A Cooperative Game for Distributed Wavelength Assignment in WDM Networks

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***Abstract*—The use of game theory in networking problems is becoming more popular given its potential to model real commercial situations where different agents try to optimize their profit. In this context, this paper proposes a novel cooperative game theory based distributed wavelength assignment method for WDM (Wavelength Division Multiplexing) networks. Experimental results show a clear improvement of the proposed method over a well recognized state of the art distributed wavelength assignment algorithm known as DIR (Destination Initiated Reservation). Thus, this new approach inspired in game theory, provides a first baseline for future work considering cooperation among competing long haul providers that may benefit from collaboration.**

***Keywords*-Game Theory; Nash Bargaining Problem; distributed RWA.**

I. Introduction

At the very core of modern telecommunications resides a very complex: the growing user base requires a solution to provide everyone with enough transmission capacity. Every solution provided to accommodate this ever growing user base, must satisfy contrasting (and in some cases contradictory) objectives: simply supplying bandwidth is not a desirable solution, one must find a way to use the bandwidth in an *efficient* way.

Emerging optical systems are deployed using WDM (Wavelength Division Multiplexing) [1]. In WDM systems, connection requests are satisfied by establishing all-optical channels between source and destination. Given a set of connection requests between two nodes and the paths connecting them, the RWA (Routing and Wavelength Assignment) problem models, as an ILP (Integer Linear Programming) [1], the assignment of every connection request between the two nodes to a free channel in a path joining them. Every resulting pair is known as a lightpath. The *wavelength continuity constraint* imposes a restriction on the lightpaths: the lightpath *must* be established using the same wavelength along the entire path [2]. Wavelength converters could be placed at the nodes to weaken this restriction; however, this is a very expensive alternative [1].

The prohibitively high computational cost of an ILP ($\mathcal{NP}$ (Non-Deterministic Polynomial)-complete) [3] discourages its use in large networks, as well as in networks with bursty traffic patterns. This non-trivial problem drives, in some sense, the research in distributed RWA schemes.

These distributed RWA schemes are, usually, based on message passing, allowing the nodes to establish the needed wavelengths by themselves, thus rendering the existence of a central node unnecessary. There are several proposed distributed RWA schemes: DIR (Destination Initiated Reservation) [4], SIR (Source Initiated Reservation) [4], among other's. The following sections present a very brief description of these schemes.

In the DIR method, the source node sends a *reservation request* message that will travel to the destination node; this message gathers information on the availability of wavelengths along the way. Once this message arrives at the destination node, an available wavelength will be chosen, and a *reservation message* will be sent. This reservation message traverses the reverse path of the *reservation request*, reserving the selected wavelength [4]. One inherent problem of DIR is that, due to the fact that information gathering and wavelength reservation are decoupled, outdated information could result in trying to reserve a wavelength that is no longer available, thus resulting in a blocked connection request.

The differences between DIR and SIR are subtle but important. The SIR method follows a somewhat more aggressive approach. There is no *information gathering* stage *per se*. A *reservation message* is sent to the destination node reserving the available wavelengths on the way. Once it reaches the destination, one of the previously reserved wavelengths is selected. This selection is announced to the source of the connection request in a message, which traverses the reverse path of the *reservation message*, announcing the selection and releasing the unused reserved wavelengths. The number of wavelengths reserved by the *reservation message* varies depending on whether a greedy or moderate approach is used. In the greedy case, every single available wavelength is going to be reserved. In the case a more moderate approach is chosen, a single wavelength is going to be reserved.

The greedy approach improves the chances for the requested connection to be established, while imposing a higher blocking risk for competing connection requests. A more moderate approach, where a single wavelength is reserved, reduces the effect on competing connection requests.

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Game Theory constitutes one of the first attempts at formalizing Economic Science. Presented as an integral work for the first time by Von Neumann and Morgenstern [5], it provides a formal framework for describing the behavior of rational and intelligent individuals. Cooperative Game Theory studies voluntary coalitions and negotiations within the Game Theoretical Framework [6].

