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Adaptive Portfolio Asset Allocation Optimization with Deep Learning

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***Abstract*—Portfolio management is a well-known multi-factor optimization problem facing investment advisors. The system described in this work can assist in automating portfolio management, and improving risk-adjusted returns. The asset allocation action recommendations were personalized to the portfolio under consideration, and were examined empirically in this work in comparison to standard portfolio management techniques. This work presents a Long Short-Term Memory approach to adaptive asset allocation, building upon prior work on training neural networks to model causality. The neural network model discussed in this work ingests historical price data and ingests macroeconomic data and market indicators using Principal Components Analysis. The model then estimates the expected return, volatility, and correlation for the selected assets. These neural network outputs were then turned into action recommendations using a Mean-Variance Optimization framework augmented to use a forward-looking rolling window technique. Testing was performed on a dataset with a 7.66 year duration. The observed mean annualized return for classical passive portfolio management approaches were 4.67%, 3.49%, and 4.57%, with mean Sharpe ratios of 0.46, 0.20, and 0.54. 10 simulations using the new Long Short-Term Memory model from this work provided a mean annualized return of 10.07%, with a Sharpe ratio of 0.98. This work provides the conclusion that a Long Short-Term Memory model can generate better risk-adjusted returns than conventional strategic passive portfolio management.**

Keywords–Recommender systems; Deep learning; Portfolio management.

I. Introduction

Whereas our prior work investigated learning causality from observing computer user actions [1], this extension of that work continues that line of research, using Long Short-Term Memory (LSTM) neural networks to model the cause and effect inherent in portfolio management decisions. The objective of this work is to develop an artificial intelligence (AI) approach for adaptive investment portfiolio management by examining the assets for a long time period and by looking and diverse asset classes on a global scale. This approach will be contrasted against conventional strategic passive investment portfolio management strategies. Adaptive asset allocation is a timely area of research, as deep learning innovations are being productized to create financial products such as robo-advisors [2], ETFs [3], and hedge funds [4]. The decision engine from [1] was replaced by the Markowitz's Mean-Variance Optimization (MVO) framework, and the learning algorithm was replaced by a LSTM model. Prior work on LSTM portfolio management [5] was extended in this work.

In recent years, there has been an increasing focus on investments into passively managed funds. These aim to replicate the market's performance rather than beating it [6]. In 2011, Burton Malkiel made the case for passive investing by empirically proving that investing in the S&P 500 during the period of 1969 to 2010 would have generated 80% more returns than the average actively managed fund [7]. This has encouraged investment managers to enjoy the benefits of diversification using cross-asset correlations within a certain risk profile [8]. The dilemma of managing a portfolio has thus become deciding what proportion of the investment should be allocated to each asset available. Modern portfolio theory (MPT) suggests the use of the MVO framework to eliminate risks that are not systematic, and determine the optimal distribution of the investment between the different assets. It is important to note here that the optimality of the asset allocation does not imply that the estimate of asset-related features is optimal. Rather, on the assumption that the asset-related parameters were error-free, the MVO asset allocation is guaranteed not to add any additional error.

Markowitz's MVO framework aims to achieve portfolio diversification while minimizing specific risks and determining

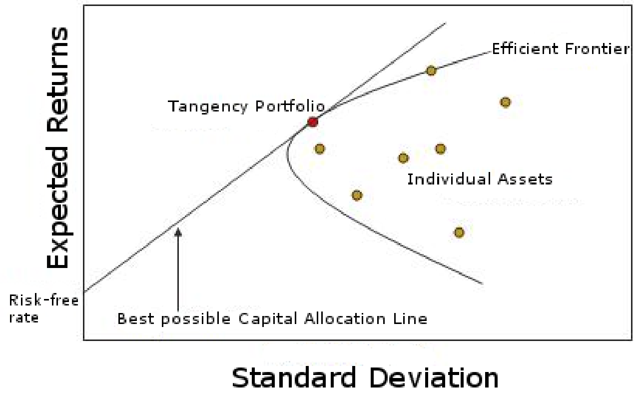

Figure 1. Markowitz Frontier (Modified from Wikimedia Commons) [12]

the risk-return tradeoffs for each asset [9]. The first step for MVO is to develop estimates of the expected returns and volatilities for each asset. These estimates can be achieved using classical indexing models such as the Capital Asset Pricing Model (CAPM) [10] or the Three-Factor Model (3FM) [11], or by using computational methods. After projecting the expected returns and volatilities for each asset, the algorithm selects the allocation of assets that has the highest expected return for a defined level of risk, or the lowest level of risk for a given level of expected return. As shown in Figure 1, the optimal portfolio lies on the hyperbolic curve called the efficient frontier.

This work examines the asset return and volatility estimates generated by LSTM Recurrent Neural Networks (RNNs). The model incorporates economic and market input features to adapt to changes in the financial markets, by regularly adjusting the asset allocation. This work used a globally diversified multi-asset portfolio consisting of 11 asset classes in a variety of different markets. Most of the previous studies have used short testing periods, while this work examined a broader testing horizon (January 2010 to August 2017).

The challenge of this work was to:

(TASK 1) Predict the return of any asset under consideration, by training an LSTM to model the asset performance.
(TASK 2) Optimize the allocation of assets within a model portfolio using the MVO framework.

The following sections review the prior art (Section II), present the causality-learning LSTM model (Section III and asset allocation model (Section IV), followed by a description of the dataset (Section V), a description of the empirical experimentation (Section VI) and presentation of the results (Section VII), closing with conclusions and future work (Section VIII).

