# *A Formal Framework for Modeling Topological Relations of Spatial Ontologies*

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***Abstract*— A spatial ontology adds to the components of a domain ontology spatial relations and spatial aspects of its concepts. Spatial relations are of two types: the metric relations and topological relations, expressing a link of interconnection between two spatial concepts. In this paper, we propose a formal modeling of topological relations of a spatial ontology. This formal model is called "meta-OntologicalOnto" which is a set of rules written in description logic. The "meta-OntologicalOnto" is used as a reference during the construction of the spatial ontology. The field of application of our work is the road domain whose ultimate goal is to obtain a road ontology named "OntoRoad".**

***Keywords- Spatial Ontology; topological relations; road domain; formal rules; meta-OntologicalOnto; OntoRoad.***

## I. INTRODUCTION

A geographical object is an object modeling a real-world phenomenon. It is described by semantic data and geometric data. The building of spatial ontologies should allow modeling of all properties of spatial objects to meet users' needs. This makes the problem of constructing spatial ontologies more complex than those of other domain ontologies.

The most known definition of ontology [1] is: ontology is a specification of a conceptualization of a knowledge domain. This definition shows that domain ontology must have a formal aspect. Spatial domain ontology consists of concepts with a spatial aspect and spatial relations. Spatial aspect of a concept means its graphic shape: Point, Line or Polygon. Spatial relations are of two types: the metric relations expressing a value of distance or proximity between two objects, as beside, near, etc.... between two objects, and topological relations expressing a link of interconnection between two spatial objects. Topological relations are characterized by the property to be preserved under topological transformations and describe whether features intersect or not, how they intersect, and concepts of overlap, neighborhood and inside. They are important for numerous practical applications that involve spatial query, spatial analysis and spatial reasoning. Referring to the ontology definition, it is essential to use formal methods to identify and define topological relations in spatial ontologies.

The formalization of spatial relations is an active research field and numerous works deal with the recognition of pertinent topological relations. The core models in this domain are the Calculus Based Model (CBM) [2], the 9-Intersections Model (9IM) [3] and the Region Connection Calculus (RCC) [4]. All these approaches satisfy the requirements that they provide a sound and complete set of topological relations between two spatial objects.

In N-intersection models, a mathematical model, called Four-Intersection Model (4IM) [5] that classifies topological relations based on the content of the four intersections between the boundaries and interiors of the two simple geometric features, was derived. A number of variants of this model were derived, including Dimension Extended Method (DEM) [2] that takes the dimension of the intersection components into account, Nine-Intersection Model (9-IM) [3] that categorizes binary topological relations based on the comparison of nine intersections between the interiors, boundaries and exteriors of the two features and finally Dimensionally Extended Nine-Intersection model (DE-9IM) [6] that introduces the dimension of the intersections into 9IM.

The binary topological relation between two objects (A and B) in [5], is based upon the intersection of A's interior (A°), boundary (əA), and exterior (A¯) with B's interior (B°), boundary (əB), and exterior (B¯). The nine intersections between the six object parts describe a topological relation that can be concisely represented by a 3-3 matrix, called the 9-intersection model.

$$\begin{pmatrix} \partial A \cap \partial B & \partial A \cap B^{\circ} & \partial A \cap B^{-} \\ A^{\circ} \cap \partial B & A^{\circ} \cap B^{\circ} & A^{\circ} \cap B^{-} \\ A^{-} \cap \partial B & A^{-} \cap B^{\circ} & A^{-} \cap B^{-} \end{pmatrix} \emptyset \; or \; \neg\emptyset \quad (1)$$

By considering the values empty (Ø) and non empty (¬ø), one can distinguish between $2^9$=512 binary topological relations. Only a small subset of them can be realized when the objects of concern are embedded [7] [19]. There is a set which provides a mutually exclusive and complete coverage of topological relations between two regions (Fig. 1), termed {Disjoint, Meet, Overlap, equal, Contains, Inside, Covers and coveredBy} [8].

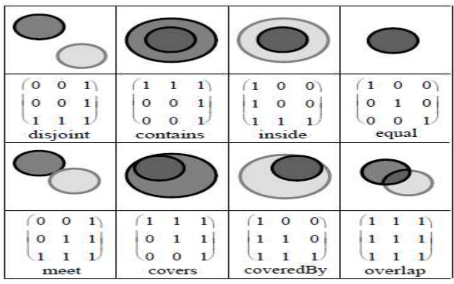


Figure 1. The eight topological relations between two spatial regions and their corresponding 9-intersection matrices [2].

