Transmission Network Expansion Planning under Uncertainty using the Conditional Value at Risk and Genetic Algorithms

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***Abstract*—The aim of this work is to study the Transmission Network Expansion Planning (TNEP) problem considering uncertainty on the demand side. Such problem consists of deciding how should an electrical network be expanded so that the future demand is ensured. We expanded the power transport problem formulation so that power losses are included in the objective function. Uncertainty is included through stochastic programming based on scenario analysis; different degrees of uncertainty are considered. Further, an explicit risk measure is added to mathematical model using the Conditional Value at Risk (CVaR). Weighting the relative importance of minimizing expansion and operational costs against the value of the CVaR simulates the attitude of the investor towards risk and shows to be of significant importance when planning the future. The problem was optimized using Genetic Algorithms. This work provided insight on how investment decisions change when considering several levels of uncertainty and risk aversion, in an extended formulation of the TNEP problem.**

Keywords–Transmission Network Expansion Planning; Genetic Algorithms; Uncertainty; Risk Analysis.

I. Introduction

The prime objective of the Transmission Network Expansion Planning (TNEP) [1] problem is to decide how should a power transmission network be expanded in order to supply a forecast value (or values) of the demand.

Future uncertainties are such a challenging issue for the various prediction methods that one must acknowledge that long-term forecasts might, and most likely will, be wrong [2]. In order to mitigate the effect of uncertainty, building not only one but several scenarios has proved to be a technique that, despite its greater computational effort, presents an reliable method to tackle the ambiguities of the future. Some of the most commonly uncertainties studied in the TNEP problem relate to: deliberate attacks upon the grid [1], [3], capacity failing of the transmission lines [4], and demand uncertainty [5]. The scenario analysis in the above works is implemented using stochastic programming [4]. Despite the usefulness of stochastic programming and scenario analysis, these approaches do not provide a risk assessment method. The work developed in [5] evaluates risk using fuzzy techniques after the optimization is done for each scenario individually, and so is not incorporated in the cost function. The drawback of this approach is that not including risk measure in the mathematical model conditions the number of scenarios that can be tested mainly to the difficult interpretation of the individual results of each scenario [3]. In this work we aim to implement a risk-based stochastic formulation of the TNEP problem that considers the concepts of *Value at Risk (VaR)* and *Conditional Value at Risk (CVaR)*.

In the business and finance sectors, the VaR summarizes the worst expected loss over a target horizon within a given confidence interval and CVaR is a measure which could be defined as the expected value of the losses worse than VaR, over the same target horizon [6]. In the TNEP problem the losses that were just referred, concern the operational costs for the different scenarios (namely generation and curtailment), under each expansion plan.

The paper is organized as follows. Section 2 describes the modeling of TNEP. Emphasis is made on modeling with uncertainty. The risk analysis used in the paper are described in Section 3. Section 4 described the genetic algorithm applied to solve the TNEP problem. The obtained results are presented and discussed in Section 5, and finally, Section 6 presents the conclusions.

II. Modeling

A. Loss Free Model

The TNEP problem is a large combinatorial problem, as many possible configurations of the network may satisfy the demand, and more importantly might have very similar or equal costs. A common approach to tackle the problem is to use the DC model formulation for the power flow equations.

Even with the simplifications considered in the DC-Model the TNEP problem constitutes a large combinatorial, non-polynomial, multi-dimensional problem, that cannot be solved either without employing further simplifications, or heuristic and meta-heuristic approaches. The classical DC-Model for the TNEP problem is presented below.

$$\min_{\theta,g,n} \sum_{\forall(i,k)\in\Omega} c_{ik}n_{ik} + \sum_{\forall i\in\Omega_g} c_{g_i}p_{g_i} \tag{1}$$

s.t.