II. Related Work

Random chance and many hidden variables can influence asset returns. It is therefore challenging to estimate the future returns for any asset, which is required in order to perform MVO asset allocation. Asset returns can be affected by economic conditions, commodities prices and political events, along with many other factors [13]. While conventional models tend to assume that relationships are linear when forecasting future returns, many of the real-world cases in the financial markets are non-linear [14]. Artificial Neural Networks (ANN) are excellent approximators of non-linear functions, and so the use of these ANN models in computational finance research has persisted. It has been known for decades that multilayer feedforward neural networks are capable of approximating any measurable function to any desired degree of accuracy [15]. The difference between ANN and other approximation methods is that it uses one or more hidden layers to transform the input variables, using a transfer function to deal with nonlinear statistical functions [16]. ANNs can analyse huge quantities of data to recognize patterns and make sense of incomplete or noisy data, and therefore provide an excellent alternative to linear models for forecasting and estimating financial time-series [17] [16].

An important part of the portfolio management process is the method of performance evaluation. Many techniques and models have been developed to evaluate portfolio performance based upon the portfolio's return-risk characteristics. The Sharpe ratio is measured by dividing the difference between the portfolio's expected return and the risk-free interest rate by the portfolio's standard deviation. The ratio computes the excess returns per unit of total risk [18]. The Treynor ratio is measured by dividing the portfolio's excess returns by the portfolio's beta (systematic risk) [19]. Jensen's alpha (α_i) is the intercept of Jensen's excess return of the single-index model, and it measures the abnormal return over holding the investment portfolio of an index fund [20]. As the various methods of weighting the portfolios have different standard deviations, this work uses the Sharpe ratio to enable comparison between results.

A number of studies have been conducted to evaluate neural networks' ability to predict financial time-series. The work available in the literature tends to focus on a specific stock market over a relatively short testing period. Freitas et al (2001) focused on the Brazilian Stock Exchange (BSE) over a 21 week testing period using an autoregressive neural network [21]. Jang & Lai (1994) focused on the Taiwan Stock Exchange (TSE) over two years of testing with a DAS network [22]. Ko & Lin (2008) had the same focus, looking at the TSE over a two-year period with a multi-layer resource allocation neural network [23]. Raei (2006) conducted a comparitive empirical study to examine the ability of a four layer Perceptron network to beat a classical portfolio that comprised of stocks from the Tehran Stock Exchange over a period of 13 months [24]. Overall, the literature lacks a strong body of comparative empirical analysis to examine the ability of machine learning (specifically LSTM neural networks) to achieve better estimates of the returns than the classical indexing models within multi-asset investment portfolios. As well, very few of them involved the use of a wide range of assets that covered different classes on a global scale. The vast majority of published work focuses on a specific market (such as a US market or Australian market) or a particular asset class (such as stock market or fixed income market).

Portfolios with exposure to global equity markets generate better risk-adjusted returns than portfolios dominated with domestic equities [25]. Empirical studies have also shown that globally diversified portfolios containing both equities and bonds outperform portfolios of equities alone [26]. Therefore, bonds should play a vital role in the portfolio.

One of the commonly-used classes of ANNs is the Recurrent Neural Network (RNN). While feed-forward networks are designed to have no feedback loops [27], RNNs contain at least one directed cycle to create an internal memory. Long Short-Term Memory (LSTM) networks are a form of RNN designed to deal with modeling temporal sequences [28]. Because of this, LSTMs can be readily applied to financial time series. Lee & Yoo (2017) looked into using LSTMs to predict potential returns of a variety of investments. They decided against using MVO to find the optimal set of assets' weights because MVO makes the simplifying and often incorrect assumption that asset returns are normally distributed [29]. Their dataset consisted of 10 top stocks in terms of market value from the S&P500 from 2004 to 2016. This work aims to expand on the use of LSTMs for asset allocation by bringing in additional market and economic data, to improve the quality of the predictions. It will also focus more on the parameters of the LSTM model itself.

III. Overall AI Model

Figure 3 presents the building blocks of the adaptive asset allocation system discussed in this work. The output from the system is a set of asset weights within an investment portfolio. The iterative process of asset allocation begins with preparing the input dataset, including historical returns of the 11 assets (ETFs) in the portfolio, and additional economic and market data that may influence future returns for each asset.

Once the input data has been prepared, Principal Components Analysis (PCA) is used to reduce the dimensionality of the additional economic and market data down from 387 features. After dimensionality reduction, only key economic and market features are passed along. This information compression helps to reduce the number of dimensions without much loss of information, going from a sparse to a dense data representation [30].

The reduced market and economic data are then passed to an LSTM RNN, along with the historical prices for the assets. Using this data, the RNN produces a prediction of the assets' future returns.

Finally, these predictions are used by the MVO model to generate the optimal weighting of the assets.

Figure 2 shows the overall algorithm of the model. Constraints a and b are chosen depending on the level of risk desired for MVO.

Input: Economic data $h0$; Market data $h1$; Assets' historical data h_2; Minimum weight constraint $a := 0.05$; Maximum weight constraint $b := 0.35$; TIP minimum weight constraint $a_{TIP} := 0.0$
Output: Optimal set of assets' weights $optimal\_asset\_allocation$

$e_0 = merge(h_0, h_1)$
$e_1 = scale(e_0)$
$reduced = PCA(e_1)$
$prices = scale(h_2)$
$returns = LSTM\_predict(reduced, prices)$
$optimal\_asset\_allocation = MVO(returns, a, b, a_{TIP})$

Figure 2. Overall LSTM-based asset allocation algorithm.

IV. Mean-Variance Optimizer

The MVO framework maximizes returns for a certain level of risk, or minimizes risk for a given expected return. MVO requires the estimations of expected returns of all included assets, their standard deviations, and the variance-covariance (or correlation) matrix in order to find the optimal asset allocations and calculate a set of efficient portfolios. The MVO method was used in this work to generate three outputs of asset allocations: CAPM-based weights, 3FM-based weights, and LSTM-based weights.