The RCC8 model [4] is composed of 8 Jointly Exhaustive and Pairwise Disjoint (JEPD) base topological relations between two spatial regions A and B. These relations are based on the binary primitive C(A,B), which means A is connected to B. In the RCC8 context, C(A,B) is interpreted as being true when the closure of A and B share a point, where A and B are viewed as sets of points. The only requirement for the relation C is that it is reflexive and symmetric. Using C(A,B) a large number of relations can be defined [4]. The set of eight relations {DC, EC, PO, EQ, TPP, NTPP, TPPi, NTPPi} constitutes the set of the RCC8 relations (see Table 1). They are invariant with respect to geometric transformations. It is possible to add more expressiveness to the RCC relations by introducing additional primitives. In [4], 23 relations are defined by adding the convex hull as another primitive. This extension allows distinguishing different types of "inside" a region. In the context of [4], a region is said to be inside another one when it is connected to its convex hull, but the regions do not overlap.

TABLE I. : SOME OF THE RELATIONS DEFINED BY C(A,B).

| Relation | Interpretation | Definition of $\mathbf{R}(A,B)$ |
|---|---|---|
| $\mathbf{DC}(A,B)$ | A is disconnected from B | $\neg\mathbf{C}(A,B)$ |
| $\mathbf{P}(A,B)$ | A is a part of B | $\forall D[\mathbf{C}(D,A) \rightarrow \mathbf{C}(D,B)]$ |
| $\mathbf{PP}(A,B)$ | A is a proper part of B | $\mathbf{P}(A,B) \wedge \neg\mathbf{P}(B,A)$ |
| $\mathbf{EQ}(A,B)$ | A is identical with B | $\mathbf{P}(A,B) \wedge \mathbf{P}(B,A)$ |
| $\mathbf{O}(A,B)$ | A overlaps B | $\exists D[\mathbf{P}(D,A) \wedge \mathbf{P}(D,B)]$ |
| $\mathbf{DR}(A,B)$ | A is discrete from B | $\neg\mathbf{O}(A,B)$ |
| $\mathbf{PO}(A,B)$ | A partially overlaps B | $\mathbf{O}(A,B) \wedge \neg\mathbf{P}(A,B) \wedge \neg\mathbf{P}(B,A)$ |
| $\mathbf{EC}(A,B)$ | A is externally connected to B | $\mathbf{C}(A,B) \wedge \neg\mathbf{O}(A,B)$ |
| $\mathbf{TPP}(A,B)$ | A is a tangential proper part of B | $\mathbf{PP}(A,B) \wedge \exists D[\mathbf{EC}(D,A) \wedge \mathbf{EC}(D,B)]$ |
| $\mathbf{NTPP}(A,B)$ | A is a non-tangential proper part of B | $\mathbf{PP}(A,B) \wedge \neg\exists D[\mathbf{EC}(D,A) \wedge \mathbf{EC}(D,B)]$ |
| $\mathbf{TPPi}(A,B)$ | B is a tangential proper part of A | $\mathbf{PP}(B,A) \wedge \exists D[\mathbf{EC}(D,B) \wedge \mathbf{EC}(D,A)]$ |
| $\mathbf{NTPPi}(A,B)$ | B is a non-tangential proper part of A | $\mathbf{PP}(B,A) \wedge \neg\exists D[\mathbf{EC}(D,B) \wedge \mathbf{EC}(D,A)]$ |

The CBM [2] offers a small set of topological relations with high expressiveness:{ Touch, In, Cross, Overlap, Disjoint} and three boundary operators, which are proved to be mutually exclusive and complete. The capability of CBM is equivalent to DE-9IM [6].

Through the analysis of literature, it can be found that the numbers of topological relations differentiated by these models can only indicate the capacities of the models but nothing about a complete spectrum of all possible topological relations. For example, there should be infinite number of topological relations for a line and a region. To overcome the deficiency of these general models, efforts have also been made to develop dedicated models to specific types of features, e.g., line-line relations [9, 10], region-region relations [11, 12] and Line-region relations [13].

The field of our works is the road domain; we are therefore interested in topological relations supposed being relevant in this domain. This paper presents the topological relations considered by our approach of building spatial ontologies and formally defines these relations.

In the first part of this paper, we present an approach of building spatial ontologies which defines a process of building adding a new step of spatialization [18] to the process of building domain ontologies [14]. In the second part of this paper, we present the formal definitions of topological relations written in descriptive logic. We end this paper with a conclusion and future works we intend to achieve.