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$$-\sum_{k\neq i} f_{ik} + p_{g_i} = d_i \tag{2}$$

$$f_{ik} - b_{ik}(n^o_{ik} + n_{ik})(\theta_i - \theta_j) = 0 \tag{3}$$

$$|f_{ik}| \le (n^o_{ik} + n_{ik})\overline{f_{ik}} \tag{4}$$

$$0 \le p_{g_i} \le \overline{p_{g_i}} \tag{5}$$

$$0 \le n_{ik} \le \overline{n_{ik}} \tag{6}$$

where n_{ik} is integer, θ_i is unbounded, (1) is the cost function divided in two parcels, the first relating to expansion costs and the second to operational costs, (2) is the flow balance constraint in each node network, (3) is the flow calculation formula, and (4)-(6) are the capacity constraints.

B. DC Model with power losses

The inclusion of power losses in a non-linear manner in the TNEP problem has already been presented in [7] and results in changes to (2) and (4) which can now be written as:

$$-\sum_{k\neq i}(f_{ik} + \frac{1}{2}h_{ik}) + p_{g_i} = d_i \tag{7}$$

$$|f_{ik}| + \frac{1}{2}h_{ik} \le (n^o_{ik} + n_{ik})\overline{f_{ik}} \tag{8}$$

where the losses h_{ik} are defined as:

$$h_{ik} = g_{ik}\theta^2_{ik} \tag{9}$$

C. DC Model with load curtailment

This paper introduces load curtailment as measure of how much active power is left unsupplied. This is an important variable when considering uncertainty, since the unexpected rise of the demand might lead to a shortage of power transmission capability.

One must notice how the cost associated with unsupplied power changes the problem in conceptual terms. If the cost of load curtailment is low, then this would pose another decision for the investor to make, whether or not to expand the network further satisfying a higher percentage of the total demand or if not leaving a higher percentage of the demand unsupplied. On the other hand if the the cost of the load curtailment is extremely hight this would cause the solutions that consider curtailed load to be extremely costly. Practically, this means that considering high curtailment costs is equivalent to search for a solution where all of the demand is met.

The model considering load curtailment and power losses simultaneously changes the problem cost function, (1), and the flow balance (7), into the following:

$$\min_{\theta,g,n,r} \sum_{\forall(i,k)\in\Omega} c_{ik}n_{ik} + \sum_{\forall i\in\Omega_g} c_{p_{g_i}}p_{g_i} + \sum_{\forall i\in\Omega_d} c_{r_i}r_i \tag{10}$$

$$-\sum_{k\neq i}(f_{ik} + \frac{1}{2}h_{ik}) + p_{g_i} + r_i = d_i \tag{11}$$

D. Model with uncertainty

Deterministic models can be transformed into stochastic optimization models that take into account the randomness of the stochastic variables and these models can be solved using stochastic programming techniques. The stochastic model that considers power losses, load curtailment and uncertainty is:

$$\min_{\theta,g,n,r} \sum_{\forall(i,k)\in\Omega} c_{ik}n_{ik} +$$

$$\sum_{\lambda\in\Lambda}\Pi(\lambda)\left(\sum_{\forall i\in\Omega_g} c_{p_{g_i}}p^\lambda_{g_i} + \sum_{\forall i\in\Omega_d} c_{r_i}r^\lambda_i\right)$$

s.t.

$$-\sum_{k\neq i}(f^\lambda_{ik} + \frac{1}{2}h^\lambda_{ik}) + p^\lambda_{g_i} + r^\lambda_i = d^\lambda_i \tag{12}$$

$$f^\lambda_{ik} - b_{ik}(n^o_{ik} + n_{ik})(\theta_i - \theta_j) = 0 \tag{13}$$

$$h^\lambda_{ik} = g_{ik}(\theta^\lambda_{ik})^2 \tag{14}$$

$$\left|f^\lambda_{ik}\right| + \frac{1}{2}h^\lambda_{ik} \le (n^o_{ik} + n_{ik})\overline{f^\lambda_{ik}} \tag{15}$$

$$0 \le p^\lambda_{g_i} \le \overline{p^\lambda_{g_i}} \tag{16}$$

$$0 \le n_{ik} \le \overline{n_{ik}} \tag{17}$$

where n_{ik} is integer, θ^λ_i is unbounded, λ is a scenario, and $\Pi(\lambda)$ is the probability of each scenario. The model given above shows that each expansion plan is evaluated for all possible scenarios.