Considering Metcalfe's law [7], one can envision a non-distant future, where coalitions among competing telecommunication companies are the norm. Pushing the idea even further, one could envision occasional negotiations, resulting in ad-hoc agreements between different companies to relay each others traffic. This convergence of once-competitors is by no means a simple task. Cooperative Game Theory provides a rich set of tools that could be used to prescribe the behavior in this ideal environment.

This paper presents a novel distributed wavelength assignment scheme based in the Cooperative aspects of Game Theory, particularly the $\mathcal{NBP}$ (Nash Bargaining Problem) [8].

This work is organized as follows: a brief introduction to the elements of Game Theory is presented in Section II, Section III maps the Distributed RWA problem to a cooperative game, Section IV presents an illustrative example and in Section V the experimental results are shown. The conlusions and further reaserch section can be found at the end.

II. Game Theory

Game Theory can be defined as the study of mathematical models of conflict and cooperation among intelligent rational decision-makers [9].

Despite the fact that the $\mathcal{NBP}$ belongs to a cooperative game theory approach, it is built upon a non-cooperative game. Therefore a distinction between both approaches is relevant. Non-cooperative game theory explores situations where players do not take into account the possibility to coordinate with each other, thus making communication between players of no benefit at all. On the other hand, cooperative Game Theory analyzes situations where players could benefit from communicating and establishing coalitions among themselves [9].

We now briefly introduce essential concepts required by the model, as presented by Myerson in [9]. A game is defined as a 3-tuple

$\Gamma = \left\langle N, (C_i)_{i\in N}, (f_i)_{i\in N} \right\rangle$ where:

- N is the set of players,
- C_i represents the set of strategies with $i \in N$,
- f_i denotes the utility function with $i \in N$.

In a game (denoted by Γ), each player $i \in N$ has a utility function (denoted by f_i) that represents their own preferences, and a set of strategies (denoted by C_i) from which to choose.

A general behavioral archetype is assumed by all models presented by Von Neumann and Morgestern: every player in a game is going to act in a way as to maximize his utility function [5]. In this context, a strategy is a complete plan of action considering every possible situation that might arise during the course of a game [5].

A. Nash Bargaining Problem

In his work, Nash presents the bargaining solution for a situation in which all players are: *i.* rational, *ii.* intelligent, *iii.* free to choose among the various possible agreements, *iv.* are not going to repudiate any choice made, and, *v.* are perfectly informed, *i.e.* every player knows everything about the game in question [8].

Nash defined the bargaining procedure for a two-player interaction explicitly. He described the negotiation as a two step game: the $\mathcal{TG}$ (Threat Game) and the $\mathcal{DG}$ (Demand Game). The first is a non-cooperative game, while the second depends on the first games and is played cooperatively as described next.

The Threat Game ($\mathcal{TG}$):

Each player values all jointly achievable plans of actions, while expecting a non-cooperative behavior of each other. From a players perspective, a threat is a strategy he is forced to choose in case the negotiation is not favorable [8].

Among various possible solution concepts for a non-cooperative game the $\mathcal{NE}$ (Nash Equilibrium) is perhaps the most widely used [6]. Nash's Equilibrium captures the stable state of a situation, considering the actions that the players take when they act rationally [10].

Formally, according to Osborne and Rubinstein [6]:

$$f_i\left(c_i^*, c_{-i}^*\right) \geq f_i\left(c_i, c_{-i}^*\right) \;\; \forall c_i \in C_i \tag{1}$$

where: c_i^* denotes the equilibrium strategy for player i, c_{-i}^* denotes the equilibrium strategies for all other players in the game and c_i denotes a unilateral deviation by player i. These actions are the best, in the sense that there is no possible unilateral deviation by any of the players involved [9].

