evaluation of graph-based methods*

The advantage of all graph-based methods is description of the topology of 3D models which is an important shape feature. In addition, representation of 3D models as topological graphs which facilitate the abstraction of these models at different levels of detail and description of local geometry at each node. [1] Other advantage of the graph-based methods is possibility of partial matching between 3D models. (Except for MRG method)

MSGs are efficient despite the large and complex graphs in the database. This method is considered insufficient for fine discrimination between 3D models, referring to the disability of capturing all properties of the adjacency matrix by the eigenvalues. [17][20]

Although skeletal graphs with using simplification of 3D, are stable to small changes in shape, but the simplification causes a loss of information affecting the discrimination power of the method. [27]

The advantage of MRG refers to the fact that geodesic distance as a continuous function is invariant against rotation and translation. Exponential computational complexity of this method has been avoided by applying coarse-to-fine strategy in the retrieval process. [29][30]

V. CONCLUSION AND FUTURE WORK

In this paper, shaped-based similarity and design retrieval methods have been discussed. Each of the discussed methods has advantages as well as disadvantages. Which method should be used in a particular application depends on the desired discrimination power of descriptor or the required efficiency of the similarity search. If only general classification of objects in a database is needed then harmonics-based or invariant-based is good choice. If partial matching between objects should be possible, then one of the graph-based methods is good choice. As a conclusion, the combination of different methods may help to achieve high discrimination power as well as efficient similarity search.

The next step will be review and discussion of the rest of the methods and classes, which decompose a shape into a signature, i.e., Statistics/probability-based methods, 3D Object Recognition-based methods, Feature Recognition (FR)-based methods, and Group Technology (GT)-based methods.

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