However, in [8], stochastic programming in itself does not rule out that riskier options are chosen considering all plausible scenarios. In fact, this stochastic formulation alone is considered to be a *Risk-Neutral* approach when dealing with uncertainty as pointed out by [1]. To address this issue the following subsection describes the risk measure used in this work, its relation to scenario analysis and stochastic programming and its inclusion in the mathematical model.

III. Risk Analysis

One of the major purposes of this work is to assess investments cost under uncertainty considering an explicit risk measure in the mathematical model. The choice then falls on how to quantify risk and the investor's respective risk attitude. In order to so effectively, a measurement is needed that provides reliable assessment on the relative risk of several different solutions for set of plausible scenarios.

A. Conditional Value at Risk

The *Conditional Value at Risk* has proven to be an useful tool in assessing risk due to its linearity and conservativeness [9]. Moreover, CVaR has been reported to outperform other risk measurements as it can readily be incorporated into any optimization problem as using the following formula [9], [10] as:

$$\widetilde{F}_\varphi(\omega_\lambda, \xi) = \xi + \frac{1}{m(1-\varphi)}\sum_{\lambda=1}^{m}\omega_\lambda \tag{18}$$

and the CVaR optimization problem as:

$$\min_{\xi \ \omega_\lambda} \ \widetilde{F}_\varphi(\omega_\lambda, \xi) \tag{19}$$

s.t.

$$\omega_\lambda \geq 0 \tag{20}$$

$$\omega_\lambda \geq f(\mathbf{x}, y_\lambda) - \xi \tag{21}$$

where m is the number of scenarios, $\mathbf{x}$ is the variable concerning the option to be taken for the project and y_λ the value of the random variable y in scenario λ. In addition, for any solution $\mathbf{x}$ and a confidence level φ, VaR is the value of ξ such that the probability of the loss not exceeding ξ is φ [11].

To solve the TNEP problem. It is important to notice that what Rockafellar defines in [9] and [10] as the losses function $f(\mathbf{x}, y_\lambda)$, relates in the TNEP problem, to the cost of a given plan after uncertainty clears, i.e., the cost for a given scenario. Minimizing the CVaR for a given expansion in the TNEP problem can then be described by the model below.

$$\min_{\xi} \ \text{CVaR} \tag{22}$$

s.t.

$$\omega_\lambda \geq 0 \tag{23}$$

$$\omega_\lambda \geq \left(\sum_{\forall (i,k) \in \Omega} c_{ik} n_{ik} + \sum_{\forall i \in \Omega_g} c_{p_{g_i}} p_{g_i}^\lambda + \sum_{\forall i \in \Omega_d} c_{r_i} r_i^\lambda \right) - \xi \quad \forall \lambda \in \Lambda \tag{24}$$

Notice that, in our model we aim to include the risk attitude of the investor. The CVaR in itself does not provide information about the attitude of the investor, it provides information about the investment necessary to supply the demand under a set of possible scenarios with a certain degree of confidence. To include the risk attitude in the objective function the TNEP problem is expanded to include the CVaR, and weights were established between the stochastic formulation and the CVaR, to reflect the relative importance of minimizing each one.

