AI-centric Proxy Design Synthesis for Non-Obvious Link/Entity and Higher-Order Network Discernment

S. Chan
VTIRL, VT/I-PAC
Orlando, USA
stevec@i-pac.tech

Abstract—**The discernment of relevant sparse and "Very Small/Non-Obvious" (VSNO) clusters within High Dimensional Data (HDD) and the operationalization of Spatio-Temporal Knowledge Graph Completion (STKGC) for High-Order Network (HON) discernment are NP-Hard. The amalgam of a Lower Ambiguity, Higher Uncertainty (LAHU)/Higher Ambiguity, Lower Uncertainty (HALU) Module (LHM), Isomorphic Paradigm (IsoP) Comparator Similarity Measure Module (ICSM2), Multi-Criteria Decision-Making Module (MCDM2), Information Fusion Module (IFM), AI Energy Consumption Module (AECM), and a bespoke Metaheuristic Algorithm Module (MAM) are delineated in this paper to show the potentiality for the concurrent treatment of VSNO, STKGC, and HON, which are essential for Advanced Analytic Technologies (AAT)/Advanced Anomaly Detection (AAD), At-the-Edge Observational Space Analysis (AOSA), and Continuous Situational Awareness (CSA). These are vital aspects for critical applications, such as, among others, network analysis (e.g., C2 systems) and maritime domain awareness. The described LHM-ICSM2-MCDM2-IFM-AECM-MAM amalgam can be operationalized by a bespoke Graph Convolutional Network (GCN)-Bidirectional Long Short-Term Memory (BiLSTM)-Graph-Attention-Network (GAT) mechanism along with a Robust Convex Relaxation (RCR)-based Deep Convolutional [Neural Network] Generative Adversarial Network (DCGAN)-Hypergraph-Induced Infimal Convolutional Manifold Neural Network (H-IICMNN)-1,2,3,4 architectural construct (GCN-BiLSTM-GAT & RCR-DCGAN-[H-IICMNN]-1,2,3,4 or GBGRDH-1,2,3,4) to address the involved NP-Hard problems.**

Keywords-Intelligent Decision-Making Systems; Artificial Intelligence; Machine Learning; Big Data; Advanced Analytics; Command and Control; Large Scale Complex Networks.

I. INTRODUCTION

The architecting of a discernment capability for Very Small/Non-Obvious" (VSNO) clusters, as well as related links (which might be of a dotted line nature) and nodes (wherein a node might also equate to discerning a Higher Order Network or HON containing other nodes and links), is a considerable feat. Various considerations, such as spatial and temporal, are often presumed; however, in many cases, this information must be appropriately fused (i.e., Information Fusion or IF), as the involved data repositories might be devoid of such temporal and/or spatial information. Data repositories with quadruple representation will likely have temporal information and those with quintuple representation will likely have both spatial and temporal information; however, those with triple representation will often not (without being extended). A determination with regards to IF must be made based upon the involved ambiguity/uncertainty, and such Artificial Intelligence (AI)-centric Intelligent Systems (IS) Decision Engineering Discernment Engines (DE2) (AI-IS-DE2 construct), which illuminate these desired Decision Engineering Pathways (DEPs), are non-trivial to design. As a nice recap, Ding reviews various involved categories: Representation (e.g., Higher-Order Networks or HONs), Prediction (e.g., Spatio-Temporal Knowledge Graph Completion or STKGC for missing links/nodes), Simulation (e.g., ambient Control Signals or CS amidst dynamic topological changes), Inference (e.g., Adaptive Criteria Weighting Systems or ACWS for non-biased/more balanced suppositions), and [Command] & Control (C2) (e.g., elastic/resilient C2) [3]. HON, C2/CS, STKGC, and ACWS have been considered in the aggregate within various works-in-progress and prior works [1][2][3]. After all, by better understanding the involved C2 (which may serve as a HON, thereby having the ability to exercise CS), it will: (1) more readily effectuate IF, (2) better leverage Advanced Analytic Technologies (AAT)/Advanced Anomaly Detection (AAD), and (3) more robustly maintain At-the-Edge Observational Space Analysis (AOSA) as well as "Continuous Situational Awareness" (CSA) for the purposes of DE2. Similarly, ACWS can inform STKGC to affirm the CS, HON, and C2.

For the C2 case, various Global Maritime Domain Awareness (MDA) frameworks have been examined, and it was noted that MDA tend to consist of various constituent Regional Maritime Situational Awareness (RMSA) networks. Typical RMSA architectural constructs generally involve "a Decision Support System (DSS) node" in a lead role and various geographically distributed DSS nodes as part of an "enclaved network" [4]. As the situation evolves, the "enclaved network may evolve," such as into a Multi-Partner Enclave (MPE), or devolve [4]. Depending upon the circumstances, the "interim lead DSS" designation may alternate temporally, as certain DSS nodes may be construed to be more apropos and/or "optimally positioned" to guide/shape the "mosaic-at-large" (e.g., MDA, in this case), such as for IF [4]. The IF criterion may potentially involve ingesting more data, but this will be informed by a particular "Lower Ambiguity, Higher Uncertainty (LAHU)/Higher Ambiguity, Lower Uncertainty (HALU) Module (LHM)" and AI Energy Consumption Module (AECM) [4][5][6].

A. LHM

With regards to the LHM, under a "Compressed Decision Cycles (CDC)" paradigm, which equates to a condition wherein time is of the essence, the LHM will actuate upon "sparse data" or "higher uncertainty…given the condition of lower ambiguity" (i.e., LAHU); this roughly equates to the situation, wherein an Isomorphic Paradigm (IsoP) is recognized, if it has occurred previously "within the historical data" [1][6][7]. In contradistinction, given a state of "higher ambiguity" (i.e., HALU), for which IsoP has not occurred before, the LHM might stipulate the need for "more data 'to lower uncertainty'" [1][5][6]. The LHM will also consider the AEC involved, and this information is provided by the AECM.

B. IsoP Comparator Similarity Measure Module (ICSM2)

The LHM is supported by a bespoke ICSM2, which ascertains the prior occurrence of IsoPs by way of facilitating the derivation of the Optimal Shapley-Nondominated Solution (OSNS) and the Optimal Corresponding (OC) "Generalized Linear 'f'-sided" (GLf) [Spherical] Fuzzy Number (FN) (SFN)" or "(OCGLfSFN)-based membership function" (by way of the Precursor [non-OC] GLfSFN or P-GLfSFN) [7]. These "best-fit approximations" lend to the IsoP determination.

1) Nondominated Solution (NS)

Wu notes that Shapley values (SVs) (various researchers have noted that Monte-Carlo, among other can be leveraged to generate the *f*-th feature along with ML model *m*, feature index *f*, number of iterations *i*, etc.) can be leveraged to transform FN-related Fuzzy Optimization and Decision Making (FODM) problems to "Scalar Optimization Problem[s]" (SOPs) that can be efficiently resolved to segue to the Nondominated Solution (NS), wherein "no one objective function can be improved without" a concurrent degradation to "the other objectives" [7][8]. The OSNS can then be ascertained.

2) OCGLfSFN-based membership function

As noted in the introduction for Section IB, regarding "best-fix approximation," Lakshmana has reported on the efficacy of the "approximations of general non-linear FNs" by way of higher-order linearized Generalized 'f'-gonal FN/SFN forms, "'such as *Tr*iangular, *T*ra*p*ezoidal,' as well as *Pe*ntagonal, *He*x*a*gonal, *He*p*t*agonal, *O*ctago*n*al, etc." [7]; these can be re-expressed as "G*Tr*FN, G*Tp*FN, G*Pe*FN, G*Hx*FN, G*Hp*FN, G*On*FN, etc., respectively" [7]. According to Velu and Ramalingam, "best-fit approximations" can be improved "when higher-order piecewise linear" FNs are utilized to approximate "non-linear information" [7][9]. Along this vein, Augustin asserts that, as one example, G*Hp*FN "can represent more intricate and nuanced degrees of uncertainty" since "certain apropos 'f'-gonal FN/SFN forms" are quite good at "preserving ambiguity" [7][10]. Ban, another advocate of this principle, has a predilection for "*Tr*iangular, *T*ra*p*ezoidal, and semi-*T*ra*p*ezoidal for the 'preserv[ing]…and weight[ing]' of ambiguity" [7][11]. The pathways for deriving the OSNS (a Multi-Objective Decision Making or MODM problem) and the selection of the 'f'-gonal FN/SFN form (a FODM and Multi-Criteria Decision-Making or MCDM problem) are informed by the ICSM2.

C. MCDM Module (MCDM2)

The ICSM2 is a constituent of the MCDM2, which is comprised of Multi-Attribute Decision Making (MADM) and MODM components, each of which has Subjective Method (SM) and Objective Method (OM) constituent elements. By well counterpoising SM with OM, selection bias can be better mitigated, and the MADM/MODM SM/OM (MMSO) amalgam facilitates the operationalization of an ACWS (that informs STKGC, etc.)

D. IF Module (IFM)

For Real World Scenarios (RWS), the MCDM problems, handled by the MCDM2, tend to be nested (e.g., marsupial drones, wherein the main MCDM for the "mother" drone is to deliver the "baby" (a.k.a., "joey") drones to the area of operations, and the joey drones then perform their various tasks, which involve distinct and disparate MCDMs), and IF-related "constituent grey" MCDM problems can be construed to be FODM that are complex "because the measures/objectives tend to" be at odds [7]. Accordingly, the facilitation/derivation of the OSNS (a constituent MODM problem) by the MCDM2's ICSM2 is central, and other contributory, value-added approaches include: (1) the "Dempster-Shafer framework" to address IF "and reasoning under uncertainty," (2) "Zadeh's Type-2 Fuzzy Set (FS) (T2FS)" for IF and tackling "the fused probability with possibility-probability information," as well as (3) "Debois and Prade's FNs" for the encapsulating of "complexity/uncertainty" [7][12][13]. Overall, the overarching intent to preserve ambiguity, uncertainty, et al., via "best-fix approximation" is maintained.

E. AEC Module (AECM)

Of note, LHM actions are shaped by the AEC information provided by the AECM. In essence, there are two counterpoising: (1) the LHM's consideration of the ambiguity/uncertainty counterpoising, and (2) the LHM's consideration of the AEC status — current/anticipated AEC. The latter is a non-trivial consideration, and historically, AEC numbers have been "skewed more towards the training side" [14]. Contemporary times have spotlighted a potential inversion, wherein the AEC for inferencing is oftentimes far greater than that for training [14][15]. This makes sense, for while a single inference "requires much less computation than that" involved in model training, "inference happens far more frequently than model training" [16]. Along this vein, Luccioni notes that "in-depth work quantifying" AEC as well as other inference-related costs "is limited" [16], and Luccioni, Desislavov, and others have asserted that "the total energy cost" for the various segments of the Artificial Intelligence (AI)/Machine Learning (ML) "model life cycle … is very rarely available" and that the AEC "per (one) inference is rarely reported" [16][17]. Ranking industrial organizations concur and posit "that inference loads will increase over time" [18]. For example, AI chip/server provider, NVIDIA, "estimates that 80-90% of the cost of neural networks lies in inference processing," and Castro furthers this by asserting that "training a particular AI model incurs a one-time cost, whereas using an AI model continues to consume energy over time" [19][20]. It then follows that most of AEC "will eventually come from inference" [19][20]. The described lack of AEC data for "the key AI stages (e.g., pre-training, fine-tuning, and inferencing)" likely constitutes a key reason for the potential "dearth of analyses on effective compute (e.g., algorithmic efficiency versus hardware efficiency)" [7]. Even when a certain model has been closely scrutinized, the phenomenon of AEC varying, such as with the involved number of parameters (e.g., "a higher number of parameters" segues to a higher AEC), has not been well studied [7][14][20]. Finally, higher requisite accuracies beget higher AECs, and AAT/AAD necessitates tasks that have an even higher AEC. This facet is oftentimes not well considered in contemporary designs.