Given n assets, MVO requires expected returns on each asset r_i, standard deviation of returns σ_i, and covariance $\sum$ to be used as inputs to generate an efficient frontier of optimal portfolios. The total portfolio return r_p can be solved with the equation

$$r_p = \omega^t r$$

where ω is an $n \times 1$ column vector of portfolio weights, ω^t is the transpose of the vector ω, and r is an $n \times 1$ column vector of assets' returns r_i. The total portfolio variance σ_p^2 can be solved with

$$\sigma_p^2 = \omega^t \sum \omega$$

The asset allocation optimization problem can then be reduced to the following:

$$\begin{aligned} \underset{\omega}{\text{minimize}} \quad & \omega^t C \omega \\ \text{subject to:} \quad & r_p = \omega^t r, \\ & \sum_{i=1}^{n} \omega_i = 1, \\ & \text{No short selling constraint, thus } \omega_i \geq 0, \\ & a < \omega_i < b \end{aligned}$$

where C is an $n \times n$ covariance matrix, r_p is the total portfolio return, a is the minimum weight constraint, and b is the maximum weight constraint. For this work, the maximum weight constraint was set arbitrarily to 35% for all of the portfolio assets. The minimum weight constraint was set arbitrarily to 5% for all portfolio assets except for TIP, which had a minimum constraint of 0%. Treasury Inflation Protected Securities (TIPs) link principal and coupon payments to the Consumer Price Index (CPI), to help protect investors from inflation [31]. They have no minimum weight constraint because they are inefficient for investors with moderate to high risk tolerance [32].

V. Data Description and Analysis

The dataset used to evaluate and compare approaches consisted of historical, economic, and market data, as well as an economic event calendar.

A. Input Data

The input data can be grouped into three categories: asset historical data, macroeconomic data, and market data. While the classical models used only the historical data, the LSTM model used all three types of input data.

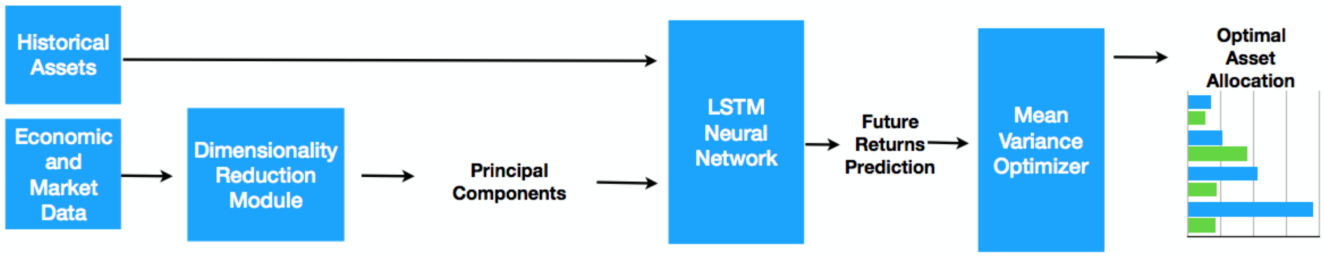

Figure 3. Block diagram for the overall asset allocation AI system [5].

1) Historical Data: Historical monthly adjusted closing prices were used to calculate the monthly expected returns and volatilities of each asset in the portfolio. The researched period was 115 months, from February 2008 to August 2017. The time-series data for each asset was obtained from Thomson Reuters (2017) [33]. Table I presents the asset classes considered for portfolio inclusion, and the functions of each asset. It can be observed that some of the portfolio's asset classes cover different stock markets to achieve global diversification. The chosen asset classes were the same as used by Wealthfront [32].

2) Macroeconomic Data: As can be seen in Table I, the portfolio's asset classes cover a variety of markets, not just the US equity market. Although the main focus is a US-based portfolio, the status of economies such as the Euro zone, the UK, Japan, and BRIC (Brazil, Russia, India and China) economies have a noticeable influence on the US market. Thus, the input dataset included as many economic indicators as possible for the US and other major economies in the developed and emerging markets, by including all of the economic indicators available on the Thomson Reuters Datastream [33].

3) Market Data: It is safe to assume that market variables generally influence asset returns. For this work, the LSTM neural network was exposed to market data consisting of major global equity market indices, commodities, major currencies' exchange rates, and the VIX index. The VIX index, also known as the market's "fear gauge", reflects the implied volatility of the S&P500 index options in the Chicago Board Options Exchange. It is a forward-looking measure of market expectations over the coming month's volatility [34]. This data should help to adjust the assets' weights during extreme or unexpected economic events over the analyzed time frame.

VI. Experiment

Classical passive portfolio management approaches CAPM, 3FM, EQWT (equally weighted portfolio rebalancing) were evaluated for this work in comparison with an LSTM-based approach. For CAPM and 3FM, the only input was the historical data of the various assets. For EQWT, the allocation in each of the 11 assets was set to 9.09%. This results in an equally weighted portfolio containing US stocks, foreign stocks, emerging market stocks, dividend stocks, real estate, natural resources, Treasury Inflation-Protected Securities (TIPS), municipal bonds, corporate bonds, emerging market bonds, and US bonds. CAPM assumes that all investors have all information at the same time, cannot influence prices, and can trade without paying for transactions [10]. 3FM assumes that value beats growth and that smaller companies do better than larger ones. Over the long term, investors must be able to handle extra short-term volatility for better long-term performance [11]. An initial version of the LSTM approach (LSTM in Table II and Figures 4 and 5), and a refined version developed through hyperparameter exploration (LSTM2 in Table II and Figures 4 and 5) are described in this work.

All computations were based on a fixed 24-month rolling window. A minimum of 5% (except TIPS have 0%) and maximum of 35% weight boundaries were used to achieve diversification and mitigate the MVO sensitivity problem. The use of multi-period optimization for CAPM and 3FM, and the rolling window method in MVO, enabled forward-looking estimations, and generated time-varying optimal weights for the portfolio assets.