B. Stochastic Model with Risk Aversion

The full optimization problem is presented below:

$$\min_{\theta, g, n, r} (1-\beta) \left[\sum_{\forall (i,k) \in \Omega} c_{ik} n_{ik} + \sum_{\lambda \in \Lambda} \Pi(\lambda) \left(\sum_{\forall i \in \Omega_g} c_{p_{g_i}} p_{g_i}^\lambda \sum_{\forall i \in \Omega_d} c_{r_i} r_i^\lambda \right) \right] + \beta(\text{CVaR}) \tag{25}$$

s.t.

$$-\sum_{k \neq i} (f_{ik}^\lambda + \frac{1}{2} h_{ik}^\lambda) + p_{g_i}^\lambda + r_i^\lambda = d_i^\lambda \tag{26}$$

$$f_{ik}^\lambda - b_{ik}(n_{ik}^o + n_{ik})(\theta_i - \theta_j) = 0 \tag{27}$$

$$h_{ik}^\lambda = g_{ik}(\theta_{ik}^\lambda)^2 \tag{28}$$

$$\left| f_{ik}^\lambda \right| + \frac{1}{2} h_{ik}^\lambda \leq (n_{ik}^o + n_{ik}) \overline{f_{ik}^\lambda} \tag{29}$$

$$0 \leq p_{g_i}^\lambda \leq \overline{p_{g_i}^\lambda} \tag{30}$$

$$0 \leq n_{ik} \leq \overline{n_{ik}} \tag{31}$$

$$\omega_\lambda \geq 0 \tag{32}$$

$$\omega_\lambda \geq \left(\sum_{\forall (i,k) \in \Omega} c_{ik} n_{ik} + \sum_{\forall i \in \Omega_g} c_{p_{g_i}} p_{g_i}^\lambda + \sum_{\forall i \in \Omega_d} c_{r_i} r_i^\lambda \right) - \xi \quad \forall \lambda \in \Lambda \tag{33}$$

with n_{ik} integer and θ_i^λ unbounded. In the objective function the stochastic formulation presented earlier and the CVaR are weighted by the parameter β. Such Parameter reflects the attitude towards risk of the investor. The the higher the β is, the more averse to risk the investor. Notice that for $\beta = 0$ the objective is reduced to the one of the stochastic formulation and so, according to [1], the investor is *Risk-Neutral*. Therefore, an investor whose attitude towards risk is very high will have a value of β very close to one. In this work we will employ a variety of values of β to study different levels of risk aversion.

IV. Genetic Algorithms in TNEP

GA belong the set of evolutionary algorithms (EA), that due to their population-based inherent nature, are able to tackle problems with a high degree of complexity [12]. For the TNEP problem an integer encoding is chosen that reflects the number of lines in the connection that such entry of the chromosome relates to, as done by Gallego in [13].

However, this information alone can only provide information on the expansion cost of the solution. In order to evaluate operational costs, namely generation and curtailment an augmented version of the common Optimal Power Flow problem was also derived in [7]. Possible load curtailment is considered at every demand node by the inclusion of a high cost generator andthe full augmented OPF is presented below. One must notice that if the lossless model is to be tested the

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term G is dropped.

$$\min_{\theta,P_g,P_r} \sum_{\forall i\in\Omega_g} \Gamma(P_{g_i}) + \sum_{\forall r\in\Omega_d} \Gamma(P_{r_i}) \tag{34}$$

s.t.

$$-\sum_{i\neq k}\left(F_{ik} + G_{ik}\frac{\theta_{ik}^2}{2}\right) + P_{g_i} + P_{r_i} = D_i \tag{35}$$

$$F_{ik} = -B_{ik}\theta_{ik} \tag{36}$$

$$|F_{ik}| + G_{ik}\frac{\theta_{ik}^2}{2} - \overline{F}_{ik} \leq 0 \tag{37}$$

$$0 \leq P_{g_i} \leq \overline{P}_{g_i} \tag{38}$$

$$\theta_r = 0 \tag{39}$$

The GA then performs a series of iterative computations in order to evolve a population of individuals (possible solutions), using the principle of *survival of the fittest*. These steps are the following: 1) Initialize the population; 2) Evaluate each chromosome; 3) Selection; 4) Crossover; 5) Mutations; 6) Replacement of the old population by the new; 7) Back to 2) until the termination criteria is met.