F. Metaheuristic Algorithm (MA) Module (MAM)

AEC is heavily impacted by the involved MA, which is operationalized by the MAM. Despite the grim backdrop depicted in Section IE, "there are opportunities to reduce AEC at the MA level," such as "at the convolutional layer," via avenues "that scale well across the AI stages" [7]. By way of contextualizing information, two of the MA bastions are: "Evolutionary Algorithms (EA) and Swarm Intelligence (SI)" [7]. It is recognized that "Holland's Genetic Algorithm (GA) is among the more popular EA" while "Kennedy & Eberhart's Particle Swarm Optimization (PSO) "is among the more popular SI" [7]. Various MA have since "been put forth, but they tend to be derivative variants of EA or SI" [7]. For many cases, the plain vanilla PSO has outperformed the derivatives [7]. This was furthered by a remark, made during a keynote session of the World AI IoT Congress; "oftentimes "purported performance assertions are more marketing than actuality;" in any case, "Nikelshpur and Tappert as well as others have successfully utilized SI, in the form of PSO, for pre-training," "Wang et al. and others have successfully used PSO for fine-tuning," and Babanezhad as well as "others have successfully used PSO for inferencing" [7][21][22][23]. As it is seemingly fit for purpose "'across the AI stages,' PSO-based MA" warrants further investigation" [7].

The aspects discussed within this paper (with utilized acronyms) are presented in Table I (which is drawn from [1]), via five parts: (1) the overarching objectives (e.g., targets, actions), (2) the functional requirements, (3) the constraints (e.g., functional, selection bias, spatial/temporal), (4) certain boundaries, and (5) the requisite components (e.g., constituent elements, which each constitute a separate system). In this way, by Section V (Conclusion & Future Work), it can be evaluated whether the proposed approach suffices in addressing the overarching objectives.

TABLE I. CONSIDERED ASPECTS OF THE LHM-ICSM2-MCDM2-IFM-AECM-MAM AMALGAM (WITH UTILIZED ACRONYMS) [1]

I. Overarching Objectives (e.g., targets, actions) & Case Studies *Decision Support & Decision-Making*
• Intelligent System (IS) • Command & Control (C2) ➢Information Fusion (IF) ⇁Multi-Partner Enclave (MPE) ➢Advanced Analytic Technologies (AAT) ⇁Advanced Anomaly Detection (AAD) ➢Continuous Situational Awareness (CSA) ➢At-the-Edge Observational Space Analysis (AOSA) • Decision Engineering Discernment Engine (DE2) ➢Decision Support System (DSS) ⇁Optimal Decision Engineering Pathway (DEP) amidst ⇁Uncompressed Decision Cycles (UDC) ⇁Compressed Decision Cycles (CDC) ⇁Fuzzy Optimization and Decision Making (FODM) ⇁Scalar Optimization Problem (SOP) ⇁Nondominated Solution (NS) ⇁Optimal Shapley-Nondominated Solution (OSNS) • Multi-Criteria Decision Making (MCDM) with constituent ➢Multi-Attribute Decision Making (MADM) ➢Multi-Objective Decision Making (MODM) ⇁Mathematical Programming Methods (MPM) ⇁Artificial Intelligence (AI)/Machine Learning (ML) methods ⇁Integrated Approaches (IA). ➢MADM/MODM each have: ⇁Subjective Methods (SMs) ⇁Objective Methods (OMs) Collectively: MADM/MODM SM/OM (MMSO) • Quality Control Program (QCP) ➢Quality Assurance/Quality Control (QA/QC) ⇁ Real World Scenario (RWS) case studies ⇁Global Maritime Domain Awareness (MDA) ⇁Regional Maritime Situational Awareness (RMSA)
II. Functional Requirements *Aspects Needed*
• Knowledge Graph (KG) ➢KG Embedding (KGE) ⇁KG Completion (KGC) ⇁Spatio-Temporal KG (STKG) Completion (STKGC) ⇁KG Reasoning (KGR) ⇁STKGR ⇁Type-Sensitive (TS) STKGR (TS-STKGR or T2S2KGR) • Discernment Facets: ➢High Dimensional Data (HDD) ⇁Sparse Solution Discernment (SSD) ⇁Very Small/Non-Obvious (VSNO) ⇁*H*DD *V*SNO SS*D* (HVD) ⇁HDD-centric Cluster Validity Index (CVI) Measures (HCM)
III. Constraints (e.g., functional, bias, temporal) *Implementation Considerations*
• High Order Network (HON) ➢ HON interactions (HONi) ⇁Hypergraphs ⇁Complex Manifolds (CMs) ⇁Simplicial Complexes (SC) ⇁ Homological Percolation Transition (HPT) ➢Control Signals (CS) ⇁Control Energy Cost (CEC), ⇁Key Control Driver Nodes (KCDN) ⇁Influence Dominating Sets (IDS) ⇁Positive Influence Dominating Sets (PIDS) ⇁Negative Influence Dominating Sets (NIDS), ⇁Bak–Tang–Wiesenfeld (BTW)

⨽Co-Evolution Networks (CEN)
⨽Elongated Temporal Span (ETS)
⨽Gramian
⨽Inverse Gramian
⨽Vanishing-Moment Recovery Matrix (VMRM)
• Multi-Layer Networks (MLN)
➢Efficient Controllability Problem (ECP)
⨽*Minimum Controllability Problem (MCP)*
• Relationship Membership Stream (RMS)
➢Probability [& Statistics] Systems Theory (PST)
➢Fuzzy Systems Theory (FST)
⨽ Fuzzy Number (FN)
⨽ Spherical FN (SFN)
⨽Optimal Corresponding (OC) Generalized Linear 'f'-sided SFN (OCGLfSFN)
⨽Generalized (G) 'f'-gonal FN/SFN forms:
⨽G *Tr*iangular FN/SFN (G*Tr*FN/G*Tr*SFN)
⨽G *Tr*a*p*ezoidal FN/SFN (G*Tp*FN/G*Tp*FSN)
⨽G *Pe*ntagonal FN/SFN (G*Pe*FN/G*Pe*SFN)
⨽G *H*e*x*agonal FN/SFN (G*Hx*FN/G*Hx*SFN)
⨽G *H*e*p*tagonal FN/SFN (G*Hp*FN/G*Hp*SFN)
⨽G *O*ctago*n*al FN/SFN (G*On*FN/G*On*SFN)
⨽Fuzzy Set (FS)
⨽Intuitionistic FS (IFS)
⨽Pythagorean FS (PFS)
⨽Neutrosophic FS (NFS), which combines Neutrosophic Set (NS) with FS.
⨽Type-2 Fuzzy Set (T2FS), as contrasted to Type-1 Fuzzy Set (T1FS)
⨽Spherical Fuzzy Sets (SFS)
⨽T-SFS (TSFS)
➢Rough Set (RS)
⨽Rough (R)-Fuzzy Set (RFS)
⨽Three-Way Soft Clustering (TWSC)
⨽Similarity Measures (SimM)
⨽Center of Gravity (COG)
⨽Radius of Gyration (ROG)
➢Grey Systems Theory (GST)
• AI/ML Implementations
➢ Robust Convex Relaxation (RCR) paradigm
⨽Constriction Factor (CF)
⨽Particle Swarm Optimization (PSO)
⨽Numerical Stability Architectural Construct (NSAC)
⨽Number of Function Evaluation (NFE)
➢Sequence of Transformations (SOT)
⨽Nonnegative Matrix Factorization (NMF)
⨽Gaussian Composite Model (GCM)
⨽Multiresolution Matrix Factorization (MMF)
⨽Corresponding WT (CORWT)
⨽Enhanced CORWT (ECORWT)
⨽Wavelet Transform (WT) which include
-Continuous Wavelet Transform (CWT), whose implementation can include CWT PyWavelet Schema (CPS)
• Explainable AI (XAI)
➢Criteria Weighting Systems (CWS), which might include MMSO, such as:
⨽Point Allocation (PA)
⨽Analytic Hierarchy Process (AHP)
⨽CRiteria Importance through Intercriteria Correlation (CRITIC)
⨽Technique of Order Preference by Similarity to an Ideal Solution (TOPSIS)
⨽VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)
⨽Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA)
while other HDD-oriented sub-space approaches include:
⨽Clustering in QUEst (CLIQUE)
⨽Merging Adaptive Finite Intervals And (MAFIA)
➢Adaptive CWS (ACWS),
• Exemplar Metrics:
➢Performance (P)
➢Consistency (C)
➢Flexibility (F)
• Neural Networks (NN)/Deep NNs (DNNs)
➢Convolutional NN (CNN)
⨽Graph Convolutional Network (GCN)
⨽Deep Convolutional Generative Adversarial Network (DCGAN)
⨽Hypergraph-induced Convolutional Manifold Network (H-CMN)
⨽Hypergraph-Induced *Infimal* Convolutional Manifold NN (H-IICMNN)
➢Graph NN (GNN)
⨽Graph-Attention-Network (GAT)
➢Recurrent NN (RNN)
⨽Bidirectional Long Short-Term Memory (BiLSTM)
➢Model Paradigm
⨽Training
⨽Fine-Tuning
⨽Inferencing
⨽Forward Passes (FP)
• Triples, Quadruples, and/or Quintuples (TQQ)

IV. Specific Boundaries

• Cluster Validity Index (CVI), which can be grouped as
➢External Measures (EMs), such as
⨽F-Measure (FM)
⨽Normalized Mutual Information (NMI)
➢Internal Measures (IMs), such as
⨽Calinski-Harabasz (CH)
⨽Davies-Boulding (DB)
⨽Ball-Hall (BH)
⨽Pakhira-Bandyopadhyay-Maulik (PBM)
⨽Trace(W) (TW)
⨽Point-Biserial (PB)
➢Relative Measures (RMs), which can be construed to be IMs:
⨽Dunn-Index (DI)
⨽Maulik-Bandyopadhyay (MB)
⨽McClain-Rao (MR)
These can also be grouped as Candidate Lists (CL):
⨽Difference-like Criteria (DLC) -> DLC CL (DCL)
⨽Optimization-like Criteria (OLC)

V. Requisite Modules (e.g., constituent elements, which each constitute a separate system)
Key Constituents

➢LAHU/HALU Module (LHM), which is comprised of
⨽Lower Ambiguity, Higher Uncertainty (LAHU)
⨽Higher Ambiguity, Lower Uncertainty (HALU) considerations
➢Isomorphic Paradigm (IsoP)
⨽IsoP Comparator Similarity Measure Module (ICSM2)
⨽Isomorphic Heuristical Pathway (IHP)
➢MCDM Module (MCDM2)
➢IF Module (IFM)
➢AI Energy Consumption (AEC) Module (AECM)
⨽Units:
⨽Joules (J)
⨽Watt (W)
⨽kilo Watt hours (kWh)
⨽Floating Point Operations (FLOPs)
⨽[computational] Efficiency (EFF)
➢Metaheuristic Algorithm (MA) Module (MAM)
⨽Evolutionary Algorithms (EA)
⨽Genetic Algorithm (GA)
⨽Swarm Intelligence (SI)
⨽PSO

Overall, this paper describes an AI-IS-DE2 construct (i.e., LHM-ICSM2-MCDM2-IFM-AECM-MAM) being advanced, whose focus is the discernment of HON so as to better understand the C2/CS at play (and vice versa); in either case, STKGC is leveraged, as is ACWS, by the underpinning Graph Convolutional Network (GCN)-Bidirectional Long Short-Term Memory (BiLSTM)-Graph-Attention-Network (GAT) along with a Robust Convex Relaxation (RCR)-based Deep Convolutional Generative Adversarial Network (DCGAN)-Hypergraph-Induced Infimal Convolutional Manifold Neural Network (H-IICMNN)-1,2,3,4 (GCN-BiLSTM-GAT & RCR-DCGAN-[H-IICMNN]-1,2,3,4 or GBGRDH-1,2,3,4), whose "IsoP scrutinization" of the IsoP repository (i.e., Isomorphic Heuristical Pathway or IHP) to ICSM2 progression is shown in Fig. 1 [7]. The ICSM2 informs the LHM that influences the involved MCDM2, which impacts the IFM.