In a two player context, the resulting equilibrium strategies $n_1 = f_1\left(c_1^*, c_2^*\right)$ and $n_2 = f_2\left(c_1^*, c_2^*\right)$ obtained with equation (1) determine the threats for player 1 and 2 respectively (threat point (n_1, n_2)).

The Demand Game ($\mathcal{DG}$): Given the threat point (n_1, n_2), it is possible to form the set of utilities for the jointly achievable set of strategies for the players in case they cooperate (set B) [8]. Now among all cases where both players could benefit mutually (reflected by set B), each player demands a strategy denoted by d_i $i \in \{1, 2\}$ with utility denoted by $b_1 = f_1(d_1, d_2) \in B$ for player 1, with the corresponding definition for player 2.

The rationality assumption forces each player to make a demand resulting in the highest possible payoff. Formally,

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both players choose their demands according to [8]:

$$argmax_{(n_1;n_2)\leq(b_1;b_2)}\left(b_1-n_1\right)\left(b_2-n_2\right) \qquad (2)$$

The solution obtained by solving (1), and then (2), is known as the $\mathcal{NBS}$ (Nash Bargaining Solution). It has the following interesting properties: *i.* it is unique, *ii.* it is Pareto efficient, *iii.* it is based on Von Neumann and Morgenstern utilities, *iv.* it is symmetric, in the sense that it does not matter which of the players is known as 1 or 2, and, *v.* is independent of irrelevant alternatives, that is, it is not affected by alternatives that would not have been chosen [9], [8]. For a more detailed exposition, the reader may refer to [11], [8].

III. RWA AS A COOPERATIVE GAME

The $\mathcal{NBP}$ allows different network operators to cooperate with each other without neglecting their own interest. In order to produce a "game" the description below is going to establish analogies between the various elements of Game Theory and those pertaining the RWA problem. After these analogies are described, the $\mathcal{NBS}$ is going to be determined.

We start the construction of the model by defining the following elements: *i.* The set N of players, *ii.* the set C of strategies and, *iii.* the Von Neumann and Morgenstern utility function f.

In this paper, the set of players is mapped to the set of optical nodes in the network, considering that each node may be operated by a different company interested in its own benefit. Of course, when cooperation is good for a company it will be willing to cooperate. This is clearly a case suited for the $\mathcal{NBP}$.

Definition 1 – *The set N of players*:
Let V be the set of vertices and E the set of links in an optical network represented by $G=(V,E)$. An enumeration of the set V is produced: every $v\in V$ is assigned an index i such that $1\leq i\leq|V|$, where i represents a player in the game. ∎

The bargaining process proposed by Nash in [8] is based in a barter scheme *i.e.*, in an exchange of goods between the players. For the purposes of this work, one can derive the set of objects by closely observing the way an optical WDM network operates. In the proposed model, the players are two adjacent nodes in the network which have cross-flow traffic to be serviced.

Definition 2 – *The set C of strategies.*:
Let $v_k, v_{k'}\in V$ represent two nodes of an optical network $G=(V,E)$. Let $e=\langle v_k, v_{k'}\rangle$ be a link between adjacent nodes, and let $\lambda\in\Lambda_e$ be a free wavelength on link e. ∎

Given a connection request t_o that involves v_k and $v_{k'}$, the available lightwaves $\lambda\in\Lambda_e$ shared by link e constitute the strategies for player v_k and $v_{k'}$. This establishes a correspondence between λ and a barter object. Each player will accept as his own the objects received, and (possibly) trade with them in a later instance. Thus, each player must decide if he is interested in *taking* an object in exchange. The chosen utility function must represent, in a reasonable manner, the interests of the players in the game. Since the the problem at hand requires the minimization of the used wavelengths (*min*-RWA), the behavior of the players should reflect this objective. In order to achieve this behavior from the players, a non-negative cost is assigned to every available wavelength in the network. This non-negative cost, motivates each user to use the set of wavelengths representing the lowest possible cost, thus maximizing his utility.