1) Evaluation: Operational costs are calculated through the augmented version of the OPF. Performing risk analysis is then possible minimizing the CVaR. Notice that if uncertainty is not considered then the cost evaluations consists in running the OPF once for the demand profile of case at study, since there are no scenarios and no possible risk.

2) Selection: Selection is a fitness based method, used with the purpose of choosing the most suited chromosomes in the population to form new individuals.

3) Crossover: Two individuals are selected from the population (applying two times the chosen selection operator) and are then recombined with a probability p_c, creating two new individuals. This is done by generating a random number $r \in [0, 1]$. If $r \leq C_r$, the two individuals are combined through crossover. The method chosen in this work is the two-point crossover.

4) Mutation: This operator selects an individual from the population with probability M_r and randomly changes one of its alleles. Having the chromosome integer variables, this consists in randomly choose one of the values possible for that variable.

5) Replacement and Elitism: Replacement is the process by which the new individuals, created with the above operators, are introduced in the population. There are cases when the fittest individual in the population is replaced by an individual with lower fitness. Therefore, the elitism reintroduces n_c copies of the best individual into the population.

V. Results

A. Deterministic approach

The network studied in this section is one of the most studied network configurations and was created specifically for the TNEP problem. [14] This nwtowrk has been the subject of various studies namely the ones presented in [14] [15] and [7]. This network consists on 3 generation nodes with a total capacity of 1100 MW, 5 demand nodes with a total demand of 760 (MW), and 15 different possible connections most of them with different line characteristics. Each entry of the chromosome corresponds to the listing of connection presented in Table I, and the constraints concerning generation capacity and the demand profile of the network are presented in Table II. In order to validate our results we will begin by employing a deterministic model, namely considering and neglecting losses.

TABLE I
LINE DATA FOR GRAVER'S EXAMPLE

Corridor	R (pu)	X (pu)	Capacity	Cost($*10^3)
1-2	0.10	0.40	100	40
1-3	0.09	0.38	100	38
1-4	0.15	0.60	80	60
1-5	0.05	0.20	100	20
1-6	0.17	0.68	70	68
2-3	0.05	0.20	100	20
2-4	0.10	0.40	100	40
2-5	0.08	0.31	100	31
2-6	0.08	0.30	100	30
3-4	0.15	0.59	82	59
3-5	0.05	0.20	100	20
3-6	0.12	0.48	100	48
4-5	0.16	0.63	75	63
5-6	0.15	0.61	78	61

TABLE II
BUS DATA FOR GRAVER'S EXAMPLE

Bus	D_i (MW)	$\overline{P_{g_i}}$ (MW)	c_{g_i} €/MW
1	80	150	10
2	240		
3	40	360	20
4	160		
5	240		
6	0	600	30

At this stage, and to have comparable results, we considered that no curtailment was possible and so, the cost of unsupplied load is very high to penalize solutions unable to supply all the demand. Notice that when using a meta-heuristic approach the inclusion of the load curtailment a more natural formulation of the problem,, since during the solution-space search the random inherent characteristics of meta-heuristic methods might find solutions where is impossible to supply all the demand. Should this occur, considering load curtailment in the mathematical model works as a penalty to the objective function rather than a possible operational cost.

Below, Tables III and IV present the results yielded by our GA when losses were neglected and considered, respectively. From these results we notice that in both cases the GA were able to find the best solution proposed thus far by any study.

Table V shows statistical results and the genetic operators with which the solutions were obtained.