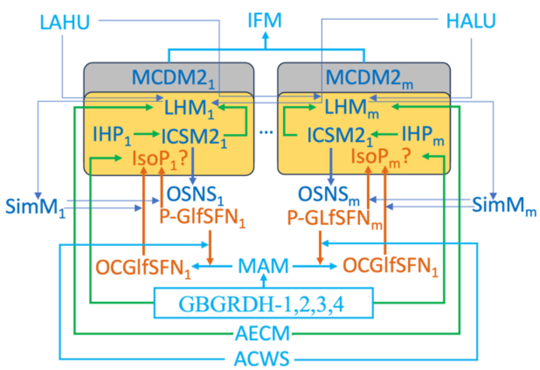


Figure 1. IsoP Scrutinization [7]

Section I provided an overview as to some the overarching objectives of the construct (in addition to some of the tactical objectives, such as IF): AAT (e.g., AAD), AOSA, and CSA for DE2, such as shown in Fig 2. The color schema alludes to the relative AEC in a ROYGBIV fashion.

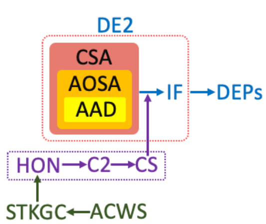


Figure 2. Exemplar Overarching Objectives

It also introduces some key modules: LHM, ICSM2, MCDM2, IFM, AECM and MAM. The paper is, subsequently, structured as follows. Section II provides background information regarding the tasks of: (1) AAT/AAD (a more granular and narrower aperture), (2) AOSA (a wider aperture with greater contextualization), and (3) CSA (an ongoing tasking of AAT/AAD and/or AOSA), and Section III presents key foundational considerations, which include the preferred OCGLfSFN, the ranking of the involved FNs/SFNs (facilitated by the ACWS), the SimM challenge for these FN/SFN, the SimMs and Distance Measures (DMs) for Spherical Fuzzy Sets (SFS) and T-SFS (TSFS), the discernment of clusters/boundaries, the spatio-temporal representation via a quintuple, the use of Type-Sensitive (TS) for RWS, the consideration of Influence Dominating Sets (IDS), ascertaining the Minimum Controllability Problem (MCP), determining the Efficient Controllability Problem (ECP), finding out the Controllability Gramian, establishing the Inverse Gramian, perceiving the phase transitions for High Dimensional Data (HDD) Clustering, and employing various HDD-centric Cluster Validity Index (CVI) Measures (HCM). Section IV provides some preliminary experimental results, and Section V concludes with some observations, puts forth some future work, and the acknowledgements close the paper.

II. Background

A case study, such as MDA, involves IF as well as AAT/AAD, AOSA, and CSA (which comprise the DE2 construct and are all affected by the degree of robustness at the pre-training, fine-tuning, and inferencing phases); these key facets are described below.

A. AAT/AAD

Tasks involving AAT/AAD will necessarily have a high AEC, as the "discernment of relevant sparse and 'Very Small/Non-Obvious' (VSNO) subspace entities within the associated High Dimensional Data (HDD)" will be involved, and more extensive use (and higher AEC) will be involved for AAD-related VSNO [14]. The color schema of Fig. 3 alludes to the relative AEC in a ROYGBIV fashion.

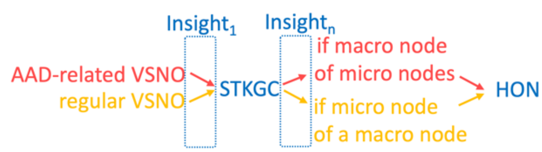


Figure 3. Relative AEC Delineation

B. AOSA

Along the vein of AEC, for AOSO, "optimizing the algorithms" seems to be an approach more in accordance with Hernandez and Brown (of OpenAI) spotlighting the fact that "effective compute" is being driven by "25x growth algorithmic efficiency" as contrasted to "the hardware efficiency gain estimate" of the "8x growth" posited by Moore's Law [14][24]. Desislavov affirms and posits that lower AEC, and improved resultants are likely "attributable to algorithmic improvements" rather than "more computing power" [14][17].

1) Pre-Training:

The import of adequately managing AEC is underscored by Hoffman's research. Hoffmann had found that several contemporary large models "are significantly undertrained," and this leads to situations, wherein "fine-tuning and inferencing AECs" are likely to be "much greater than expected" [14][25]. In fact, "it turns out that the 'convolutional layers'" [a.k.a., Conv] of the Convolutional

Neural Network (CNN) tend to have the 'highest AEC'" (compared to the fully connected layers, and increased filter sizes at the Conv tend to segue to an increased number of parameters, higher computational complexity, and higher AEC), and to address this point, approaches for the reduction of AEC have included "SqueezeNeXt, which builds upon SqueezeNet, or SqueezeNet itself, which utilizes 1x1 convolutions (rather than 3x3), by way of example" [14][26][27]. SqueezeNet is often used in conjunction with a "Fire Module" (i.e., 1x1 convolution filters comprising a "squeeze convolution layer" and an "expand layer" consisting of 1x1 as well as 3x3 convolution filters) [14][26][27]. A "Fire Module" or "Fire Layer" is referred to as FL, and when a series of FLs is used, it can be referred to as SFL. A Modified Squeezed DCGAN You Only Look Once (YOLO) version 3 (v3) Implementation (MSYv3I) can incorporate these notions, as shown in Figure 4. In addition, alternative transformations (distinct and disparate from the Conv or more energy-efficient convolutions with alternative numerical methods) can be leveraged.

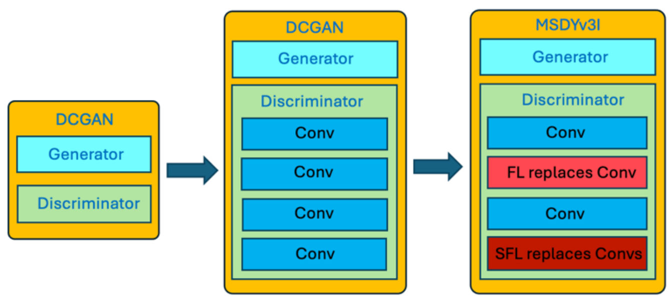


Figure 4. MSYv3I of FL and SFL

2) Fine-Tuning:

Fine-tuning can be construed to be "additional training of the pre-trained model on a more task-specific subset of the original dataset," which substantially changes the pre-trained model, such as via an ACWS, "so as to better conform with the dataset and/or task-at-hand" [14]. In addition, "Wang, Knack, and others note that with regards to AEC, there is a 'need to study fine-tuning…separately from' pre-training 'and inference workloads'," for while pre-training AEC costs can be relatively steady, fine-tuning and inferencing AEC costs can "vary greatly," as they are dependent upon the task-at-hand, the desired accuracy, and the robustness of the pre-training [14][28][29]. This is especially the case when the deployed model is undertrained (as contrasted to being overtrained "past the Chinchilla optimal") [14].

3) Inferencing:

Moving from fine-tuning to inferencing, it is generally recognized that the works of "Li et al. as well as Canziani et al. are among the first robust examinations regarding inference costs" [14][17], and several interesting observations are put forth; for example, Canziani notes that it has become current "practice to run several trained instances of a given model over multiple similar instances" [14][30]. This "practice is known as model averaging," usually involves an ensemble of Deep Neural Networks (DNNs), and "dramatically increases the amount of computation required at inference time to achieve" the specified accuracy (or, given the involved ambiguity/uncertainty, the degree of quantitative exactitude) [14][30]. Clark puts it quite well by noting that the involved AI first needs to "'understand…the query then 'thinks' of an answer," which "thereby increas[es] the involved AEC" [14][31]; this is consistent with a statement made at a keynote session of the World AI IoT Congress: "it's not just about detecting but understanding." For this ensemble paradigm, the "AEC is multiplied accordingly" segueing to a much higher multiple for the aggregate AEC [7].

C. CSA

Taking the proxy case of MDA, the likelihood/probability of a large model size, high accuracy requirements for the involved AAT/AAD, and a high number of Forward Passes (FP) is quite elevated [14]. Also, as "MDA is typically associated with mission-critical activities," it has been noted that "the involved fine-tuning" will "likely be extensive and ongoing" [14]. Moreover, CSA will "necessarily involve higher-order AEC tasks" (e.g., AAD) that will necessitate high FPs [14][16]. As MDA "and other similar mission-critical" RWS "are likely prioritizing quality of results," the various involved modules are likely to be calibrated accordingly [14].

With regards to the overall involved AEC, researchers have noted that "Desislavov's estimates tend to be quite close to reported measured values," so "the Desislavov approach is adopted," for this paper, "wherein inference Floating Point Operations (FLOPs) are focused upon and 'the efficiency metric FLOPS per Watt' (Watt=W) is re-expressed as 'FLOPS per Joule' (Joule=J), so as to express the [computational] Efficiency (EFF) and AEC, such as in (1):

$$\text{EFF in units: FLOPS/W=[FLOPs/s]/[J/s]=FLOPS/J} \quad (1)$$

$$\text{AEC=FP/EFF in units: [FLOPs]/[FLOPs/J]=J)" [14][17].}$$

Inference AEC is task-dependent, and as one simple example, the task of "object detection" (e.g., in an AAT case) has "a higher AEC than image classification," and aberrant object detection (e.g., in an AAD case) has an even higher AEC [14][16]. Experimentation by Lucioni had found that inference AEC tended to be much higher than expected, and "the 'mean and standard deviation of inference energy' in kiloWatt hours (kWh) 'per 1,000 queries' was 542% and 2000% greater, respectively" [14][16][17]. Lucioni further found that the "utilization of 'multi-purpose models for discriminative tasks' had a higher AEC when 'compared to task-specific models for these same tasks'," and the differential was quite high: "2-3x to 5-7x" [14][16][17]. Furthermore, any requisite "increase in accuracy," such as throughout the course of AOSA and/or CSA, can segue to "a dramatic" "increase in the required FLOPs for" FP [14][17].

III. THEORETICAL FOUNDATIONS & CONSIDERATIONS

A. Preferred OCGLfSFN

Continuing from Section I, a goal of the involved MA (for which the MAM is responsible) is to ascertain the OCGLfSFN. Prior to segueing to this OC form, there is a precursor non-OC form (i.e., P-GLfSFN), as previously delineated in Fig 1. For example, Augustin acknowledged the predilection for G*Hp*FN for its ability to "represent more intricate and nuanced degrees of uncertainty" while Ban favored G*Tr*FN, G*Tp*FN, and semi-G*Tp*FN for the preservation of ambiguity and weighted ambiguity [7][32][33]. Whatever the preferred form, the choice of the precursor Generalized 'f'-gonal FN/SFN form (e.g., P-GLfSFN) affects the efficacy of the utilized "defuzzification method" (i.e., "the transformation of a FN/SFN into a crisp form") [7]. The significance of this centers upon the intricacy that "as the LHM contends with the counterpoising of" ambiguity/uncertainty, the precursor non-OC form, which best preserves ambiguity, is likely to be optimal for facilitating/deriving the OCGLfSFN.

B. The Ranking of FNs/SFNs

There are numerous "ranking methods for the discussed pre-cursor [non-OC form] Generalized "'f'-gonal FN/SFN form (e.g., P-GLfSFN), and the appropriate selection" is central [7]. For example, Velu and Ramalingam noted that "a ranking method which works very well for" G *He*x*a*gonal FNs/SFNs "may have some shortfalls when it is extended for" G *O*ctago*n*al FNs/SFNs [7][8]. Similarly, "a ranking method which works very well for" G *O*ctago*n*al FNs/SFNs might have "some shortfalls when it is used for" *Tr*iangular or *T*ra*p*ezoidal FNs/SFNs [7][34]. In any case, the ranking mechanism (facilitated by the ACWS) informs the precursor non-OC to final OC form.