Each model (CAPM, 3FM, EQWT, LSTM1, LSTM2) performed a rebalancing step on a monthly basis from January 2010 to August 2017, to maintain the target asset allocation scheme as determined by the investor's risk tolerance. The initial portfolio value for each approach was set to 100 USD, and recalculated monthly according to the consequences of the recommendations from the model for each approach. This traditional back testing methodology enables simple and straightforward comparisons of absolute returns. However, other statistical properties such as the Sharpe Ratio were observed to capture the risk adjusted return. Recall that a higher Sharpe Ratio generally indicates a more attractive investment.

The LSTM network from [5] (LSTM1) used the Mean Absolute Error (MAE) loss function. It used PCA to reduce the dimensionality of the market and macroeconomic data to 70 dimensions. It had a layer width of 16 and one hidden layer.

To develop the LSTM2 model reported in this work, the research team experimented with both MAE and Mean Squared Error (MSE) to observe which loss function would generate increased risk-adjusted returns. A variety of other parameters were experimented with as well. The number of dimensions outputted by the PCA function ranged from 10 to 200. The batch sizes used in the model were further explored, with values ranging from 1 to 512. Other values modified were the size of each epoch, the dropout rate, the width of the model, and the number of layers. Finally, early stopping was implemented to see if it would have a significant impact.

TABLE I. 11 assets considered for portfolio inclusion, and a description of their key benefits. The 10 year Sharpe ratio for each asset was obtained from Yahoo Finance for the period covering 2008 to 2018.

Assets Class	Ticker	Investment	Functions	Sharpe Ratio
US Stocks	IVV	iShares S&P500 Index ETF	Investment growth, long-term inflation protection, and tax efficiency	0.68
Developed Market Stocks	EFA	iShares MSCI EAFE Index ETF	Investment growth, long-term inflation protection, and tax efficiency	0.26
Emerging Market Stocks	EEM	iShares MSCI Emerging Market Index ETF	Investment growth, long-term inflation protection, and tax efficiency	0.24
Dividend Growth Stocks	VIG	Vanguard Dividend Appreciation Index Fund ETF	Investment growth, long-term inflation protection, and tax efficiency	0.74
US Government Bonds	SHY	iShares 1-3 year US treasury Bond ETF	Income, low historical volatility, diversification	0.75
Corporate Bonds	LQD	iShares iBoxx $ Inv Grade Corporate Bond ETF	Income, low historical volatility, diversification	0.74
Emerging Market Bonds	EMB	iShares JPMorgan USD Emerging Markets Bond ETF	Income, diversification	0.69
Municipal Bonds	MUB	iShares National Muni Bond ETF	Income, low historical volatility, diversification, tax efficiency	0.71
TIPS	TIP	iShares TIPS Bond ETF	Income, low historical volatility, diversification, inflation protection	0.45
Real Estate	VNQ	Vanguard REIT Index Fund ETF	Income, diversification, inflation protection	0.40
Natural Resources	XLE	Energy Select Sector SPDR Fund	Diversification, inflation protection, tax efficiency	0.22

The result of the described hyperparameter search was a model that used PCA to reduce market and macroeconomic data to 150 dimensions. The LSTM network had 3 hidden layers with a width of 128, and made use of early stopping.

LSTM1 and LSTM2 were both trained iteratively over a 92 month period, predicting the assets' future returns for the next month. These predictions were then passed along to the MVO calculations to reshuffle the asset weights. The training period increased by one month with each prediction. Each model was fitted using the Adam optimizer. This simulation was run ten times for each network.

VII. Performance Evaluation

The performance of classical passive portfolio management approaches (EQWT, CAPM, 3FM) and the active AI-based approaches described in this work (LSTM and LSTM2) were measured in terms of statistical properties (Table II) and percentage returns over time (Figure 4 and Figure 5). As can be seen, the results significantly outperformed both the classical methods and the previous LSTM model. Specifically, the observed mean annualized return for LSTM2 was 10.07%, whereas the other approaches generated less than half: 4.67% (CAPM), 3.49% (3FM), and 4.57% (EQWT), and 4.18% (LSTM). This much higher return was generated for a much improved risk-adjusted return. The mean Sharpe ratios of CAPM (0.46), 3FM (0.20), EQWT (0.54), and LSTM (0.43) were significantly lower that that of LSTM2 (0.97).

The results obtained for the conventional methods were in line with expectations. Whereas the return of individual assets under consideration may have been high (e.g. IVV returned over 100% growth), the risk of holding only that asset is unattractive in terms of the Sharpe ratio. Not knowing the future, any successful strategy should create risk-adjusted returns above the risk-free rate (e.g. fixed income treasury bonds) in order to justify the risk involved in following the strategy. LSTM2 was able to deliver high risk adjusted returns by holding growing assets with low volatility. The model was not explicitly trained to avoid volatility. Rather, it is likely that the training data led the model to converge upon a solution that allocates assets assessed to provide high return with high confidence into the portfolio. It is probable that the LSTM2 model has keyed in on low volatility by maximizing the probability of the expected return for each asset. In other words, the backpropagation "push" on the LSTM2 model to make high confidence asset allocations naturally discounts high volatility assets, adjusting their expected returns downward due to the relatively lower confidence.

Figure 4 shows that the return generated by LSTM2 was higher than the other approaches for all of the computed performance metrics. A more in-depth look is provided in Figure 5, where the growth of the portfolio returns for each simulation is tracked. The assets considered for the portfolio were the same for all models. The implication of a higher Sharpe ratio is that the volatility of the underlying assets held by LSTM2 was low. Another observation is that the returns for LSTM2 covered a broader range of outcomes than the single layer LSTM1. The higher variance in outcomes is tied in part to the fact that each month in the simulation requires the retraining of a new LSTM model, which trains using a new random seed in the random number generator. The large differences in the results of LSTM2 imply that chance does play a role in the future returns predictions. The future returns estimates approximate an unknown result with some error. This error aggregates each month resulting in different holdings for the models with different random seeds.