B. Stochastic Cases

In this section, the optimization problem tackled through the GA is the one described from (25) to (33). For this purpose 30 scenarios were built randomly with a normal distribution and standard deviations of: 0.1, 0.2 and 0.3 from the mean value. Probabilities for each scenario were considered to be

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TABLE III
EXPANSION PLAN FOR GRAVER 6-BUS - LOSSES NEGLECTED

Method	Corridor	Number of Lines	Expansion Cost(10^3€)
Heuristic [14]	4-6	2	130
	3-5	2	
	2-3	1	
Genetic [13]	4-6	3	110
	3-5	1	
Improved Heuristic [7]	4-6	3	110
	3-5	1	
Proposed Genetic	4-6	3	110
	3-5	1	

TABLE IV
EXPANSION PLAN FOR GRAVER 6-BUS - CONSIDERING LOSSES

Method	Corridor	Number of Lines	Expansion Cost(10^3€)
MILP [15]	4-6	2	140
	3-5	1	
	2-6	2	
Improved Heuristic [7]	4-6	2	130
	3-5	1	
	2-6	1	
	2-3	1	
Proposed Genetic	4-6	2	130
	3-5	1	
	2-6	1	
	2-3	1	

equal, and Risk Attitude (β) was considered to have values of: 0; 0.25; 0.5; 0.75 and 1.

In order to consider a cost of curtailed load that will allow the investor to choose between curtailed load for a subset of scenarios and further network expansion we employ a method used by Van Mieghen in [16] where such cost is based on the concept of *Critical Fractile* (C_f). The *Critical Fractile* expresses the optimal service probability and depends on unit cost to unit return [16] or in TNEP, unit cost to unmet unit penalty. Practically it means that if the value of C_f is set a priori, differently for each demand node, it will also impose a different value of c_{r_i} for each demand node. To transpose these concepts from the work of Van Mieghen to the TNEP problem, we propose a curtailment cost based not only on the Critical Fractile but also on the average line cost and capacity as follows:

$$c_{r_i} = \frac{\overline{c}/\overline{F}}{1 - C_f} \tag{40}$$

$$C_{f_i} = 0.5 \qquad \forall i \in \Omega_d \tag{41}$$

where $\overline{c}$ is the average cost, $\overline{F}$ the average transmission capability. Fig. 1 shows the evolution of the cost function value as the risk aversion and standard deviation vary.

Notice how a standard deviation of 0.3 yields a much costlier solution, specially for higher degrees of Risk Aversion. This results show how for medium networks the effect of uncertainty can be extremely constraining, specially when many nodes have different mean values of demand. Secondly we present in Table VI the configurations that yielded the above costs.

TABLE V
GENETIC PARAMETERS AND STATISTIC RESULTS FOR GRAVER'S NETWORK

Assumption	P_z	M_r	C_r	Mean €*10^5	Std Dev €*10^5	Min €*10^5
No Loss	150	0.04	0.9	1.4453	0.1935	1.2668
Loss	100	0.05	0.9	1.5519	0.1756	1.4716

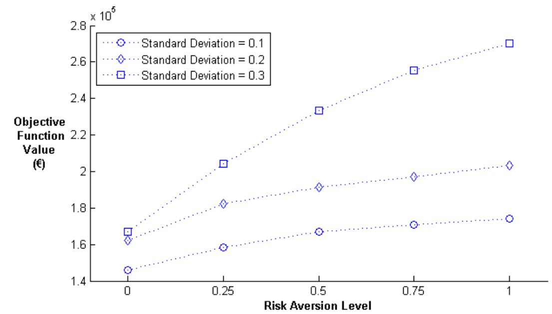

Figure 1. Results for Graver's Network under Uncertainty

Here we see that the solutions found for smaller values of standard deviations are in fact very similar to the ones obtained under the lossless-line assumption. Notice however, that now we considered possible curtailment and so decision of considering load which is not supplied is possible contrarily to before. It is also interesting to notice how the the connections built with an increasing degree of uncertainty and with and increasing level of risk aversion are in fact reinforcements of the configurations achieved before, meaning that for this particular case there is a set of connections which is extremely important when expanding the network.

Nevertheless, Fig.1 shows only the objective function value considering different values of standard deviation and risk aversion. Here would also be interesting to present how expansion costs and the risk measure evolve with the give varying parameters. Figs.2 and 3 show such evolutions.