C. The Similarity Measure (SimM) Challenge for FN/SFN

With regards to the ranking methods referred to in Section IIIB, the underpinning measures typically involve various SimMs. Gogoi & Chutia noted that while there are a myriad of methods (each with advantages/drawbacks), "a universally accepted 'silver bullet'" SimM "for ascertaining the similarity between" FNs/SFNs "does not necessarily exist" [7][35]. They also noted that a "'literature survey reveals that most of the' SimM 'are being developed based upon'" the following parameters: "geometric distances, height, area, perimeter, 'Center of Gravity (COG),' 'Radius of Gyration (ROG),' etc." [7][35]. It was noted in [35] that for various studies, with the exception "of Hejazi et al. (2011)," certain "glass ceiling" SimM methods (e.g., "failing to 'give reasonable similarity between pairs' of FNs when one FN 'is identical for both the pairs'") "are being carried forward" into contemporary works [7][35]. This is reminiscent of our prior finding that certain bugs/issues in various frameworks/libraries/toolkits, such as made available via assorted developer platforms, were being carried forward into various projects/papers. To aggravate matters, "FNs are simply a special case of a" FS, and "beyond FS, there" are other FS variations; these include the Atanassov Intuitionistic Fuzzy Set (IFS), Pythagorean Fuzzy Set (PFS), and Neutrosophic Fuzzy Set (NFS) [7][36][37]. The IFS, which is often leveraged for "coalition decision-making," is comprised of constituent elements that "have both membership function u and nonmembership function v, such that $u + v <=1$, and hesitation margin h, such that $u + v + h = 1$" [7]. Other situations are better addressed by PFS, "wherein $u + v >=1$ (or $u + v <=1$) and $u^2 + v^2 + h^2 = 1$" [7][36]. Yet other cases are better handled by NFS, which combines "FS with NS" [7][37]; delving into this, Das notes that while FS addresses "uncertainty" by the utilization of "membership grade," Smarandache's NS tackles "uncertainty using truth, indeterminacy, and falsity member grades" [7][37]. Furthermore, Ashraf, Gundogdu & Kahraman, Mahmood, etc. have "contributed to the notion of…SFS, which 'is the generalized structure over' the referenced FS (e.g., IFS, PFS, and NFS)" [7][38].

D. SimMs and Distance Measures (DMs) for SFS/T-SFS

Various SimM approaches have been adapted for the SFS ecosystem, as noted by Zhang, and Wei observes, by way of example, that a plethora of "SimMs for SFS 'based on the cosine and cotangent function' have been" put forth [7][39]. Likewise, certain combinatorials, such as "Jaccard, Exponential, Square root cosine for SFS," etc., have been employed as pragmatic implementations of SimMs [7][40]. With regards to DMs, "Donyatalab and others have examined 'Minkowski, Minkowski-Hausdorff, Weighted Minkowski and Weighted Minkowski-Hausdorff distances for SFSs'" [7]. Overall, there have been numerous SimM and DM advances, and among these, researchers, such as Wu, have "focused on the T-SFS," which is a "'specific case of NS'" (a.k.a., "n-hyper SFS") [7][39][40]. According to Wu, T-SFS is quite adept in contending with "uncertainty information" and "can handle information that SFSs…cannot process" [7][40]. Accordingly, the SimMs/DMs of T-SFS show promise for higher efficacy.

E. Discernment of Clusters/Boundaries

Despite the prospective high promise, the use of SimMs and DMs for clustering should also have concomitant methods of *Boundary* Detection (BD) for enhanced efficacy. As a case in point, regardless of the type of Knowledge Graph (KG) involved, they are often incomplete. Along this vein, researchers have criticized "Static KG (SKG) for neglecting temporal information" [41][42]. Others have critiqued Temporal KG (TKG) for neglecting spatial information [41][43]. Accordingly Spatio-Temporal KG (STKG) seems promising, and Ye notes that for the associated "KG Completion (KGC)," such as that of STKG Completion (STKGC), "discriminative methods (a.k.a., conditional methods that discern *boundaries* among labels, classes, etc.) endeavor to, by way of example, 'predict the possible label' (e.g., node name, line segment name, etc.)" [44]. Wei further clarifies this by noting that "discriminative methods focus on discerning elements of the" involved Triples, Quadruples, and/or Quintuples (TQQ) to "'efficiently construct large-scale' KGs, 'which often require'" "an ensemble," "multiple models" "and/or cascading succession of models" [41]. According to Zeb, the objective is to "undertake KGC" (e.g., STKGC) to "determine the 'unobserved' and/or complete the observed TQQ, thereby facilitating sufficient/efficient inference" [45]. Along this vein, "robust KGC can, potentially, facilitate reducing the inference load and the associated AEC" [41].

F. Spatio-Temporal Representation with a Quintuple

With regards to the referenced STKGC, "Nayyeri suggests that 'spatio-temporal facts can be represented as a quintuple' (s/h, p/r, o/t, l, τ), 'where l reflects the location information (spatial)'" rather than simply using the triple or quadruple representation [43]. "Nayyeri further notes that the 'quintuple representation in the form' (?, p/r, o/t, l, τ),(s/h, ?, o/t, l, τ), (s/h, p/r, ?, l, τ), (s/h, p/r, o/t, ?, τ) or (s/h, p/r, o/t, l, ?) 'is especially suitable because for each incomplete quintuple, four of the five elements are always present'" [43]. In support of Nayyeri's assertion, "Dihedron algebra" is known to be 'a rich 4D algebra of hypercomplex spaces' that can operationalize geometric operations in higher dimensions" [43].

G. Type-Sensitive (TS) for RWS

The TQQ issue (e.g., the extraction potential, or lack thereof, of the triples, quadruples, and quintuples of the TQQ amalgam) is further explored, and "according to Zhang, many of the 'current models have difficulty distinguishing representations of the same entity or relation at different timestamps'" [46]. He refers to this phenomenon as an "entity type information gap" [2]. As a very simplistic example, the relation 'invent,' 'devise'," modify, hybridize, etc., "could connect head entities of type 'AI company'," tech startup company, ML firm, etc. "to tail entities of type 'AI algorithm'," ML algorithm, AI/ML technique, ML method, etc. [2]. Pertaining to "cases such as this, He [et al.] points out that this apriori knowledge" regarding "entity type information" and/or "relation" connectors can provide insights into the likely and apropos "entity type information" for the unknown tail entity. After all, "its position in the vector space should not be far away" [47]. This paradigm constitutes the essence of being "Type-Sensitive" (TS)," and a correlation is made by He et al. that for RWS, "an entity tends to belong to multiple types'" [2][47]. Accordingly, the TS approach, such as TS-STKGC, "might better lend towards RWS" [2].

H. The Consideration of Influence Dominating Sets (IDS)

As noted in Sections III E through G, boundary, spatio-temporal, and TS distinctions help to clarify the sets at play; this is vital for the consideration of IDS, which are typically divided into two groupings: Positive Influence Dominating Sets (PIDS) and Negative Influence Dominating Sets (NIDS). Both PIDs and NIDs must be taken into consideration for the overarching IDS, which is often contextualized by "the Bak–Tang–Wiesenfeld (BTW) sandpile effect of non-equilibrium systems" [41]. Particularly in the case of "Multi-Layer Networks" (MLN)," which pervades RWS, "Grilli had found that HON-related IDS" "interactions have a stabilizing influence within LSCN, and the existence of HON nicely explains many RWS" [41][48][49]. While understanding HON-related IDS, it is useful to undertake the resolving of certain problems to progressively contextualize the state of and/or the prospective controllability. In the course of understanding HON-related IDS, it is useful to undertake the resolving of certain problems to progressively contextualize the state of and/or the prospective controllability. Of note, this involves the progression from the resolving of the Minimum Dominating Set Problem (MDSP) (which centers upon "determining the smallest dominating set of a given graph") to that of the Minimum Controllability Problem (MCP) (which centers upon ascertaining a pragmatic dominating set — that might not necessarily be the smallest — for a given graph).

I. Minimum Controllability Problem (MCP)

First, with regards to MCP, Nguyen articulates the distinction that MDSP is "more suited for a static" Large Scale Complex Network (LSCN), and in contrast, Terasaki points out that solving the MCP is "more suited for dynamic LSCN" [41][50]. Lin adds to this by discussing the notions of a connection condition, which is referred to as a "Critical Connection Component" (CCC) and represents "an *infimal* strongly connected component" as well as rank condition; the rank equates to the number of singular values and the condition is the ratio of *max:min* singular values [41][51][52]. According to Lin, the involved/studied "system is structurally controllable if and only if [iff] a connection condition…and a rank condition…are both satisfied" [41][51][52]. However, as noted by Alizadeh, an approach that "ensures controllability" that is "equivalent to solving a combined maximum matching" (for which Berge's Lemma might put it best—maximum matching is achieved if and only if [iff] there is no augmenting path) is a different matter entirely [41][53].

J. Efficient Controllability Problem (ECP)

Second, progressing from MCP to ECP, Gokler notes that ECP might be the more practical problem to contend with [41][54]. According to Lindmark, the ECP "contends with minimizing the number of requisite CS" and "the requisite Control Energy Cost (CEC)" [41][55]. CS and CEC should be considered in tandem, for "Chen asserts that 'if the number of" CS 'is small, the' CEC 'demanded…could be prohibitively high'" per CS [41][56]. In contrast, "the CEC 'is reduced exponentially as the number of' input CS increases" [41][56]. An extraordinarily elevated CEC would not be practical to achieve, and "controllability for only a limited temporal span" or window may not meet the mark for the envisioned sustained C2 [41][56]. For all intents and purposes, "practical controllability has the criteria of persistence over an Elongated Temporal Span (ETS) so as to be able to effectuate actual/effective control when needed/desired" [41][56]. As noted by Gao, "the optimality problems at hand" "could be construed to center upon" "an optimal number of CS (CS_{opt})" functioning as IDS "on an optimal number of Key Control Driver Nodes (KCDN) ($KCDN_{opt}$)" at a reasonable/sustainable "optimal CEC (CEC_{opt}) over an ETS_{opt}" for effective/efficient control of a LSCN [41][57]. To effectively discern CS, KCDN, CEC, and ETS, the various MLNs and involved HONs should be posited. This discernment pathway leverages: (1) the informative nature of Co-Evolution Networks (CEN), and (2) the insights provided by complex manifolds" [41][56].

1) Informative Nature of CEN

First, information derived from adjacent networks or those at other levels (e.g., MLN) "can be quite informative" from a CEN perspective, particularly if network enlargements/enhancements were made to support the other involved network(s). Likewise, if an outage of one network affected another, certain other suppositions can be made [41]. RWS include KGC facilitation "by, say, knowledge of" adjacent network(s) (e.g., communications network) "interoperating"/co-evolving "alongside an involved" LSCN (e.g., "power grid"). Other examples of intertwined networks (which might be mutually reinforcing and/or opposing) include that of a milk kinship network and tribal political network (i.e., CENs) [41]. Oftentimes, "knowledge of the involved networks across various jurisdictional/functional demarcation boundaries" can be invaluable for STKGC [41].

2) Informative Nature of Complex Manifolds (CM)

Second, researchers, such as Battiston, have examined both "the informative nature of CENs" and "the stabilizing influence of HON" [58]. Battiston, Vazquez, and Sun & Biaconi note that "HON topologies can be" expressed by hypergraphs, even more "complex hypergraphs ('hypergraphs of hypergraphs' or chygraphs)," and "multiplex hypergraphs ('a set of hypergraphs…with the same set of vertices')," respectively [41][58][59][60][61][62]. Extending this point, Ding suggests a more "robust characterization" of "HON topologies" can be effectuated by an amalgam of CMs [41][63]. The CM is described by Voisin as having "'complex-valued coordinates (called holomorphic coordinates)' assigned to positions on a manifold" [41][64]. As noted in [41], "CMs can provide invaluable insights, and 'a physical system embedded on a twisted topological complex manifold' can bring out 'fundamental physical properties of an unknown system,' such as 'if and when' a 'system is undergoing a *phase transition*'" [41][65]. When CMs are considered against the "BTW principle and set against the described LSCN controllability/uncontrollability optimality problems," such as $CEC_{opt,}$ "the impact of existing HON topologies" becomes much clearer [41][63][64].