Figure 6 shows the average percentage returns of LSTM1 and LSTM2 along with the classical models, and the percentage returns of each individual asset in the portfolio. As can be seen, LSTM2 makes the best use of the more profitable assets, while still keeping a much lower Sharpe ratio than investing solely in those stocks. Figure 7 shows how the various Sharpe ratios did over time; each point is the rolling Sharpe ratio measured over the past year. LSTM2 consistently has a higher Sharpe ratio than the other methods.

It can be helpful to see how the different methods fared over the course of various economic events, to compare the performance and risk associated with each strategy. Figure 8 shows the percentage returns of the various methods over the course of the US downgrade in August of 2011 [35]. LSTM2 had the strongest recovery after the event, and as can be seen in Figure 9, and also maintained the highest rolling Sharpe ratio during this period of increasing market risk. Because there is a 5% minimum holdings requirement in the MVO for LSTM2, it is expected that the Sharpe ratio will broadly track market trends. Note in Figure 9, and more generally in Figure 7, that LSTM2 tracked the overall trend in the market, while reducing the risk exposure over other approaches for most of the simulated backtest period. During the time period of the US downgrade event, the LSTM2 model balanced the portfolio to favor US stocks, emerging markets, corporate bonds, real estate, and natural resources (IVV, EEM, LQD, VNQ, XLE). This balance of assets recovered slightly better during the recovery than other approaches. In addition, the LSTM2 model generated more consistent risk-adjusted returns than the other approaches.

LSTM2 also fared well during the European debt crisis of June 2012 [36]. It had a similar return on investment to the other methods (Figure 10), but was the only method to maintain a positive rolling Sharpe ratio throughout (Figure 11). During the time period of the European debt crisis, the LSTM2 model balanced the portfolio to favor stocks (developed market, emerging market, and dividend growth), corporate bonds, and real estate (EFA, EEM, VIG, LQD, VNQ). This balance of assets created a similar return as compared to the other approaches. Post crisis, the model shifted more into real estate, signalling a recovery in housing that was perhaps unrelated to the debt crisis in Europe.

During the Taper Tantrums of May 2013 [37], CAPM actually outperformed both LSTM models, both in percentage return and in rolling Sharpe ratio (Figures 12 and 13). This demonstrates that sometimes the LSTM2 model misjudges market movements and suffers in terms of risk exposure and returns as a result. This is the expected observation, as many of these crises are unforeseeable, and took the broader market by surprise. The Taper tantrums were a good example of market distortions driven by regulator action, rather than market fundamentals. This type of intervention should not be easily predicted by LSTM2, and indeed it underperformed during this period, generating a small positive return with a lower Sharpe ratio than CAPM and 3FM. During the time period of the Taper tantrums, the LSTM2 model balanced the portfolio to favor stocks (US, emerging markets, and dividend growth), TIPs, and real estate (IVV, EEM, VIG, TIP, VNQ). This balance of assets had worse returns than CAPM, though it still outperformed both the 3FM and LSTM1 methods. The LSTM2 model consistently had a lower Sharpe ratio than the classic approaches over those months. During this period, the model seems to have predicted a rise in inflation, and hedged by getting the portfolio into TIPs. This turned out to be wrong, but gives some interesting insight into what the model was expecting.

In Figure 14, LSTM2 can be seen to outperform the other models during the Russian financial crisis of December 2014 [38]. During the time period of the Russian financial crisis, the LSTM2 model balanced the portfolio to favor stocks (US, emerging markets, dividend growth), municipal bonds and real estate (IVV, EEM, VIG, MUB, VNQ). This balance of assets away from EFA (developed markets including European countries with Russian exposure) resulted in a much faster recovery after the event than the classic models. It also generated a significantly higher rolling Sharpe ratio (Figure 15).

Over the course of the China market crash of August 2015 [39], LSTM2 generated a negative return, and had a negative rolling Sharpe ratio, but still outperformed the other models (Figures 16 and 17). During the time period of the China market crash, the LSTM2 model balanced the portfolio to favor US stocks, dividend growth stocks, municipal bonds, real estate and natural resources (IVV, VIG, MUB, VNQ, XLE). The model was invested in MUB (municipal bonds) prior to the crash, and sold out to favor IVV and VNQ (US stocks and real estate) during and after the crisis. This balance of assets resulted in higher returns than other approaches, and fewer losses during the crisis. In addition, the LSTM2 model generated more consistent risk-adjusted returns than the other approaches as measured by the Sharpe ratio. The model seems to have predicted that the bond market yields would shrink due to government intervention, implying a shift in assets away from bonds and into stocks and other assets. This idea is further supported by the continuous drop in consistent returns among all of the observed strategy as shown in Figure 17.

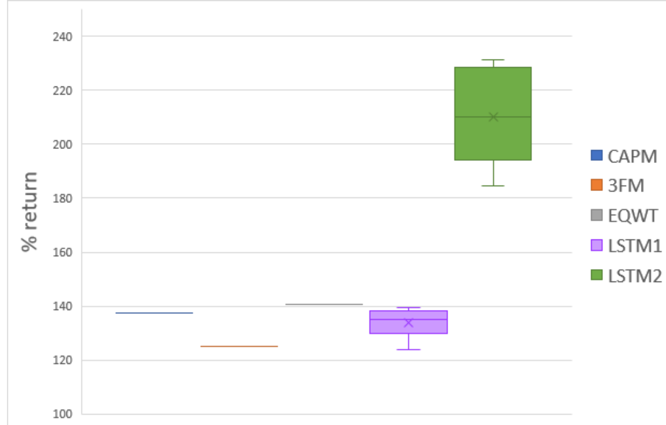

Figure 4. A comparison of the percentage return for each portfolio management method over 10 simulations

Finally, when Brexit happened in June of 2016 [40], the 3FM model had better percentage returns, but the LSTM2 model maintained a higher rolling Sharpe ratio over the same period (Figures 18 and 19). During the time period of Brexit, the LSTM2 model balanced the portfolio to favor US stocks, developed market stocks, emerging market stocks, real estate and natural resources (IVV, EFA, EEM, VNQ, XLE). There is significant UK exposure in EFA, and the model allocated away from EFA in the month following the crisis. In effect, the model missed the risk, as one would expect for such a low probability event. The asset allocation for LSTM2 resulted in lower returns than the 3FM method, but the LSTM2 model generated more consistent risk-adjusted returns than the other approaches as measured by the rolling Sharpe ratio.