As one can see, for any degree of uncertainty, we observe a decreasing in the CVaR value as risk aversion increases. An expected result as risk becomes more important to minimize when comparing to the expansion costs that, as also observable from Fig.2 increase as the aversion towards risk is higher.

Also these evolutions show that for some risk aversion levels (namely $\beta = 0.75$ and $\beta = 1$) the values of both expansion costs and CVaR are constant even though the value of the objective function in Fig.1 increases. This is explained by the different weights present in the objective function for different levels of risk aversion which let us conclude that even with an increasing aversion towards risk there was no solution that could further decrease the value of CVaR.

VI. CONCLUSIONS

This paper studies the Transmission Network Expansion Planning problem considering uncertainty on the demand side. Uncertainty is included considering scenario analysis. An explicit risk measure is incorporated in the mathematical model using the Conditional Value at Risk (CVaR). The problem

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TABLE VI
GA GRAVER'S NETWORK

σ	Corridor	β				
		0	0.25	0.5	0.75	1
0.1	4 - 6	3	3	2	2	2
	3 - 5	1	1	1	1	1
	2 - 6	0	0	2	2	2
0.2	4 - 6	3	2	2	2	2
	3 - 5	1	1	2	2	2
	2 - 6	0	2	2	2	2
0.3	4 - 6	1	2	2	2	2
	3 - 5	1	1	2	2	2
	2 - 6	2	2	2	3	3

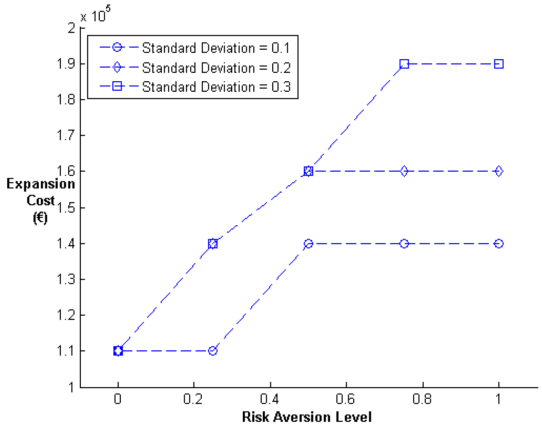

Figure 2. Expansion Costs for Graver's Network under Uncertainty

is optimized using genetic algorithms. Our results present concrete values for investment, ranging from a slightly risk-averse investor to an extremely risk-averse one. These results show that the variation in investment is steeper between a risk-neutral investor and a slightly-averse investor, since in the former no importance is given to the Conditional Value at Risk. We noticed that, even if the value of the cost function increases, certain levels of aversion yield the same expansion plan, meaning that in fact for this problem only a subset of risk attitudes of those considered are relevant to the study. From this perspective, this work also presents information on which risk aversion levels are enough to study risk attitude in TNEP, and in general values for $\beta = \{0, 0.5, 1\}$ show satisfactory differences that provide reliable information on how investment will increase with risk aversion. Another aspect of employing a risk analysis in TNEP is the cost of curtailed load. An increasing aversion towards risk was observed when a higher investment is needed.

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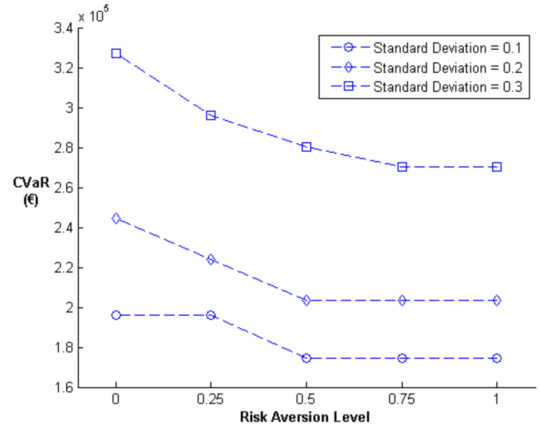

Figure 3. Conditional Value at Risk for Graver's Network under Uncertainty

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