Definition 3 – *Cost function*:
Given a set Λ_{e_j} of wavelengths for a link $e_j\in E$ in the network represented by G, the cost function is defined as:

$$\eta_{e_j}:\Lambda_{e_j}\mapsto\mathbb{N} \qquad (3)$$

This function defines a mapping between every wavelength and its position in the radioelectric spectrum for the L window (1565 nm to 1625 nm) defined in ITU-T G.696.1. ∎

The link load, represents the common cost incurred by the players in sharing a particular link. This is similar to the concept of tolls in public roads: the cost is shared between the drivers. The link load is defined as the number of paths that, given a set of pending connection requests, share a particular link. Formally:

Definition 4 – *Link load*:
given a set of paths P in the network represented by G, the load ξ_{e_j} of a link $e_j\in E$ is defined as the number of paths $p_m\in P$ using link e_j

$$\xi_{e_j}=|\{p_m\in P\mid e_j\in p_m\}| \qquad (4)$$

∎

The concept of neighborhood, which is defined as the set of links in a path joining the origin and destination of the connection request, represents the concept of *bounded rationality* [9].

Definition 5 – *Neighborhood*:
Let $p_m\in P$ be a path joining nodes $v_k, v_{k'}\in V$. The neighborhood for node v_k is defined as a subset $\omega_{p_m}\subseteq p_m$. ∎

The utility presented below is adapted for our work from the one presented by Bilò, Moscardelli and Flammini in [12]:

Definition 6 – *Utility function*:
The utility function is defined as $f_{t_0}:T_{v_k}\mapsto\mathbb{N}$, where:

$$f_{t_o}=\sum_{e_j\in\omega_{p_m}}\frac{\eta_{e_j}\left(\lambda\right)}{\xi_{e_j}} \qquad (5)$$

where: t_o represents a pending connection request, e_j represents a link in the network G, ω_{p_m} represents the neighborhood, $\eta_{e_j}\left(\lambda\right)$ represents the cost function for wavelength λ and ξ_{e_j} denotes the load of link e_j ∎

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The correspondence between the $\mathscr{NBP}$ and an optical network is now complete. However, merely defining the game between a pair of nodes is not enough. In order to complete the proposed model, it is necessary to define which of all possible nodes in G will have the opportunity to barter. For the purposes of this work, the matching of two players that conforms a game is based on Dijkstra's shortest path algorithm [13].

Definition 7 – *Matching technology*:
Let the nodes $v_1, v_2, v_3, \ldots, v_q \in V$ be in graph G, and given a connection request $t_0 = \langle v_1, v_q \rangle$ with v_1 and v_q as origin and destination nodes respectively. The shortest path $p_m = < v_1, v_2, v_3, \ldots, v_q > \in P$ joining the nodes v_1 and v_q produces a "game" confronting v_1 and v_2. After this game is played, v_2 is paired with v_3, and so on. Thus, given a connection request, a sequence of barter games is created. ∎

Once a player is matched, the bargaining process follows the one defined by Nash in [8]. Once the solution for the $\mathscr{TG}$ is calculated, the $\mathscr{NBS}$ gives the solution for the $\mathscr{DG}$, and based upon it, the bartered wavelength are assigned.

In summary, when a node belonging to a company (a player of the game) has traffic to be serviced, it will look for a path (using Dijkstra's shortest path algorithm). Once the player found a path (and therefore, its neighbor), he will negotiate the needed wavelengths.

IV. Illustrative example

The following example was extracted from a particular simulation run, which was chosen with the specific intent to reveal some details of the proposed model. In particular, the initial instance corresponds to a simulation of the network represented in Figure 1, with 120 Erlangs of uniformly-distributed traffic, under the assumption that the traffic does not end during the entire simulation run. In order to simplify the following example, the capacity of the fiber links was increased until 0% blocking probability was reached.

According to the proposed matching technology, two adjacent nodes in the network can interact if at least one of them has cross-flow traffic to be serviced, as illustrated in Figure 2.