Most returns generated by LSTM2 were generated during times of relative stability (non-crisis periods), as one would expect from a predictive model trained on historical data.

VIII. Conclusion

This work presented a neural network based approach to adaptive asset allocation recommendations. It combined LSTM and MVO models with data on historical prices, macroeconomic data, and market indicators. These signals were processed (e.g. scaling, PCA, rolling window) and then applied to address the portfolio management problem. The trained model was able to estimate the returns for a selected group

TABLE II. Statistics for the classical passive portfolio management approaches (EQWT, CAPM, 3FM) and active AI-based approaches (LSTM and LSTM2). The back testing period was from January 2010 to August 2017.

Statistics (Annualized)	CAPM	3FM	EQWT	LSTM	LSTM2
Mean Return	4.67%	3.49%	4.57%	4.18%	**10.07%**
Std Dev.	9.69%	10.64%	8.08%	9.26%	**10.13%**
R-f Average	0.24%	0.24%	0.24%	0.24%	**0.24%**
Sharpe Ratio	0.46	0.20	0.54	0.43	**0.98**

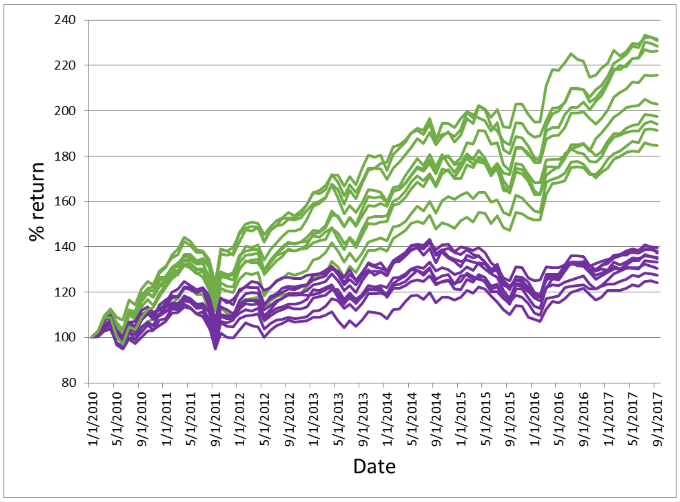

Figure 5. Percentage return comparison between the 10 simulations for LSTM1 (purple) and LSTM2 (green)

of diversified assets, and make investment recommendations with higher returns than traditional portfolio management approaches. The described system can assist in automating portfolio management. The asset allocation action recommendations were personalized to the portfolio under consideration, and were examined empirically in this work in comparison to standard portfolio management techniques. This work provides an empirical conclusion that an LSTM can provide better risk-adjusted returns than conventional strategic passive portfolio management approaches.

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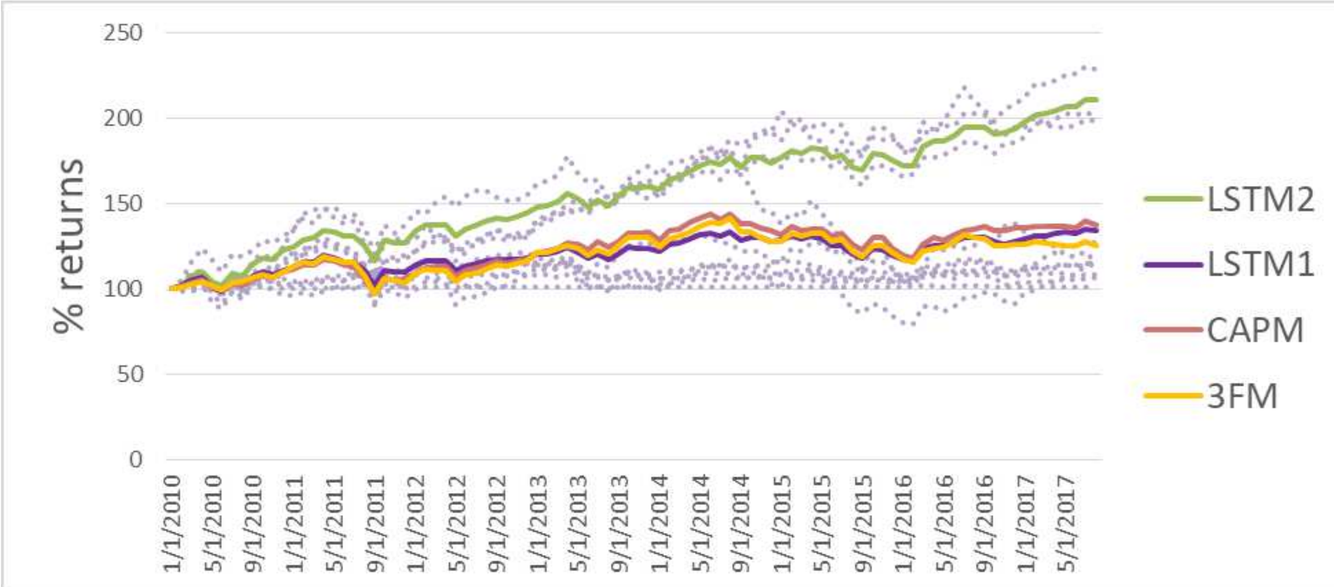

Figure 6. The % return for each underlying asset during the testing period was reported with purple dots. The average % returns for LSTM1 and LSTM2 were overlaid onto the graph. The % returns of CAPM and 3FM were also overlaid onto the graph.