K. Controllability Gramian

Prior to arriving at CEC_{opt}, it is often useful, certainly for quality assurance/quality control purposes, to obtain "the minimum CEC (CEC_{min}), which Klickstein asserts, "can be characterized by the controllability Gramian" [41][66]. For the discussed case, wherein C2 can be achieved, the "Gramian matrix should be well-behaved" [41][67]. In other words, the "sensitivity to perturbation" (i.e., "the condition number") and the CEC is not prohibitively large [41][67]. In contrast, when the Gramian matrix is ill-conditioned, C2 is not able to be effectuated, and the condition number is indeed prohibitively large [41]. In essence, "for the latter case, the LSCN is not able to arrive at the 'final state in the prespecified time within a predefined precision'" [41][67]. Lindmark notes that the handling of the "Gramian matrix is paramount, as some strategies involve 'comput[ing] in closed form...when the time of the transfer tends to infinity' and physical controllability will not manifest" [41][55]. As noted by Zhou, this "accentuates the case for CS augmentation and/or accelerant approaches" to enhance the probability of actual C2 'as contrasted to theoretical, mathematical controllability'" [41][67][68]. The expectations for more robust and accurate controllability are particularly high for the case of dense/homogeneous LSCN (vice sparse/heterogeneous LSCN) with similar sub-LSCN [41][68]. Along the same vein of moving to quintuples to, potentially, operationalize the spatio-temporal aspect, "Zhang noted that temporal LSCN, which exhibit link temporality" — something akin to "'attaching a virtual driver node to that link'— tend to be more physically controllable" [41][69]. The sequitur thought is that spatio-temporal LCNS will likewise, ostensibly, be more physically controllable. Taking these collectively, in essence, from the vantage point of CS, "if one set of Key Control Driver Nodes (KCDNs) "can influence another set of KCDNs" so as to not only influence the involved LSCN, "but also peer LSCN," higher/lower-order LSCN, and/or HON "to a particular interim state, it then follows that the ultimate desired state is more likely to be attained" [41].

L. Inverse Gramian

Continuing the point of CS, according to Ludice, the likelihood of success for CS_{opt} (e.g., "CS base candidate set and/or CS augmentation set") is intricately tied to the "diffusiveness/permeability of the LSCN," which "constitutes a potential indicator of the susceptibility for LSCN controllability" [41][70]. As another indicator, when the susceptibility for LSCN controllability is high, the inverse Gramian exists [41]; when the susceptibility for LSCN controllability is low, the inverse Gramian does not exist [41]. On this latter note, "a corresponding Vanishing-Moment Recovery Matrix (VMRM) is a suitable approximation of the inverse Gramian," as it "guarantees *n* vanishing moments of wavelet tight framelets" [41][71]. Abebe further notes that "as the number of vanishing moments increases, the polynomial degree of the wavelet also increases," so there are lockstep characteristics [41][72]. Grochnig asserts that "the potential advantage of this is that" "'wavelet tight frames can,' therefore, 'be derived from any multiresolution analysis'" [41][73]; "this segue[s] to the discerning of" the collective phenomena regarding the LSCN, and two other aspects are also important in this regard: (1) Percolation, and (2) Giant Component.

1) Percolation of a LSCN

Sun defines Percolation as positing "the fraction of nodes in the Giant Component' of a LSCN" [41][61]. In essence, Percolation is evidenced when the average node degree >1 and is indicative of the manifestation of the "Giant Component."

2) Giant Component of a LSCN

Sun notes that a "Giant Component" is a CCC that encompasses a substantive portion of the involved graph's vertices, and a "non-zero Giant Component" is required for the discerning of "'collective phenomena…emerging from' diffusiveness/permeability, etc." [41][61].

M. Phase Transitions for HDD Clustering Discernment

The IDS traits and stabilizing effect of HON underpin many RWS phenomena. These HON are comprised of "'both hypergraphs and Simplicial Complexes' (SC)" [41][61]. Zhang asserts that HON interactions (HONi) "'shape collective dynamics differently in hypergraphs and' SC" and points out that HONi "'increase[s] degree heterogeneity in' SC" while HONi 'decrease[s] degree heterogeneity in [random] hypergraphs'" [41][74]. Of significance, "the amalgam 'insights provided by these two constituent elements of HON are not dissimilar to the insights gleaned via the paradigm of CEN.'" [41]. Lee focuses upon SC and notes that it "has been 'shown to reveal a rich *phase [transition]* diagram' for 'link percolation'" (e.g., the impairing of network connectedness via the deactivating of the involved link/node) [41][61][75]. Insights into the "likeness in structure" (i.e., homological) and into the activation/de-activation of nodes/links temporally (i.e., percolation) is indicative of the involved HON topology. Accordingly, Lee notes that the Homological Percolation Transition (HPT) is also insightful since it denotes the emergence of a burgeoning cluster as the number of SCs increases [41][61][75]. Lee also notes that the CM expression for the hypergraph can also be insightful, and it can be combined with the "the value-added proposition of the SC and hypergraph interplay (in a 'CEN'-like fashion)" for revealing the HON phenomena [41][61].

N. HDD-centric Cluster Validity Index (CVI) Measures (HCM)

Axiomatically, robust HDD clustering is central to VSNO ascertainment. In turn, CVI are vital measures in assessing the resultant "optimal" number of clusters. The CVI measures suitable for HDD (i.e., HDD CVI Measures or HCM) are categorized into: "External Measures (EMs), Internal Measures (IMs), and Relative Measures (RMs)" [1]. EMs have the value-added proposition of capitalizing upon "cluster structures/resultants from data sources not necessarily intrinsic to the clusters and data at-hand" [1]. IMs have the advantage of capitalizing upon the affinity aspect (e.g., cohesion/compactness) that "exists predominantly within the clusters and data at-hand" [1]. Researchers, such as Vendramin posit that "RM can be construed to be a subset of IM" [1][76] [77][105][106].

IV. EXPERIMENTAL RESULTS AND DISCUSSION

Section IIIA described how a key goal of the involved MA is the derivation of the OCGLfSFN, and this process is, in turn, heavily dependent upon the precursor non-OC Generalized "'f'-gonal FN/SFN form(s) selected (e.g., P-GLfSFN). This choice is very much informed by the ranking method leveraged (informed by the involved MODM, such as OSNS), as discussed in Section IIIB. Underlying the ranking method are the validity measures utilized, and this typically involves SimMs, such as for SFS ("'the generalized structure' over' 'IFS, PFS, and NFS'") [7][38]; this aspect is described in Section IIIC. Section IIID furthers this by noting that SimMs are used in conjunction with DMs for both SFS and the more robust T-SFS extension. Accordingly, experimentation involving SimMs/DMs for SFS/T-SFS seems apropos, and this is reviewed in Section IVA.

Likewise, Section IIIE notes that the SimMs/DMs should necessarily be accompanied by *boundary* detection (e.g., among classes, etc.), such as via STKGC discriminative methods. Section IIIF notes that the quintuple representation can well capture spatio-temporal information that the quadruple (i.e., temporal information only) and triple (i.e., no temporal information) are unable to; hence, the quintuple representation is well suited for STKGC. Section IIIG then discusses the "entity type information gap" and introduces the notion of TS. While Sections IIIE/F/G helped to clarify the sets at play (via boundary, spatio-temporal, and TS distinctions), Section IIIH notes the consideration of IDS, via PIDS and NIDS. Section III I/J reviews the various considerations, such as IDS (for the MCP and ECP), and articulates the notion of CEC. Section IIIK notes that prior to determining CEC_{opt}, the ascertainment of the precursor CEC_{min} can be useful, and it is well reflected by the controllability Gramian. Section IIIL notes that if the involved LSCN is indeed controllable, the inverse Gramian will exist; along this vein, the VMRM is an accepted approximation of the inverse Gramian, and this sets the stage for the architectural construct to be utilized. It is also noted that as the prospective controllability LSCN becomes clearer, the Percolation and Giant Component lend toward that understanding. Section M notes that the SC and hypergraph interplay also well contribute to that understanding. While the multiplex hypergraph is quite revealing regarding the collective phenomena, SC is quite revealing with regards to *phase transitions*, and the HPT (which provides further *transition insights*) is particularly informative. Overall, the use of *phase transitions* for HDD clustering discernment is articulated. Section IIIN delineates HCM and notes that it is a critical facet for gauging HDD clustering, which is at the core of VSNO determination. Accordingly, experimentation involving the choice of HCM seems fitting, and this is reviewed in Section IVB.

Finally, the architectural construct is reviewed in Section IVC. There were two principal reasons for certain experimental testbed architectural modifications: (1) the consideration of DCNN performance degradation amidst large intra-class variations, and (2) the necessity of shifting to a GBGRDH-1,2,3,4 construct so as to leverage the "more task-specific H-IICMNN *infimal* convolution mechanisms to serve as 'efficient solvers'" for the 1,2,3,4 functional roles [41][116]. Central to the overarching architectural construct was the LHM, as it impacted the use of the involved Non-Operational Data (NOD), Situational Awareness Data (SAD), and Operational Data (OD). The baseline NOD data contextualizes the OD [5]. Likewise, SAD also contextualizes the OD “in an ongoing fashion,” such that when sufficient OD enriches the IHP repository, the LHM may decide that it is not necessary “to actuate upon” further OD under CDC conditions [5]. When OD is contextualized “by SAD as well as NOD,” it lends to the IHP [5]. The IHP, in turn, is underpinned by an Inherent Uncertainty Construct (IUC). The items of this paragraph are all discussed in Section IVC with an accompanying Fig. 7.

A. Experimentation with SimMs/DMs and SFS/T-SFS

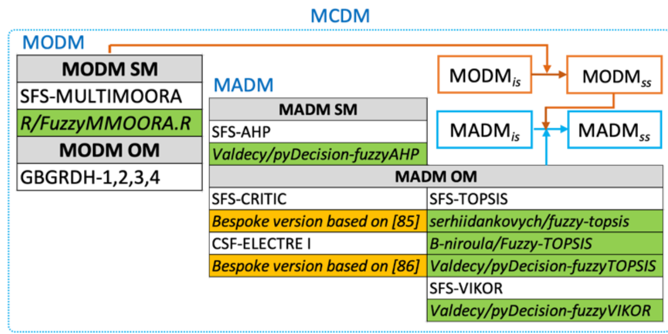
Preliminary experimental forays involved an examination of certain SimMs and DMs discussed in this paper and within the literature [78]. Table II depicts some of the affirmed resultants, and with regards to the color coding, “green denotes comparable performance, and orange signifies that the comparison was inconclusive” [7]. The entries include experimentation by Gundogdu & Kahraman as well as Sharaf, who collectively leveraged “Hwang & Yoon’s Technique of Order Preference by Similarity to an Ideal Solution (TOPSIS) and Opricovic’s VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) methods” [7][79].

TABLE II. EXEMPLAR AFFIRMED SIMM AND DM RESULTANTS [7]

		1	2	3	4	5
1	SFS-TOPSIS (Gundogdu and Kahraman)					
2	SFS-VIKOR (Gondogdu and Kahraman)					
3	SFS-TOPSIS (Sharaf)					
4	SFS-VIKOR (Sharaf)					

Then, TOPSIS and VIKOR as well as other MMSO methods were organized into their relevant categories. For example, “with regards to MADM SM, Ortega et al. proposed using SFS with Saaty’s Analytic Hierarchy Process (AHP)” [7][80]. For MADM OM, apart from TOPSIS and VIKOR, Kahraman’s use of “SFS with Diakoulaki et al.’s CRiteria Importance through Intercriteria Correlation (CRITIC) method” is presented along with Akram’s use of “Roy et al.’s ÉLimination Et Choix Traduisant la REalité (ELECTRE) method as pertains to Complex SFS (CSF)” [7]; “the extended method is referred to as CSF-ELECTRE I” [7][81]. For MODM SM, Gundogdu’s use of “SFS with Brauers & Zavadskas’ Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA)” is listed [7][82]. For MODM OM, Hanine put forth that “Mathematical Programming Methods (MPM), ML, as well as Integrated Approaches (IA)” should be incorporated into the considered approach [7][83][84]. Along this vein, as the utilized GBGRDH-1,2,3,4 architectural construct “qualifies as such (e.g., MPM/ML/IA), it was utilized for the MCDM OM” [7]. As noted in [83], the construct was devised such that the “MODM solution set ($MODM_{ss}$) facilitates the MADM input set ($MADM_{is}$) to $MADM_{ss}$ progression” [83]. In the spirit of open-source experimentation and leveraging open-architecture and various open-source pathways, various packages (e.g., frameworks/libraries/toolkits) from “Github and other repositories were experimented with” [7], and a sampling of the various MMSO packages/sortings are shown in Table III.