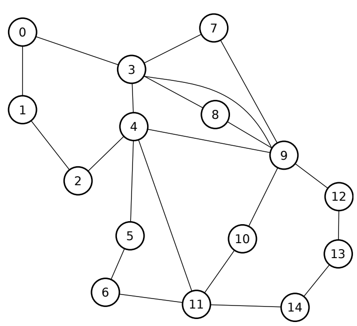

Figure 1. Pacific Internet.

Since nodes 5 and 6 share a link (Figure 1), one needs to analyze the set of pending lightpaths requests for both nodes, represented by multisets T_5 and T_6 respectively, and the set of paths in the network. Multisets are required, since multiple connection requests to the same destination may coexist in the same node; however, the establishment of one such connections, is of no relevance to the others.

$$T_5 = \{6, 9, 13, 4, 9\} \tag{6}$$

$$T_6 = \{3, 0, 3, 1, 0, 3, 4, 4, 8, 13, 8\} \tag{7}$$

Each request is represented by the intended destination node of the pending connection request (see Figure 2).

The set of paths (calculated using Dijkstra's algorithm as shown in [13]) for each node (P_5 and P_6 for nodes 5 and 6, respectively) is a set of ordered n-tuples representing the nodes a lightpath will have to traverse on its way to its destination. The paths needed by nodes 5 and 6 to satisfy their respective outstanding connection requests (T_5, T_6) are shown in (8) and (9). The only paths required are those that connect the source node with the destination of every pending lightpath request.

$$P_5 = \{\langle 5,6\rangle, \langle 5,4,9\rangle, \langle 5,6,11,14,13\rangle, \langle 5,4\rangle\} \tag{8}$$

$$\begin{aligned} P_6 = \ & \{\langle 6,5,4,3\rangle, \langle 6,5,4,3,0\rangle, \langle 6,5,4,2,1\rangle, \\ & \langle 6,5,4\rangle, \langle 6,5,4,3,8\rangle, \langle 6,11,14,13\rangle\} \end{aligned} \tag{9}$$

The set of barter objects in the game is defined considering sets T_5, T_6, P_5, P_6. In this case, for example, node 6 has three pending connection requests to node 3 (shown in T_6), each requiring one individual lightwave. According to the paths in set P_6, these pending requests need to go through node 5. Thus, node 6 requires three lightwaves passing through node 5 in order to fulfill all three requests to node 3. Node 5 takes advantage of this need, and uses the required lightwaves as object of barter. The full set of barter objects for node 5 and 6 are shown in (10) and (11).

$$Objects_5 = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8\} \tag{10}$$

$$Objects_6 = \{\lambda_1, \lambda_2\} \tag{11}$$

The strategies are based in these barter objects, as defined in the previous section.

The size of the neighborhood was defined as 1 for this example, which means that for set P_5, the neighborhood includes only the first link of every path used by node 5. For example, for path $p_1 = \langle 5,6\rangle \in P_5$, $\omega_{p_1} = \{\langle 5,6\rangle\}$, for path $p_2 = \langle 5,4,9\rangle \in P_5$, $\omega_{p_2} = \{\langle 5,4\rangle\}$. The other neighborhoods, as well as those for node 6, are obtained following a similar reasoning.

Since the only links that are going to be included when calculating the utility function are those in the neighborhood of each path the connection traverses, only the load of the links in the neighborhood are going to be considered. In this

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particular example (according to set T_5), link $\langle 5,6 \rangle$ has to carry the traffic destined to nodes 6 and 13 (see Figure 2). This would result in a link load of 2 (*i.e.* $\xi_{\langle 5,6 \rangle} = 2$).