## II. APPROACH OF BUILDING A SPATIAL ONTOLOGY

Several studies have been carried on methods for building ontologies [21]. We propose an approach for building spatial ontologies that defines a process of building with two application phases: meta-modeling phase and modeling phase. The first phase of meta-modeling defines, for each step, a meta-model that will serve as a reference then during the second phase of modeling of the process. The proposed process is based on the steps of the building process of domain ontology [14] and adds an intermediate step of definition and explanation of spatial relations within geographic applications. To this end, our approach is based on the following four steps: conceptualization, Spatialization, ontologization and Operati onalization. (Fig. 2) presents the process of building spatial ontologies.

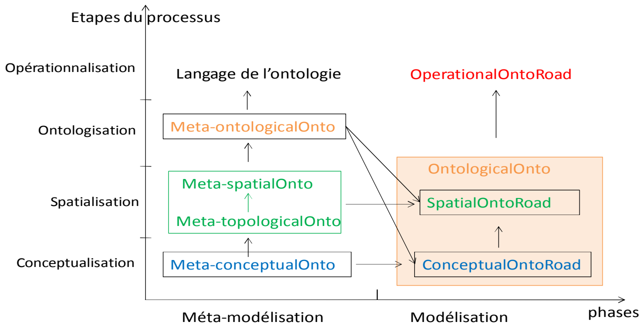


Figure 2. Process of building spatial ontologies.

The first phase of meta-modeling is to define, for every step of ontology construction, a meta-model that will serve as reference in the subsequent phase. The conceptualization step is to identify a body in the knowledge domain and to clarify the conceptual nature (concepts, relations, properties and relations of concepts, rules, constraints, etc.) of extracted knowledge from the corpus. The use of object-oriented paradigm for the conceptualization of the geographical world has been widely discussed in the literature [17]. It consists of definition of geographical object, their attributes and their relations. We proceeded to study the nature of knowledge may exist in a spatial ontology and we considered that it is important that a spatial ontology consists of a set of concepts characterized by their names and relations between concepts. Relations are those supported by the Unified Modeling Language (UML) [23] namely association, aggregation, composition and generalization.

The spatialization step is to give tow points: a spatial dimension to the concepts of ontology and to clarify the spatial relations between them. The spatial aspect of a concept is reflected in the graphic form of this concept: Line, Point or Polygon. Then we classify the ontology concepts according to their graphic shapes. A Point

 ISBN: 978-1-61208-269-1 

is characterized by a name, an abscissa and an ordinate. A Line is characterized by a name, two points: an initial section (is) and an end section (es), a direction, a lenght and a height. A Polygon is characterized by a name and at least three points which form its extremities. Spatial relations are of two types: metric relations and topological relations. Metric relations fall into two classes: distance relations that express a value of distance and unit of measure; and proximity relations that express an approximate distance between two spatial objects. We characterize a topological relation between two spatial concepts by the graphic form of the intersection of these two concepts. To represent these relations we use the formalism of connection to express that two geographical entities share a geographical space. Each topological relation is modeled graphically in [15] to show the graphic shape of the intersection of two spatial objects. (Table 2) lists all topological relations supported supported by our approach.

TABLE II. SUPPORTED TOPOLOGICAL RELATIONS.

| Graphic Form / Topological relation | Point /Point | Point /Line | Point/ Polygon | Line/ Line | Line/ Polygon | Polygon/ Polygon |
|---|---|---|---|---|---|---|
| Equality | x | | | x | | x |
| Extremity | | x | | | | |
| Inclusion | | x | x | x | x | x |
| Connection | | | x | | x | x |
| Jonction | | | | x | | |
| Joint | | | | x | | |
| Meet | | | | x | x | |
| Adjacency | | | | x | x | |
| Superposition | | | | x | | |
| Partial-Recovery | | | | | | x |

These relations are represented as a graph called "Meta-topologicalOnto" (Fig. 3). In this meta-model, classes represent the graphic shapes of one concept including: Point, Line or Polygon; and relations represent the topological relations considered in our ontology. After considering the spatial relations of our ontology we proceeded to define the spatial meta-model "meta-SpatialOnto" which extends the conceptual model [16] with spatial relations.