TABLE III. EXEMPLAR MMSO IMPLEMENTATIONS [1][7]



As indicated in [7], it was non-trivial “to appropriately adapt FNs to SFNs and SFSs,” and for “several cases, modifications of the involved method had to be” scrutinized against the ongoing work delineated in the literature. Among other exemplars, the “modification of CRITIC to a fuzzy paradigm was based upon, among others, Pamucar’s work” [7][85]; “modification of ELECTRE I to a fuzzy paradigm was predicated upon, among others, Sevkli’s work” [7][86]. For other instances, various “conversion guides” (e.g., “Amidi & Amidi R-Python”) and “online converters” (e.g., “CodeConvert’s Online R to Python Converter”) were utilized, such as for the “FuzzyMMoora function” (e.g., “R/FuzzyMMOORA.R”) [7]. Then, for the ensuing examination, three key metrics were utilized: (1) Performance (P) (“which is highly dependent upon the” Numerical Stability Architectural Construct or NSAC), (2) Consistency (C) (a useful indicator for both NSAC and the Number of Function Evaluation (NFE) (an indicator of the convergence rate — “e.g., a small NFE depicts a faster convergence rate”), and (3) Flexibility (which can be indicative of the potential for “adaptation, hybridization, etc.”) [7]. Certain “comparative evaluations” were conducted, and some “interim findings are reflected in” Table IV [7].

TABLE IV. EXEMPLAR MMSO BENCHMARKING [7]

	P	C	F		P	C	F		P	C	F
SFS-AHP				SFS-CRITIC				SFS-MULTIMOORA			
				SFS-TOPSIS							
				SFS-VIKOR				GBGRDH-1,2,3,4			
				CSF-ELECTRE I							

For Table IV, the color coding (is as follows. Red denotes worse performance while the darker shade of green denotes better performance; the progression of colors follows the ROYGBIV sequence and indicates the gradations of performance. With regards to the interpretation of Table IV, “for the conditions set within this paper, as delineated in Table IV above, it seems that SFS-MULTIMOORA and the” GBGRDH-1,2,3,4 (which supplanted the prior construct) “warrant further investigation” [7]. “For those cases, wherein the MMSO conjoined multiple packages, the various pairings were designed to be well counterpoised" by way of "complementary structures (e.g., PA is matrixed while AHP is hierarchical) and/or roles (e.g., criteria weights can be derived by CRITIC and, subsequently, ranked by TOPSIS)" [7]. The value-added proposition of an ACWS has long been put forward by researchers, and a well-counterpoised MMSO construct operationalizing an ACWS, which might buttress STKGC (a weight-aware task) is invaluable.

B. Experimentation with Selecting Apropos HCMs

Laborde, Vitelli, and others have asserted that Sparse Solution Discernment (SSD) has not been sufficiently "explored prior to the writing of this paper" and that SSD, such as that pertaining to VSNO "subspace clusters" in HDD, remains in a "fairly nascent state" [1][87][88]. With regards to VSNO clusters in HDD (i.e., *H*DD *V*SNO SS*D* or HVD), SSD is NP-Hard. This should be of no surprise since, throughout the years, researchers have articulated ("and the literature is rife with examples") the "sensitivity of prototypical clustering classifiers" "to the placement of the initial seeds, noise, and the lackluster efficacy when confronted by varying cluster sizes, densities, shape[s], etc." [89][90][91]. The robustness of the HVD is based upon the buttressing "workflow sequences, the involved measures," and "the efficacy by which similitude is gauged" [1]. Along this vein, Wang, Govaert, Nadif, etc. posited that "insight could be gleaned from the relationships among the subspace elements, such as that of submatrices (e.g., homogeneous subsets of data)" [1][91]. Taking a different approach, Majdara, Li, Xianting, etc. "proposed density-based approaches" [1][92]. Different still, Zhao, Du, Lu, etc. "put forth grid-based approaches," and "still others have introduced hybridized approaches [1][92][93]. For example, Agrawal et al. introduced a density-based and grid-based approach referred to as Clustering in QUEst (CLIQUE)," and "as a follow-on enhancement to CLIQUE," Nagesh et al. introduced "Merging Adaptive Finite Intervals And (MAFIA)" [1][93][94]. Yet "other approaches include those that are Wavelet Transform (WT)-based," and this should be of no surprise, since WT are a recognized method "to summarize high-dimensional data in a few numbers" [1][83].

The HVD mechanism is also "underpinned with soft clustering," ("this is contrasted to 'hard clustering, wherein there is classification into only one cluster'"), and "this provides the requisite versatility of more granular and variegated classification" [1]. This "characterization of soft versus hard clustering should be reminiscent of" Type-2 Fuzzy Set (T2FS), as opposed to Type-1 Fuzzy Set (T1FS), "which only accommodates membership invariableness" [1]. With regards to the soft clustering, "Three-Way Soft Clustering (TWSC) 'nicely suffices for the [HVD] purposes at hand,'" as it has the nuance of having "samples in the positive region as belonging to the cluster, samples in the boundary region as partially belonging to the cluster, and samples in the negative region as not belonging to the cluster" [1][96].

Moving from HVD to HCM, Tavakkol et al. have noted, "to the best of our knowledge, there is not any" HCM "in the literature that is designed for uncertain objects and can be used for validating the performance of uncertain clustering algorithms" [1][104]. To set the stage for the HCM exploration, Section IIIN noted that "RM can be construed to be a subset of IM, which can be construed to encompass 'Optimization-like Criteria' (OLC) and 'Difference-like Criteria' (DLC), and 'RM can refer, in particular, to DLC, wherein a baseline reference can be established and utilized to determine relative improvement(s) over a certain time frame'" [1][105][106].

Along this vein, previously, Milligan and Cooper had undertaken an examination "the IM/RM of 'McClain-Rao' (MR) as an OLC," but in a potential difference of findings, "Vendramin et al., among others, found that MR 'performed significantly better (eight times more accurately)' when transforming DLC to OLC (e.g., better results) prior to any evaluation" [1][106]. Along this vein, an exploration was initiated to determine "which HCMs have been considered for facilitating the DLC Candidate List (DCL)" [1]. The premise is that "if the classification related to DLCs can be augmented, and the involved transformations, such as that of DLC to OLC, can be accelerated," then HVD can be enhanced [1]. "The need for a robust HCM apparatus is underscored by Tavakkol, Vendramin, and others," and some of the findings are reflected in Table V [1].

TABLE V. HCM EXPERIMENTATION FACETS FOR DLC TO OLC CANDIDACY [1][5]

I	II	III	IV	V	VI	VII	VIII	IX
MR	DLC	Min;Elbow	WB	$O(nN^2)$	✓	✓	✓	
BH	DLC	Max_{diff};Elbow	W	$O(nN)$				
DI	OLC	Max	WB	$O(nN^2)$		✓	✓	
PBM	OLC	Max	WBD	$O(n(k^2+N))$				✓
TW	DLC	Max_{diff} ;Elbow	W	$O(nN)$				
PB	OLC	Max	WB	$O(nN^2)$				✓

Regarding Table V, Column I lists certain HCMs: MR, Ball-Hall (BH)," Dunn Index (DI), "Pakhira-Bandyopadhyay-Maulik (PBM), Trace(W) (TW), and Point-Biserial (PB)" [1]. Then, Column II lists "the DLC/OLC presort, as presented by Vendramin and Liu" [1][106][107]. Column III indicates the leveraged method to determine optimality, via "Min" ("the smallest index value") and "Max" ("the largest index value") [1][108]; regarding "Max," "Max_{diff} refers to the optimal K segueing to the maximum difference 'between…successive slopes'" [1][63][107][108][109]. Column III also notes various "inflection points;" these inflections are denoted by "elbows" (e.g., positive concavity) and "knees" (e.g., negative concavity), as applicable. Then, Column IV indicates "Within-cluster (W), Between-cluster (B), and full Dataset (D)," in accordance with "Powell's convention/nomenclature," and Column V provides the "computational complexity" [1][109][110][106][111]. Those columns with relatively strong performance "for the various normal distributions, increasing degree of overlap, global optimum, as well as paradigms that are generally affirmed (and/or are affirmed by other benchmarks) are checked off for the pertinent cells of Columns VI, VII, VIII, and IX, respectively, and commonalities are green highlighted" [1].

C. *Experimental Testbed Architectural Modifications*

As noted in Section IV, the VMRM had set the stage for the architectural construct to be utilized. Section IVB had also noted that HVD approaches included density-based, grid-based, and WT-based. As Continuous WT (CWTs) "are particularly amenable to time series analysis," can well handle "wavelet tight frames with *n* vanishing moments," and have "successive convolutional layers (which contain the cascading of ever smaller 'CWT-like' convolutional filters)," "CWTs are the preferred WT embodiment" for the experimentation within this paper [1][41][71][83]. It is also readily operationalized aboard the GBGRDH-1,2,3,4.

Other considerations included the fact that the very nature of HVD involves VSNO and "large intra-class variations," and "Jin noted that when a prototypical Deep Convolutional Neural Network (DCNN) is confronted with 'large intra-class variations, the performance of the traditional [D]CNN models degenerates dramatically'" [41][112]. Thus, *as one principal reason*, the prior instantiation was modified to that of a GBGRDH-1,2,3,4 was to take the DCNN performance degradation aspect (amidst large intra-class variations) into consideration. This rationale will be addressed in segments. Also, given the C2/CS impetus and the notions of HON as well as MCP/ECP, it was more pragmatic to supplant the prior DCNN approach "with the more task-specific H-IICMNN *infimal* convolution mechanisms to serve as 'efficient solvers,' as noted by Lambert" [41][116]. Thus, *as a second principal reason*, the original instantiation was modified to that of a GBGRDH-1,2,3,4, wherein the H-IICMNN-1 would now fulfill the role "as the key solver for the involved RCR optimization problems," H-IICMNN-2 would now fulfill the role "as the key solver for the non-convex problems inadvertently spawned by the RCR," H-IICMNN-3 would now fulfill the role "as the key solver for certain modified involved functions," H-IICMNN-4 would now fulfill the role "as a numerical stability stabilizer for the construct," "and a DCGAN" would now fulfill the role "as a mitigator against mode failure" [41]. This paradigm is shown in Fig. 5 below. Hence, the RCR-DCGAN-[H-IICMNN]-1,2,3,4 aspect has been addressed.

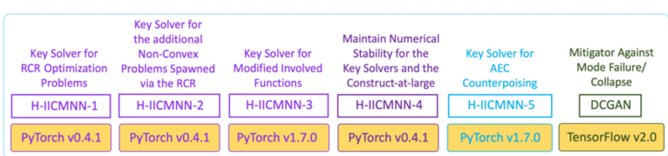


Figure 5. GBGRDH-1,2,3,4 Construct with H-IICMNN functional roles [1][14]

With regards to Fig. 5, H-IICMNN-1 was tasked with ensuring high Quality of Service (QoS) for the involved RCR optimization problems, which require consistent numerical stability. For this reason, PyTorch v0.4.1 was chosen. H-IICMNN-2 was tasked with handling additional non-convex problems that were inadvertently spawned via H-IICMNN-1. H-IICMNN-3 was tasked with handling various modified functions that have been previously shown to produce errant results due to signature and dependency issues. PyTorch v1.7.0 was deemed to be acceptable in this regard. However, H-IICMNN-4 was tasked with internal training for the GBGRDH-1,2,3,4 construct's overall stability, thereby mitigating against known numerical instability issues arising from the use of PyTorch v1.7.0; hence PyTorch v0.4.1 was utilized for H-IICMNN-4. As a TensorFlow v2.0 DCGAN implementation has been shown to exhibit consistent stability, it served in a complementary fashion (as an additional generator) so as to assist in mitigating against "mode failure" (a.k.a. "mode failure/collapse" or the "Helvetica Scenario"), which occurs when adversarial NNs, that are undergoing contemporaneous training, experience an aberrant convergence or simply fail to converge [2][1].