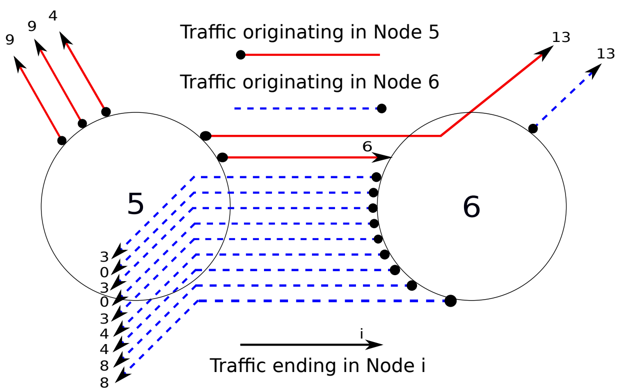

Figure 2. Traffic example.

There is one item left in the model, in order to have the full data needed to compute the utility function: costs have to be assigned to every object of barter for every player. As an example, the costs for both lightwaves needed by node 5 (*i.e.* node's 6 objects of barter) are: $\eta_{\langle 5,6 \rangle}(\lambda_1) = 1, \eta_{\langle 5,6 \rangle}(\lambda_2) = 2$, according to the definition of η previously introduced in (3).

The utility function for a connection request of node 5, $t_1 = 6 \in T_5$, is:

$$f_{t_1} = \sum_{\langle 5,6 \rangle} \frac{\eta_{\langle 5,6 \rangle}}{\xi_{\langle 5,6 \rangle}} = \frac{\eta_{\langle 5,6 \rangle}}{\xi_{\langle 5,6 \rangle}} = \frac{1}{2}$$

The corresponding utilities for the pending connection requests for node 6 are calculated in a similar way. It is important to notice, that a player may choose to barter one or more items simultaneously; therefore, if player 5 exchanges the lightwaves needed for two connection requests (*i.e.* to nodes 6 and 13) in one encounter, the resulting utility is the sum of the individual utilities.

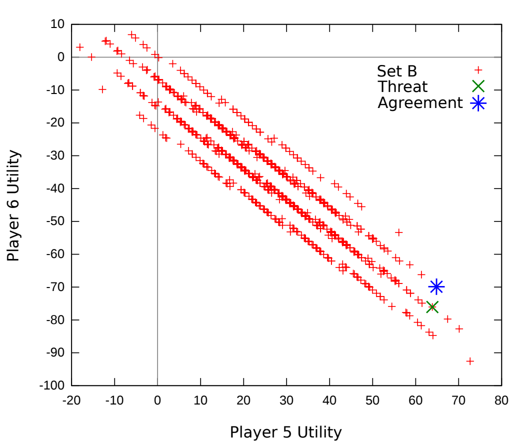

Figure 3. Agreement.

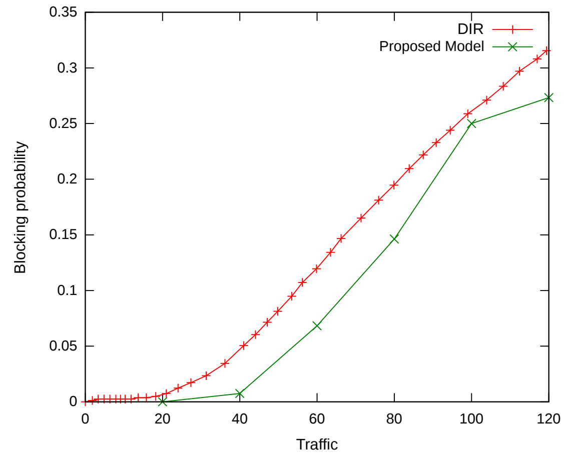

Figure 4. Comparison between DIR and the proposed model - PACnet.

Figure 3 shows the feasible set B, obtained by calculating all pairs of utilities for every pending connection request of both players in the Game. The green and blue points represent the threat point and the achieved agreement, respectively.

V. Experimental results

Given that we could not find in the literature, any work presenting a scenario where networks from different operators coexist, the comparisons were made using a single network. This does not introduce any bias in the possible extension of the presented model. The following simulation results where obtained by simulating lightpath establishment using the PACnet (Pacific Internet) network. This network is shown in Figure 1 and was extracted from [14].