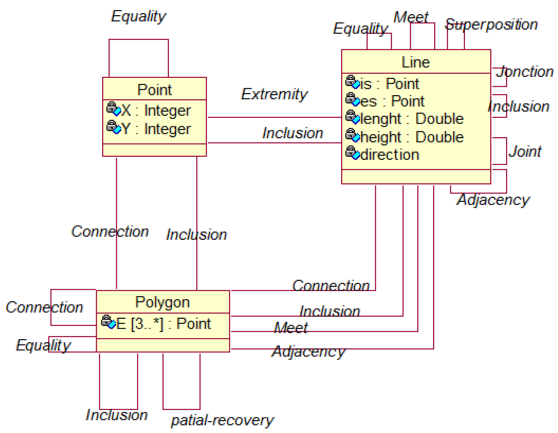


Figure 3. Topological meta-model of a spatial ontology: "Meta-topologicalOnto".

The third step of the process of building a spatial ontology is the ontologization step. It consist to model in a formal language the domain properties, the objective is to obtain a model in which almost all the ambiguities inherent in natural language are lifted. Finally, the operationalization is to make operational or functional ontology. First, must select the ontology language and the tool to build the ontology.

The modelling phase depends on the domain of the ontology and consists to define, for each step of the process of building, a model containing the concepts and relations of the domain. The result of this phase is a model of a spatial ontology instantiable depending on the user data. We chose the road domain as field of application of our work. The result ontology is called "*OntoRoad*".

We focus in this paper on the metamodeling phase of the ontologization step of process of building a spatial ontology, especially we focus on topological relations of spatial ontology. Next section will present the formal definitions of supported topological relations.

## III. FORMAL DEFINITIONS OF TOPOLOGICAL RELATIONS

Various studies have formally defines topological relations between spatial objects [19] [20] [22]. However, new needs of expressing these relations otherwise have arisen, including the need to express the type of the graph entity resulting of the intersection of the objects involved in the topological relation.

We define usage rules and we formally written topological relations of the ontology using the descriptive logic, the set of these rules and these formal definitions is called a formal meta-model of a spatial ontology "meta-OntologicalOnto".

We define the graphical shapes of a spacial object namely: Point, Line and Polygon as structures of objects. Thus we write:

**Rule 1**: a Point is characterized by a name of string and an x and y coordinates of integer:

$$\text{Point:} \left\{ \begin{matrix} \text{name:} & \text{String} \\ \text{x :} & \text{entier} \\ \text{y:} & \text{entier} \end{matrix} \right\} \quad (1)$$

**Rule 2**: A Line is characterized by a name of string, by the properties ds: start of section and fs: end of section of Point, a height of integer and a direction that takes a value of the Direction Set which is defined by:

Direction = {East, Weast, North, South, North-East, North-Weast, South-East, South-Weast}

$$\text{Line:} \left\{ \begin{matrix} \text{Name:} & \text{String} \\ \text{ds:} & \text{Point} \\ \text{fs:} & \text{Point} \\ \text{direction:} & \text{Direction} \\ \text{heigt:} & \text{Integer} \end{matrix} \right\} \quad (2)$$

**Rule 3:** A Polygon is characterized by a name of string and five extremities of Point: e1, e2, e3, e4 and e5.

$$\text{polygon:}\left\{\begin{matrix} \text{name} & \text{String} \\ \text{e1 :} & \text{Point} \\ \text{e2 :} & \text{Point} \\ \text{e3 :} & \text{Point} \\ \text{e4 :} & \text{Point} \\ \text{e5 :} & \text{Point} \end{matrix}\right\} \quad (3)$$

To define the formal definitions of spatial relations we consider: C1, C2: two spatial concepts; P1, P2: two variables of Point; G1, G2: two variables of Polygon; L1, L2: two variables of Line. We also consider the sets: E1and E2, where E1= {e1, e2, e3, e4, e5} representing the extremities of G1 and E2 = {e'1, e'2, e'3, e'4, e'5} representing the extremities of G2.

**Rule 4:** Equality (C1, C2) is a relation which holds between two spatial concepts C1 and C2 if and only if the intersection of C1 and C2 is equal to the concept itself. Then, we write:

$$\begin{gathered}\mathrm{Equality(C1,C2)} \Leftrightarrow (\mathrm{C1} \cap \mathrm{C2} = (\mathrm{C1} \vee \mathrm{C2})) \wedge \\ \begin{pmatrix}\mathrm{C1,C2{:}\,Point} \ \wedge \mathrm{C1.name} = \mathrm{C2.name} \wedge \\ \mathrm{C1.x} = \mathrm{C2.x} \wedge \mathrm{C1.y} = \mathrm{C2.y}\end{pmatrix} \vee \\ \begin{pmatrix}\mathrm{C1,C2{:}\,Ligne} \ \wedge \mathrm{C1.name} = \mathrm{C2.name} \wedge \\ \mathrm{C1.ds} = \mathrm{C2.ds} \wedge \mathrm{C1.fs} = \mathrm{C2.fs}\end{pmatrix} \vee \\ \begin{pmatrix}\mathrm{C1,C2{:}\,Polygone} \wedge \mathrm{C1.name} = \mathrm{C2.name} \ \wedge \\ \mathrm{C1.E1} = \mathrm{C2.E2}\end{pmatrix}\end{gathered} \quad (4)$$