With regards to the GCN-BiLSTM-GAT, researchers have affirmed the various facets of the amalgam. For example, "Zhang affirmed the 'expressive power' of GCN" [118]. "Siami-Namini affirmed the use of the BiLSTM for its 'better predictions,' such as 'in longer prediction horizons' over 'regular LSTM-based models'" [2][119][120]. Hou had found that Graph-Attention-Networks (GATs) well serve as "neighborhood aggregators to learn the entity and relational features of the central entity neighborhoods" [2][121]. "Hamilton affirmed the use of the GAT for its computational efficiency" [122]. "Hou furthers this by noting that the BiLSTM-GAT amalgam can 'capture the interaction features between multi-relational facts and…temporal information' along with a relation-specific weighting schema (as an encoder-decoder structure)" [2][121].

Along the vein of Explainable AI (XAI), a contribution of the bespoke task-specific H-IICMNN was to enhance discernment via a more balanced operationalization since, in the case of RWS, "the Gaussian assumption usually does not hold" [41][123]. In contrast to the [moderate-tailed] Gaussian distribution, the long-tail distribution tends to be prevalent "in KGs", and "strongly unbalanced data with a long-tail is ubiquitous in numerous domains and problems" [41][124][125][126]. Moreover, "learning with unbalanced data causes models to favor head classes," and this is indeed the case for the long tail in RWS [41][125][126]. Hence, the utilized STKG Embedding (STKGE), to achieve the STKGC, needs to be well balanced across both classes (i.e., head and tail), and Table VI demonstrates a possible layered approach.

TABLE VI. PREDICTING HEAD/TAIL ENTITIES FOR KGE TECHNIQUES [2]

	Predicting Head Entity				Predicting Tail Entity			
KGE Technique	1-to-1	1-to-N	N-to-1	N-to-N	1-to-1	1-to-N	N-to-1	N-to-N
TransE								
TransH								
TransR								
TransD								
DistMult								
TransET								

For Table VI, the color coding (like that of Table IV) is as follows. Red denotes worse performance while the darker shade of green denotes better performance; the progression of colors follows the ROYGBIV sequence and indicates the gradations of performance. In it of itself, Table VI only sets the stage, for there are more Complex Relationships (CR) that "extend beyond 1-to-N, N-to-1, and N-to-N," such as 1-1-N, N-1-1, 1-M-1, N-1-N, N-M-1, 1-M-N, and N-M-N [2]. Lin points out, such as in the case of one exemplar dataset, "there are 485,661 triples of 1-1-N, 520,476 triples of N-1-1, 211,457 triples of 1-M-1, and 26,943 triples of N-1-N, and the number of head entities with 1-M-N…is 10,143" [2][127]. Therefore, the Table VI relationships of 1-to-1, 1-to-N, N-to-1, and N-to-N must be re-examined in the context of the CRs "to better discern the varied 'relations between a pair of entities'" [2][127]. In addition, "as noted by Cai, 'generating discriminative negative samples is essential since failing to do so may hardly improve the model or even cause gradient vanishing' (wherein the associated gradient may become so small and tend toward the point of 'vanishing,' which then obviates any weighting schema to effectuate updates)" [2][42][129].

Ultimately, certain promising techniques were extended, via He et al.'s framework, the "Type-augmented Knowledge [Graph] Embedding (TaKE)," which "can be combined with any traditional KGE models" "under no explicit type [of] information supervision" and can facilitate "both type constraint and type diversity with low time and space complexity" [2][41][128]. Leveraging this approach, the STKG Embedding (STKGE) "with a Type-Sensitive (TS) extension" becomes TS-STKGE (a.k.a., T2S2KGE). With T2S2KGE as the "generic form," wherein the involved "KGE is replaced with the extended model," T2S2-DistMult and T2S2-ComplEx ("wherein ComplEx is an extrapolation of DistMult") are formulated [2][41][129][130]. In a similar fashion, "T2S2-HyTE (an extension of HyTE, which is an extension of TransH) was inferior to that of T2S2-Hybrid-TE (wherein Hybrid-TE is a hybridization of TransD and HyTE)" [41][129][130]. Ultimately, T2S2-ComplEx and T2S2-Hybrid-TE were utilized for the T2S2KGE, and the performance is noted in Table VII [41][130].

TABLE VII. HEAD/TAIL PERFORMANCE FOR T2S2KGE TECHNIQUES [41]

	Predicting Head Entity				Predicting Tail Entity			
T2S2KGE Technique	1-to-1	1-to-N	N-to-1	N-to-N	1-to-1	1-to-N	N-to-1	N-to-N
T2S2-ComplEx								
T2S2-Hybrid-TE								

For Table VII, the color coding (like that of Tables IV and VI) is as follows. Orange denotes worse performance while the darker shade of green denotes better performance; the progression of colors follows the ROYGBIV sequence and indicates the gradations of performance of the listed STKGE Techniques against various types of KG relationships (e.g., 1-to-1, 1-to-N, N-to-1, and N-to-N) [41].

The application of the pertinent KGE technique and the ensuing KGC that is construed to be a part of the IUC rubric for informing "the HVD, which further informs the IHP" is part of the ongoing Validation/Dynamic Fine-Tuning (VDFT) process employed [5]. As noted in [5], central to the described workflow is the utilization of "Zadeh's Fuzzy Systems Theory" with regards to "T2FS (a.k.a., IUC-1a)" and "'Rough-Fuzzy Set' (RFS) '(a.k.a., IUC-2a)'," "which is an extension of IUC-1a and "Pawlak's Rough Set (a.k.a., IUC-1b)" [5]. Significantly, "IUC-2a can well accommodate the notion of an affiliation, 'but not necessarily absolute inclusion'" [5][130]. In furtherance of this, "Deng's Grey Systems Theory (a.k.a., IUC-2b) can enhance the precision of IUC-2a" [5]. With regards to utilization, IUC-2b can be utilized, "if the relationship/membership (e.g., entity, attribute, etc.) is discontinuous" [5].

On the contrary, if the relationship/membership is continuous, "then other Probability [& statistics] Systems Theory approaches might be utilized, such as Information Entropy Methods (a.k.a., IUC-3), whose strength resides in ascertaining 'unknown attribute weights'" [7][131]. Simply put, "whether the relationship/membership is discontinuous or continuous (e.g., pulsed, rather than continuous), it can still be construed as a Relationship/Membership Stream (RMS)" [5]; the RMS, in the context of MCDM (e.g., MADM/MODM), is shown in Fig. 2. In addition, the "Dempster-Shafer framework" can be useful for considering multiple membership functions, and "Debois and Prade" extend this to "family of membership functions" [132][133].

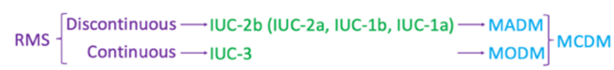


Figure 6. RMS Paradigms for the IUC [5]

Overall, the utilization of Zadeh's T2FS, Gundogdu's "rendition of SFS" ("which is quite useful for multi-dimensions"), and Yao's approach collectively segue to TWSC, which is one of the constituent elements of T2FS-SFS-TWSC (TST) amalgam [5]. This is contextualized within Fig. 7.

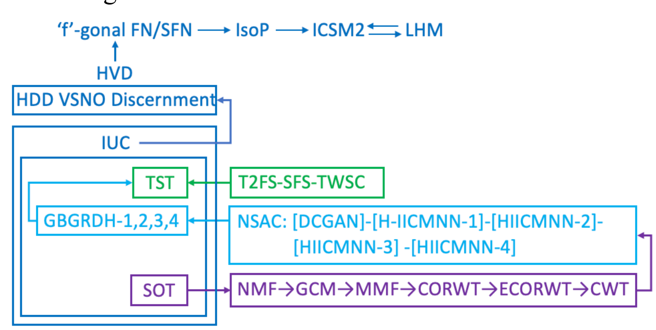


Figure 7. IUC with Constituent Elements [5][14]

The NSAC is a mainstay of the GBGRDH-1,2,3,4 construct, and the utilized SOT progresses from "a Nonnegative Matrix Factorization (NMF) to a Gaussian Composite Model (GCM), which then proceeds to a Multiresolution Matrix Factorization (MMF) that is characterized by its intrinsic ability to ascertain the multiscale structure and appropriately characterize the wavelets for a multiresolution representation" [5][134][135]. This then progresses "to yield MMF's Corresponding WT (CORWT) and the ensuing Enhanced CORWT (ECORWT)," and a "'translation-invariant CWT PyWavelet schema is utilized to implement/transform the ECORWT to the desired CWT' ('which is then used for the wavelet space-based mapping in preparation for HVD')" [5]. This segues to "a more robust IUC," IHP, ACWS, and "MMSO construct" [5]. Fig. 7 depicts the described IUC.

Finally, a review of Section IV is best summarized as follows — the posited construct consists of an interesting amalgam of (1) a LHM, (2) apropos HCM that are data uncertainty-centric, (3) an ICSM2 to gauge similitude, via the discernment of VSNO clusters in HDD (a.k.a., HVD) (wherein HVD is supported by TST), that is informed by the LHM (and vice versa), and (4) a MCDM2, underpinned by an (5) ACWS (operationalized by the MMSO), to leverage entropy weights. The LHM-ICSM2 and MCDM2 should be of no surprise, as they are denoted within the LHM-ICSM2-MCDM2-IFM-AECM-MAM amalgam. The significance of the ACWS and MMSO have been underscored in Sections IC and IVA. Likewise, the import of the HCM and DLC have been previously illuminated in Section IVB. The relationship between HVD and HCM was reviewed in Section IVB, and the role of TST within the IUC was delineated in this Section IVC.

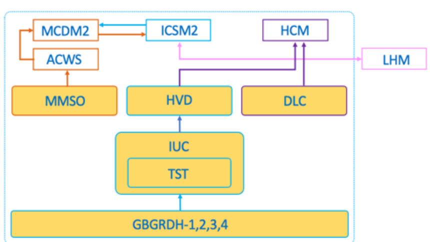


Figure 8. MDCM2-ICSM2-HCM Amalgam with Buttressing Elements [1]

A scrutinization of the GBGRDH-1,2,3,4 architectural construct was conducted to ascertain whether various assertions made in the literature were valid. For example, Medina posited "that the use of Convolutional Neural Networks (CNNs) reduces the False Positive Rate (FPR)" [83][137]. As another example, Moradi asserted that "the use of LSTMs addresses the gradient vanishing issue" [83][138]. The GBGRDH-1,2,3,4 "incorporates these lessons," among others [83]. As in [83], "prototypical ML libraries (e.g., Keras, Scikit-learn, etc.) were utilized," and "experimental variations included PT, Tensorflow (TF), Caffe, Caffe2, and SciPy" [83]. Consistent with various works-in-progress and prior works, "PT and TF" were the preferred implementations. The GBGRDH-1,2,3,4 construct "was evaluated against" other known conventional methods, and a sampling of the "classification results are shown" in Table VIII [83].