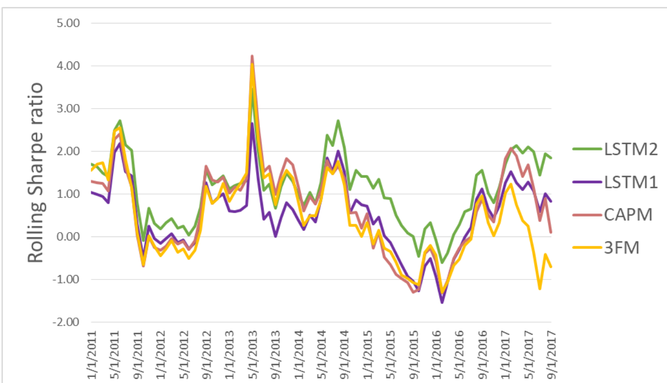

Figure 7. Average rolling Sharpe ratios for LSTM1 and LSTM2 along with rolling Sharpe ratios for CAPM and 3FM

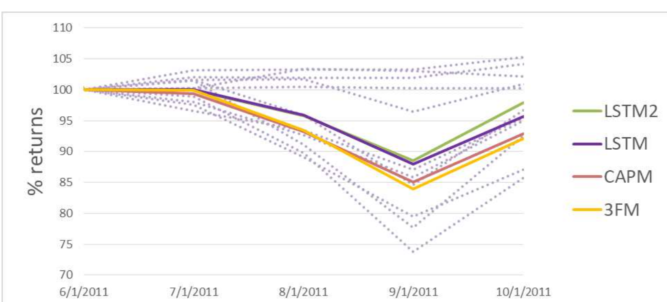

Figure 8. Percentage returns during the timeframe of the US downgrade of August 2011. The % return for each underlying asset during the testing period was reported with purple dots.

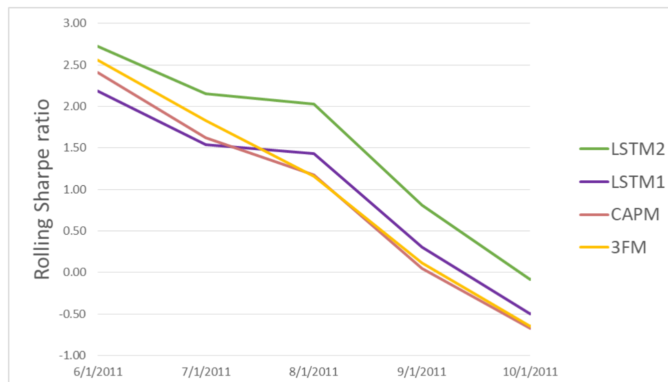

Figure 9. Rolling Sharpe ratios during the timeframe of the US downgrade of August 2011

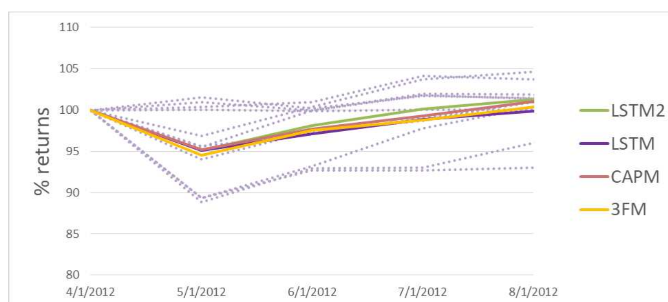

Figure 10. Percentage returns during the timeframe of the European debt crisis of June 2012. The % return for each underlying asset during the testing period was reported with purple dots.

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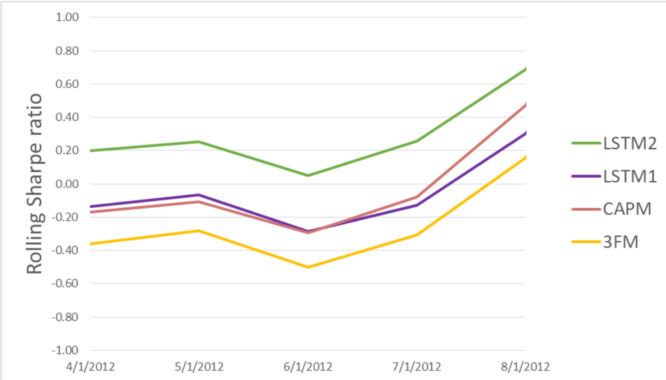

Figure 11. Rolling Sharpe ratios during the timeframe of the European debt crisis of June 2012

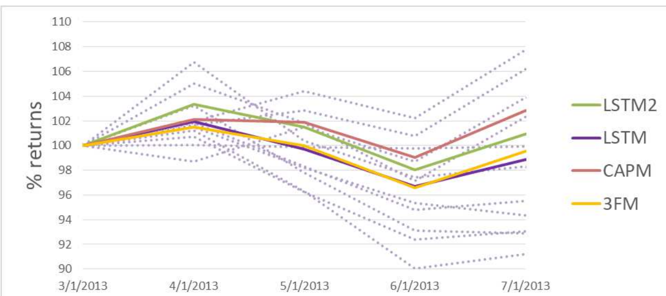

Figure 12. Percentage returns during the timeframe of the Taper tantrums of May 2013. The % return for each underlying asset during the testing period was reported with purple dots.