Equality(C1, C2) is transitive :

$$\mathrm{Equality(C1,C2)} \wedge \mathrm{Equality(C2,C3} \Rightarrow \mathrm{Equality(C1,C3)} \quad (5)$$

Equality(C1, C2) is symmetric:

$$\mathrm{Equality(C1,C2)} \Leftrightarrow \mathrm{Equality(C2,C1)} \quad (6)$$

**Rule 5:** Extremity(P1, L1) is a relation which holds between two concepts P1 and L1 of respective graphic shapes Point and Line, if and only if the intersection of P1 and L1 is equal to P1 and P1 is one extremity of L1. Then, we write:

$$\mathrm{Extremity(P1,L1)} \Leftrightarrow \mathrm{L1} \ \cap \mathrm{P1} = \mathrm{P1} \ \wedge (\mathrm{Equality(L1.ds,P1)} \vee \mathrm{Equality(L1.fs,P1)}) \quad (7)$$

Extremity(P1, L1) is not transitive.
Extremity(P1, L1) is not symmetric.

**Rule 6:** Inclusion (P1, L1) is a relation which holds between two spatial concepts P1 and L1of respective graphic shapes Point and Line if and only if the intersection of P1 and L1 is equal to P1 and the function Linear(P1, L1.ds, L1.fs) is satisfied. Linear( ) is a mathematical function which checks that three points are linear. This function is defined as follows: considering P1, P2 and P3 three points with respective coordinates (x, y), (x ', y') and (x", y");

$$Linear(P1,P2,P3) \Leftrightarrow (x - x'') * (y' - y'') - (x' - x'') * (y - y'') = 0 \quad (8)$$

Then, we write:

$$Inclusion(P1,L1) \Leftrightarrow P1 \ \cap L1 = P1 \ \wedge Linear(P1, L1.ds, L1.fs) \quad (9)$$

Inclusion(P1, L1) isn't transitive.
Inclusion(P1, L1) is not symmetric.

**Rule 7:** Connection(P1, G1) is a relation which holds between two spatial concepts P1 and G1 of respective graphic shapes point and Polygon if and only if the intersection of P1 and G1 is equal to P1 and P1 belongs to the extremities of G1. Then, we write:

$$Connexion(P1,G1) \Leftrightarrow (P1 \cap G1 = P1) \wedge P1 \in G1.E \quad (10)$$

Connection(P1, G1) isn't transitive.
Connection(P1, G1) isn't symmetric.

**Rule 8:** Inclusion(P1, G1) is a relation which holds between two spatial concepts P1 and G1 of respective graphic shapes point and Polygon if and only if the intersection of P1 and G1 is equal to P1 and the relation Inclusion (P1, Rect(G1) ) is true.

In order to check if a point belongs to a polygon G1, we use the concept of the minimum bounding rectangle which is defined as the smallest rectangle containing the geometry of an object. The sides of the rectangle can be oriented parallel to the x axis and the y axis, we obtain the minimum bounding rectangle x, y. The bounding polygon of G1is named Rect(G1) defined by [Xmin, Xmax, Ymin, Ymax] Then we write:

$$\mathrm{Inclusion(P1,G1)} \Leftrightarrow \mathrm{P1} \cap \mathrm{G1} = \mathrm{P1} \wedge (\mathrm{Xmin} < P1.x < \mathrm{Xmax}) \quad \wedge \quad (\mathrm{Ymin} < P1.y < \mathrm{Ymax}) \quad (11)$$

Inclusion(P1, G1) is not transitive.
Inclusion(P1, G1) is not symmetric.

**Rule 9:** Inclusion (L1, L2) is a relation which holds between two spatial concepts L1 and L2 of line graphic shapes is verified if and only if the intersection of L1 and L2 is equal to L1; and the extremities of L1 admit an Inclusion () relation with L2. Then, we write:

$$Inclusion(L1,L2) \Leftrightarrow (L1 \cap L2 = L1) \wedge Inclusion(L1.ds, L2) \ \wedge Inclusion(L1.fs, L2) \quad (12)$$

Inclusion(L1, L2) is transitive.

$$Inclusion(L1, L2) \wedge Inclusion(L2, L3) \Rightarrow Inclusion(L1, L3) \quad (13)$$

Inclusion(L1, L2) is not symmetric.