The following simulation results where obtained by simulating lightpath establishment using the PACnet network. Connection request pairs (origin and destination nodes) where chosen using a uniform probability distribution.

The problem solved corresponds to the well-known static-RWA problem, where all requests are known in advance and they are assumed to exist for the whole duration of a particular simulation. Traffic sets range from 0 to 120 Erlangs in 20 Erlangs step increment. For every step, 10 uniformly distributed traffic sets where generated, *i.e* 10 sets of 20 Erlangs, 10 sets of 40, and so on. Figure 4 shows the average of the blocking probability obtained by simulating the network with each of the 10 sets of connection requests for every traffic increment in the network.

Comparison with DIR Figure 4 presents a comparison between the proposed method and state of the art algorithm DIR, as presented by Lu, Xiao and Chlamtac in [4] and According to the results presented in [15], DIR outperforms SIR, making it necessary only to compare the performance of the proposed model with DIR.

The parameters used for the comparison presented in Figure 4 are the same as those used by Lu, Xiao and Chlamtac in [4], *i.e.*: *i.* each link is composed of two opposed

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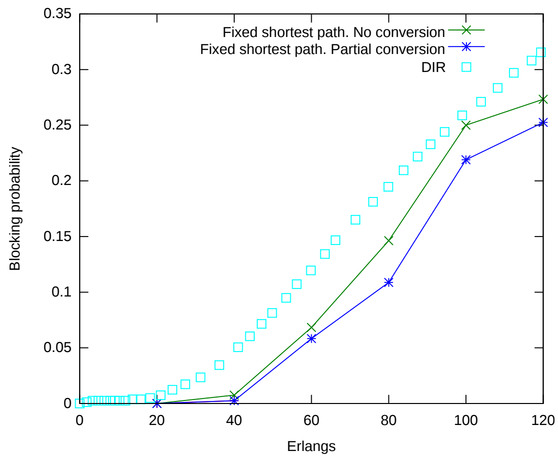

Figure 5. Comparison between DIR and the proposed model. Wavelength conversion - PACnet.

unidirectional fibers, with 8 lightwaves per fiber, *ii.* static traffic, and, *iii.* fixed shortest path routing.

As shown in Figure 4, the proposed model presents a clear improvement, in terms of blocking probability, over SIR and DIR for non-bursty traffic. As an informal model validation, one can observe in Figure 5, the improvement when using wavelength conversion, under the same simulation instance as that of Figure 4.

VI. Conclusion

A novel model, which is based upon the principles of Cooperative Game Theory, has been presented for the first time to our knowledge.

By comparing simulation results with the performance obtained from DIR, a non-trivial improvement was found. However, the most important contribution obtained from this work does not necessarily lie on quantitative blocking probability improvement, but rather on the model itself. The distributed RWA problem, where nodes from competing companies can benefit from cooperation, can clearly be modeled as a cooperative game, particulary a $\mathcal{NBP}$.

Most state of the art Distributed Wavelength Assignment algorithms account for some sort of *good faith* from other nodes in the network for assigning lightpaths. It is the case of both DIR and DRCLS (Distributed Relative Capacity Loss), proposed by Zang, Jue and Mukherjee in [16]. In a strictly interconnected scenario, this assumption would delay, and in some cases even impede, the detection of ill-intentioned nodes in the network.

One has to keep in mind that this work is a first attempt at introducing Game Theory concepts, not only to solve the current problem of Wavelength Assignment, but also to account for the inevitable evolution of the deployed networks. A plethora of work lies ahead to obtain a fully tested procedure, to cite a few:

- Exploring utility functions using genetic algorithms.
- Extending simulations with nodes using different utility functions.
- Exploring non-symmetric solutions.
- Considering topologies formed by interconnected long haul providers.
- Analyzing the dynamics and complexity of the proposed method and comparing it to state of the art distributed wavelength assignment methods.

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