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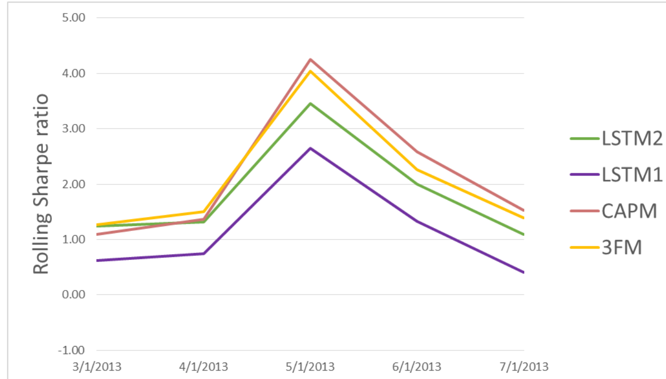

Figure 13. Rolling Sharpe ratios during the timeframe of the Taper tantrums of May 2013

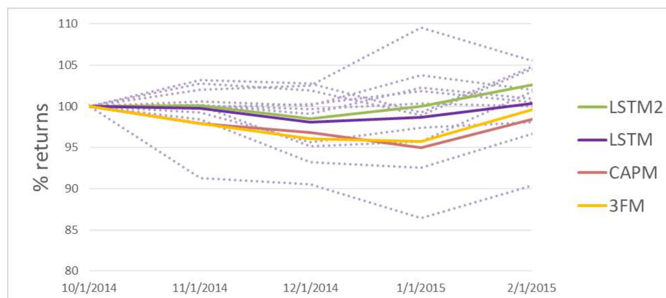

Figure 14. Percentage returns during the timeframe of the Russian debt crisis of December 2014. The % return for each underlying asset during the testing period was reported with purple dots.

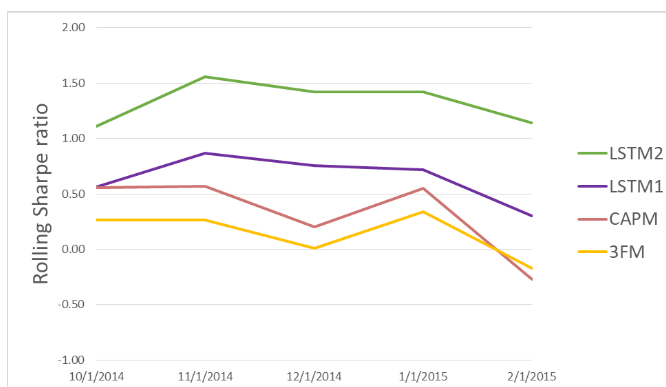

Figure 15. Rolling Sharpe ratios during the timeframe of the Russian debt crisis of December 2014

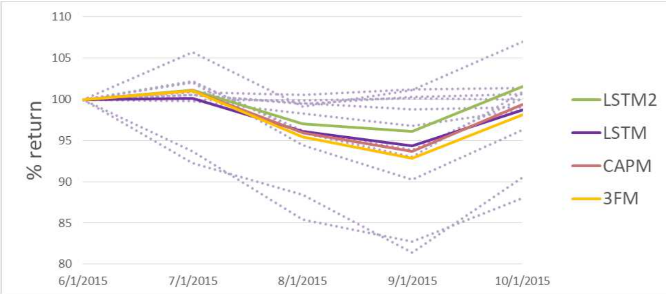

Figure 16. Percentage returns during the timeframe of the China market crash of August 2015. The % return for each underlying asset during the testing period was reported with purple dots.

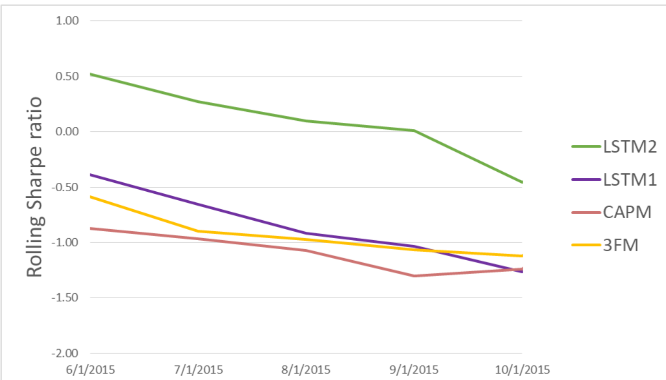

Figure 17. Rolling Sharpe ratios during the timeframe of the China market crash of August 2015

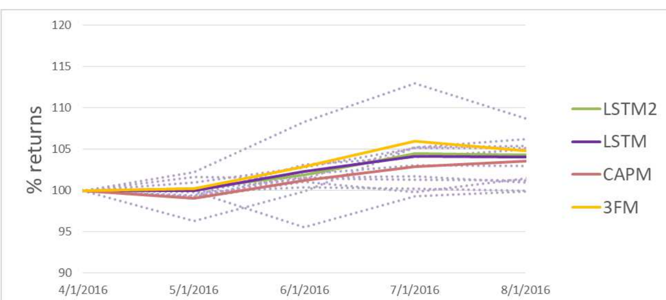

Figure 18. Percentage returns during the timeframe of Brexit, June 2016. The % return for each underlying asset during the testing period was reported with purple dots.

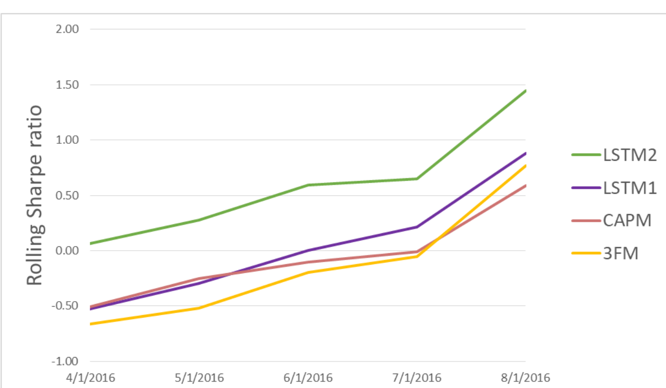

Figure 19. Rolling Sharpe ratios during the timeframe of Brexit, June 2016