**Rule 10:** Joint(L1, L2) is a relation which holds between two spatial concepts L1 and L2 of Line graphic shapes is satisfied if and only if the intersection of L1 and L2 is equal to a point P1 which is the extremity of both L1 and L2. Then, we write:

$$Joint(L1, L2) \Leftrightarrow (L1 \cap L2 = P1) \wedge Extremity(P1, L1) \wedge Extremity(P1, L2) \quad (14)$$

Joint(L1, L2) isn't transitive.
Joint(L1, L2) is symmetric :

$$Joint(L1, L2) \Leftrightarrow Joint(L2, L1) \quad (15)$$

**Rule 11:** Junction (L1, L2) is a relation which holds between two spatial concepts L1 and L2 of line graphic shapes is verified if and only if the intersection of L1 and L2 is equal to a point P1which is neither an extremity of L1 nor of L2. Then, we write:

$$Junction(L1, L2) \Leftrightarrow (L1 \cap L2 = P1) \wedge Inclusion(P1, L1) \wedge Inclusion(P1, L2) \wedge \neg Extremity(P1, L1) \wedge \neg Extremity(P1, L2) \quad (16)$$

Junction(L1, L2) isn't transitive.
Junction(L1, L2) is symmetric :

$$Junction(L1, L2) \Leftrightarrow Junction(L2, L1) \quad (17)$$

**Rule 12:** Meet(L1, L2) is a relation which holds between two spatial concepts L1 and L2 of Line graphic shapes is satisfied if and only if the intersection of L1 and L2 is equal to a point which is equal to one extremity of L1. Then, we write:

$$\mathrm{Meet}(\mathrm{L1}, \mathrm{L2}) \Leftrightarrow (\mathrm{L1} \cap \mathrm{L2} = \mathrm{P1}) \wedge \mathrm{Extremity}(\mathrm{P1}, \mathrm{L1}) \wedge \mathrm{Inclusion}(\mathrm{P1}, \mathrm{L2}) \quad (18)$$

Meet(L1, L2) isn't transitive.
Meet(L1, L2) isn't symmetric.

**Rule 13:** Adjacency (L1, L2) is a relation which holds between two spatial concepts L1 and L2 of line graphic shapes is verified if and only if the intersection of L1 and L2 is equal to a line L3 and the L3 extremities are equal to L1 or L2 extremities. Then, we write:

$$\mathrm{Adjacency}(\mathrm{L1}, \mathrm{L2}) \Leftrightarrow \mathrm{L1} \cap \mathrm{L2} = \mathrm{L3} \wedge \{\mathrm{L3.ds}, \mathrm{L3.fs}\} \equiv (\{\mathrm{L1.ds}, \mathrm{L1.fs}\} \vee \{\mathrm{L2.ds}, \mathrm{L2.fs}\}) \quad (19)$$

Adjacency (L1, L2) isn't transitive.
Adjacency (L1, L2) is symmetric:

$$\mathrm{Adjacency}(\mathrm{L1}, \mathrm{L2}) \Leftrightarrow \mathrm{Adjacency}(\mathrm{L2}, \mathrm{L1}) \quad (20)$$

**Rule 14:** Superposition (L1, L2) is a relation which holds between two spatial concepts L1 and L2 of line graphic shapes is verified if and only if L1 has a height different to zero and L1 extremities belong to L2. Then, we write:

$$\mathrm{Superposition}(\mathrm{L1}, \mathrm{L2}) \Leftrightarrow (\mathrm{L1.height} \neq 0 \wedge \mathrm{Inclusion}(\mathrm{l1.ds}, \mathrm{L2}) \wedge \mathrm{Inclusion}(\mathrm{L1.fs}, \mathrm{L2}) \quad (21)$$

Superposition(L1, L2) isn't transitive.
Superposition(L1, L2) isn't symmetric.

**Rule 15:** Inclusion (L1, G1) is a relation which holds between two spatial concepts L1 and G1 of respective graphic shapes line and polygon is satisfied if and only if the intersection of L1 and G1 is equal to L1 and L1 extremities admit Inclusion( ) relation with G1. Then, we write:

$$\mathrm{Inclusion}(\mathrm{L1}, \mathrm{G1}) \Leftrightarrow \mathrm{L1} \cap \mathrm{G1} = \mathrm{L1} \wedge \mathrm{Inclusion}(\mathrm{L1.ds}, \mathrm{G1}) \wedge \mathrm{Inclusion}(\mathrm{L1.fs}, \mathrm{G1}) \quad (22)$$

Inclusion(L1, G1) is not transitive.
Inclusion(L1, G1) is not symmetric.

**Rule 16:** Meet(L1, G1) is a relation which holds between two spatial concepts L1 and G1 of respective graphic shapes Line and polygon is satisfied if and only if the intersection of L1 and G1 is equal to a point which is one of the extremities of L1 and the other extremity of L1 does not admit Inclusion( ) relation with G1. Then, we write:

$$\mathrm{Meet}(\mathrm{L1}, \mathrm{G1}) \Leftrightarrow \mathrm{L1} \cap \mathrm{G1} = \mathrm{P1} \wedge \big((\mathrm{L1.ds} = \mathrm{P1} \wedge \neg\mathrm{Inclusion}(\mathrm{L1.fs}, \mathrm{G1})\big) \vee \big(\mathrm{L1.fs} = \mathrm{P1} \wedge \neg\mathrm{Inclusion}(\mathrm{L1.ds}, \mathrm{G1})\big) \quad (23)$$

Meet (L1, G1) is not transitive.
Meet (L1, G1) is not symmetric.

**Rule 17:** Adjacency (L1, G1) is a relation which holds between two spatial concepts L1 and G1 of respective graphic shapes line and polygon is satisfied if and only if the intersection of L1 and G1 is equal to a line L2 which extremities belong to G1 extremities and the extremities of L1 do not admit Inclusion( ) relation with G1. Then, we write:

$$\mathrm{Adjacency}(\mathrm{L1}, \mathrm{G1}) \Leftrightarrow \mathrm{L1} \cap \mathrm{G1} = \mathrm{L2} \wedge \exists\{\mathrm{e1}, \mathrm{e2}\} \subset \mathrm{E1} \wedge \{\mathrm{L2.ds}, \mathrm{L2.fs}\} = \{\mathrm{e1}, \mathrm{e2}\} \wedge \neg\mathrm{Inclusion}(\mathrm{L1.ds}, \mathrm{G1}) \wedge \neg\mathrm{Inclusion}(\mathrm{L1.fs}, \mathrm{G1}) \quad (24)$$

Adjacency (L1, G1) is not transitive.
Adjacency (L1, G1) is not symmetric.

**Rule 18:** Connection(L1, G1) is a relation which holds between two spatial concepts L1 and G1 of respective graphic shapes line and polygon is satisfied, if and only if the intersection of L1 and G1 is equal to a line L2 and at least one of the extremities of L1 doesn't admit *Inclusion*() relation with G1. Then, we write:

$$\mathrm{Connection}(\mathrm{L1},\mathrm{G1}) \Leftrightarrow \mathrm{L1} \cap \mathrm{G1} = \mathrm{L2} \wedge \mathrm{Inclusion}(\mathrm{L2.ds},\mathrm{G1}) \wedge \mathrm{Inclusion}(\mathrm{L2.fs},\mathrm{G1}) \wedge \exists \mathrm{P}: \mathrm{Point}\ (\mathrm{P} \in \{\mathrm{L1.ds},\mathrm{L1.fs}\} \wedge \neg\, \mathrm{Inclusion}(\mathrm{P},\mathrm{G1})) \quad (25)$$

Connection(L1, G1) is not transitive.
Connection(L1,G1) is not symmetric.

**Rule 19:** Inclusion (G1, G2) is a relation which holds between two spatial concepts G1 and G2 of polygon graphic shapes if and only if the intersection of the two concepts G1 and G2 is equal to G1 and all the extremities of G1 admit an Inclusion( ) relation with G2. Then, we write:

$$\mathrm{Inclusion}(\mathrm{G1},\mathrm{G2}) \Leftrightarrow \mathrm{G1} \cap \mathrm{G2} = \mathrm{G1} \wedge \forall\ \mathrm{e} \in \mathrm{E1}\ \ \mathrm{Inclusion}(\mathrm{e},\mathrm{G2}) \quad (26)$$

Inclusion(G1,G2) is transitive :

$$\mathrm{Inclusion}(\mathrm{G1},\mathrm{G2})\ \wedge \mathrm{Inclusion}(\mathrm{G2},\mathrm{G3}) \Rightarrow \mathrm{Inclusion}(\mathrm{G1},\mathrm{G3}) \quad (27)$$

Inclusion(G1,G2) is not symmetric.

**Rule 20:** *Connection*(G1, G2) is a relation which holds between two spatial concepts of polygon graphic shapes if and only if the intersection of G1 and G2 is equal to a point belonging to the extremities of both G1 and G2, the other extremities of G1 do not belong to G2 and other extremities of G2 do not belong to G1. Then, we write:

$$\mathrm{Connection}(\mathrm{G1},\mathrm{G2}) \Leftrightarrow \mathrm{G1} \cap \mathrm{G2} = \mathrm{P1}\ \wedge (\mathrm{P1} \in \mathrm{G1.E} \wedge \mathrm{P1} \in \mathrm{G2.E}) \wedge \left(\forall \mathrm{e} \in \mathrm{E1} - \mathrm{P1}, \neg \mathrm{Inclusion}(\mathrm{e},\mathrm{G1})\right) \wedge \left(\forall \mathrm{e} \in \mathrm{E2} - \mathrm{P1}, \neg \mathrm{Inclusion}(\mathrm{e},\mathrm{G2})\right) \quad (28)$$

*Connection* (G1, G2) is not transitive.
Connection (G1, G2) is symmetric:

$$\mathrm{Connection}(\mathrm{G1},\mathrm{G2}) \Leftrightarrow \mathrm{Connection}(\mathrm{G2},\mathrm{G1}) \quad (29)$$

**Rule 21:** Adjacency (G1, G2) is a topological relation which is satisfied between two spatial concepts G1 and G2 of polygon graphic shapes if and only if the intersection of G1 and G2 is equal to a line L1 and the extremities of L1 belong to the extremities of both G1 and G2. Then, we write:

$$\mathrm{Adjacency}(\mathrm{G1},\mathrm{G2}) \Leftrightarrow \mathrm{G1} \cap \mathrm{G2} = \mathrm{L1} \wedge \exists\ \{\mathrm{e1},\mathrm{e2}\} \subset \mathrm{E1}, \{\mathrm{e1},\mathrm{e2}\} \subset \mathrm{E2} \wedge \{\mathrm{L1.ds},\mathrm{L1.fs}\} = \{\mathrm{e1},\mathrm{e2}\} \quad (30)$$

Adjacency(G1,G2) is not transitive.
Adjacency(G1,G2) is symmetric:

$$\mathrm{Adjacency}(\mathrm{G1},\mathrm{G2}) \Leftrightarrow \mathrm{Adjacency}(\mathrm{G2},\mathrm{G1}) \quad (31)$$

**Rule 22**: Partial-Recovery(G1, G2) is a topological relation which holds between two spatial concepts G1 and G2 of polygon graphic shapes if and only if the intersection of G1 and G2 is equal to a polygon G3 and there is at least one extremity e1 of G1 admitting Inclusion() relation with G2 and there is at least one extremity e'1 of G2 admitting an Inclusion() relation with G1 and {e1, e'1} ⊂ E3 (Extremities of G3). Then, we write:

$$Partial-Recovery(G1,G2) \Leftrightarrow \mathrm{G1} \cap \mathrm{G2} = \mathrm{G3} \wedge \exists\ \mathrm{e1} \in \mathrm{E1}, \mathrm{e1} \in \mathrm{E2}, \{\mathrm{e1},\mathrm{e'1}\} \subset\ \mathrm{E3}, \mathrm{Inclusion}(\mathrm{e1},\mathrm{G2}) \wedge \mathrm{Inclusion}(\mathrm{e1},\mathrm{G1}) \quad (32)$$

Partial-Recovery (G1,G2) isn't transitive.
Partial-Recovery (G1,G2) is symmetric:

$$Partial-Recovery(G1,G2) \Leftrightarrow Partial-Recovery(G2,G1) \quad (33)$$

## IV. CONCLUSION AND FUTURE WORK

In this paper, we presented an approach of building spatial ontologies, which defines a process of building realised in two phases: the meta-modelling phase and the modelling phase. Then, we detailed, the meta-modeling phase of the ontologization step of the process of building a spatial ontology. The result of this step is an ontological meta-model "meta-OntologicalOnto" representing the formal definitions of topological relations of the ontology using description logic. In future work, we will define the modeling phase of the building process of a spatial ontology. This phase depends on the field of study and refers to the meta-modeling phase to define the models of the ontology. Our approach is applied to the road domain to give as a result a road ontology named "OntoRoad" which will be instantiated with data from several geographic areas of Sfax city in Tunisia in purposes of geo-localization.

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