TABLE VIII. CLASSIFICATION RESULTS OF VARIOUS ML METHODS [83]

Methods	Models	ACC
"Prototypical ML methods"	"Support Vector Machine (SVM)" [139]	"83.8%" [83]
	"Hidden Markov Models (HMM)" [140]	"87.3%" [83]
	"Random Forest (RF)" [141]	"91.43%" [83]
	"k-Nearest Neighbor (KNN)" [142]	"97.17%" [83]
"Prototypical DLNN methods"	"CNN, CNN Bidirectional (Bi)LSTM hybrid" [143][144]	"93.3-96.2%" [83]
	"RNN, RNN BiLSTM hybrid" [145][146]	"95.5-97.8%" [83]
Posited bespoke GBGRDH-1,2,3,4 construct	GCN-BiLSTM-GAT & RCR-DCGAN-[H-IICMNN]-1,2,3,4	98.4%

Although "N-fold cross-validation," as a classification error measure, "was applied to the" seven "classifiers depicted" in Table VIII, since the utilized schema is rooted in the use of an ACWS, "the subtle intent of cross-validation becomes somewhat moot" [83]. By way of explanation, "if the involved data samples were utilized to train the involved CNN, the ensuing weights and bias values would tend to overfit and segue to 'sub-optimal performance against previously unseen data'" [83]. The standard approach to offset this overfitting "is to separate the data into training data (e.g., 80%) and test data (e.g., 20%)" and settle upon a suitable counterpoising, but the use of ACWS negates this [83]. For the experimentation herein, the more conversative approach of "utilizing an artificially suppressed number of training iterations (as a higher number [of] yields seemingly enhanced performance)," to better emulate an RWS, was adopted [83].

To assess the GBGRDH-1,2,3,4 construct from the vantage point of the "efficacy of [the] RCR, from a layer-wise and overall perspective," Fig. 9 serves as a good reference point.

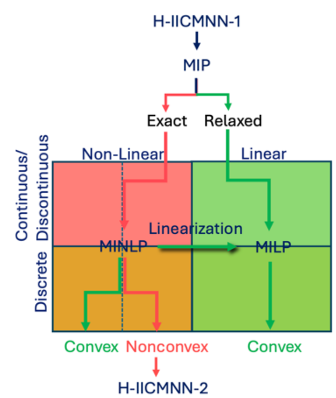


Figure 9. MIP to MINLP and MILP Pathways

When H-IICMNN-1 undertakes its task, the associated Mixed Integer Programming (MIP) can proceed via an exact or relaxed pathway with corresponding verifiers — exact (i.e., complete) or relaxed (i.e., incomplete). Exact verifiers are typically predicated upon Mixed Integer Non-Linear Programming (MINLP), "Branch-and-Bound, as well as Satisfiability Modulo Theories" while relaxed verifiers are "typically predicated upon Mixed Integer Linear Programming" (MILP) "or Mixed-Integer Convex Programming" [83]. In the optimal case, the RCR segues to the depicted green convex pathways noted in Fig. 9. If the RCR inadvertently spawns a Nondeterministic Polynomial (NP)-hard Nonconvex problem, then H-IICMNN-2 is assigned to handle that paradigm. Ultimately, "there are two core aspects of RCR: (1) the actual RCR implemented at each layer," and (2) "the verifier operationalized to ascertain robustness both layer-wise and overall [83][147]. These aspects are "central to the" GBGRDH-1,2,3,4 construct, "which has the counterpoised goals of the tightest possible relaxation" [83][147]. For the experimentation herein, the GBGRDH-1,2,3,4 construct "was able to achieve comparable [Accuracy] ACC to other well-known methods," such as those presented in Table VIII [83]. As in [83], "despite the fact that the posited bespoke method did not achieve the 98.9% rate (with a false positive rate of 4.5%) reported by Alam et al.," the GBGRDH-1,2,3,4 construct exhibits sufficient promise to warrant further examination [83].

V. Conclusion & Future work

The main output of this synthesis paper is that of a posited AI-IS-DE2 construct (i.e., an LHM-ICSM2-MCDM2-IFM-AECM-MAM amalgam) to illuminate desired DEPs; in essence, it introduced an innovative approach that contributed towards the analysis of high-dimensional data and knowledge graph completion. An overarching goal was to contribute to the challenge of discerning VSNO to better and more efficiently (e.g., AEC considerations) perform certain functions, such as AAD/AOSA/CSA as well as IF, for the purposes of DE2. The related goal was to contribute to the challenge of better effectuating STKGC to enhance the discerning of HON. This discernment process included leveraging CENs and CMs (which both help contextualize HON topologies, for which CMs, SCs, hypergraphs, and HPTs lend to transition insights and that of "collective phenomena") for the STKGC task. This discernment of HON not only informs the AAD/AOSA/CSA as well as IF tasks, but also better contextualizes the CS/C2 at play and vice versa. To facilitate the aforementioned, an ACWS to inform STKGC (to affirm the involved CS, HON, and C2) as well as a H-IICMNN approach was used; this approach also assisted in optimizing the model averaging/ensemble used to minimize the AEC. Also, at the core of the IF is the LHM, which is informed by the ICSM2 (and vice versa). In turn, the ICSM2 both informs and is informed by the MCDM2.

The role of the SVs is recapped in Fig. 10, and Section I's Table I may be referenced for the reader's convenience. It illuminates how the SV facilitates the FODM (i.e., MCDM problem) to SOP progression. The SOP can then be resolved to yield the NS, and this then segues to the OSNS. The ICSM2 informs both the FODM (the head of the FODM-SOP-NS-OSNS progression) as well as the OSNS (the tail of the FODM-SOP-NS-OSNS progression). The ICSM2 is underpinned by the IsoP, which is, in turn, reliant upon the IsoP. The IsoP efficacy is dictated by the OCGLfSFN and the P-GLfSFN. The P-GLfSFN to OCGLfSFN progression is facilitated by the involved SimMs/DMs/BD and the ACWS utilized. The ACWS is operationalized by the MMSO and the MA at play.

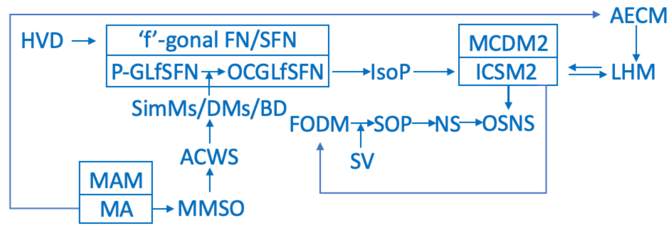


Figure 10. MDCM2-ICSM2-HCM Amalgam with Buttressing Elements [1]

As alluded to in Section IVB Table 5, HVD and HCM are intricately related. Accordingly, only those HCM with reasonable efficacy and low computational complexity were selected. In this way, practical implementation can be effectuated for RWS in a scalable way. This is applied throughout; hence, the integration of multiple bespoke modules does not introduce any unanticipated significant computational complexity (apart from the inadvertent spawning of further NP-hard nonconvex problems from the RCR). Furthermore, not only do the resource demands not necessarily increase, but it can, potentially, contribute towards lessening the resource requirements (e.g., energy) via an energy-aware computing interference-optimized metaheuristic approach, and this was previously discussed in [149].

A. *Principal Contributions*

The generalizability of the proposed approach is high, as it can be applied to the Observe, Orient, Decide, Act (OODA) cycle for various C2 systems. The LHM determines whether further observation is necessary and undertakes the orientation. The MCDM2 undertakes the decision and proceeds accordingly (i.e., facilitates the action). The other modules serve in a support role. For example, the ICSM2 performs the IsoP comparison and informs the LHM. Likewise, the AECM assesses the energy needed/available and informs the LHM. The MAM underpins the IsoP comparison. For the prevalent case of nested MCDMs, the involved MDCM2s inform the IFM, which then feeds into the next MCDM construct. Gomes and other researchers, such as Elmhadhbi, Aqqad, and Zhang note that C2 agility is critical and is "used in a variety of situations, such as disaster response, wildfire management, and power outage mitigation, to mention a few" [4][148][150][151]. They also note that "since decision speed is a crucial parameter, human involvement should be reduced to the Decision and Action phases of the OODA cycle. This segues to the analyses of the involved Socio-Technical System [STS] rubric (which encompasses "humans-in-the-loop"); hence, this affords the opportunity to scrutinize both human and machine biases.

B. Future Work

As Gomes and others note, future "automated systems will likely use concepts, such as" AI "to process incoming data…and present best option[s]" [148]. These future systems are also envisioned to "select optimal networks" (e.g., Opportunistic Networks) and adapt according to the "operational and network status." For critical matters, such as disaster response and wildfire management, shortening decision cycles is vital, and "for this reason, human-machine interaction is a promising topic for future research" with regards to C2 agility [148]. Accordingly, more experimentation regarding the LHM, MCDM2, and related modules seems warranted to advance the areas of information and decision enhancement. Also, more time needs to be spent resolve various signature and dependency issues for certain packages derived from Github and other repositories. To date, this has included implementations in PyTorch, Tensorflow, Caffe, Caffe2, and SciPy.

ACKNOWLEDGMENT

This paper is part of a series of papers under a Quality Control Program (QCP) implemented by the Quality Assurance/Quality Control (QA/QC) unit — attached to the Underwatch initiative of VTIRL, VT — for I-PAC.

REFERENCES

[1] S. Chan, "The Triumvirate of an Adaptive Criteria Weighting Methodology, Isomorphic Comparator Similarity Measure, and Apropos High Dimensional Data Cluster Validation Index Measures for the Ascertainment of Bespoke Dynamic Fuzzy Lists," in *The Sixteenth Int. Conf. on Future Comput. Technol. and Appl. (Future Computing)*, Venice, Italy, 2024, pp. 8-17.

[2] S. Chan, "Cascading Succession of Models for an Enhanced Long-Tail Discernment AI System," in *IEEE World AI IoT Congress*, Seattle, WA, 2024, pp. 393-401.

[3] J. Ding et al., "Artificial Intelligence for Complex Network: Potential, Methodology and Application," *Arxiv.org* [Online]. Feb 2024. Available: https://doi.org/10.48550/arXiv.2402.16887.

[4] S. Chan, "Prototype resilient command and control (C2) of C2 architecture for power outage mitigation," in *IEEE 10th Annu. Inf. Technol. Electron. Mobile Commun. Conf.*, Vancouver, Canada, 2019, pp. 0779-0785.

[5] S. Chan, "AI-Facilitated Dynamic Threshold-Tuning for a Maritime Domain Awareness Module," in *The 2024 IEEE Int. Conf. on Industry 4.0, Artif. Intell., and Comm. Technol*., Bali, Indonesia, 2024, In Press.

[6] S. Chan, "Resilient Decision Systems and Methods," U.S. Patent 11,862,977, Jan 2 2024.

[7] S. Chan, "AI-Facilitated Selection of the Optimal Nondominated Solution for a Serious Gaming Information Fusion Module," in *IEEE Games Media Entertainment (GEM)*, Turin, Italy, 2024, pp. 1-6.

[8] S. Nayak, "Multiobjective Optimization," Fundamentals of Optimizaiton Techniques with Algorithms, pp. 253-270, 2020.

[9] L. Velu and B. Ramalingam, "Total Ordering on Generalized 'n' Gonal Linear Fuzzy Numbers," *Int. J. of Comput. Intell. Syst*., vol. 16, pp. 1-19, Feb 2023.

[10] E. Natarajan, F. Augustin, M. Kaabar, C. Kenneth, and K. Yenoke, "Various defuzzification and ranking techniques for the heptagonal fuzzy number to prioritize the vulnerable countries of stroke disease," *Results in Control and Optim*., vol. 12, pp. 1-30, Sep 2023.

[11] V. Nayagam and J. Murugan, "Hexagonal fuzzy approximation of fuzzy numbers and its applications in MCDM," *Complex & Intell. Syst.*, vol. 7, pp. 1459-1487, Feb 2021.

[12] A. Kaltsounidis and I. Karali, "Dempster-Shafer Theory: How Constraint Programming Can Help," in *Inf. Process. and Manage. of Uncertainty in Knowl.-Based Syst*. (IPMU), Lisbon, Portugal, Jun 2020, pp. 354-367.

[13] X. Sang, Y. Zhou, and X. Yu, "An uncertain possibility-probability information fusion method under interval type-2 fuzzy environment and its application in stock selection," *Inf. Sci*., vol. 504, pp. 546-560, Dec 2019,

[14] S. Chan, "Inference-Optimized Metaheuristic Approach for a Prospective AI Training/Inference Inversion Paradigm in Optimized Energy-Aware Computing," in *IEEE World AI IoT Congress*, Seattle, WA, 2024, pp. 370-379.

[15] L. Leffer, "The AI Boom Could Use a Shocking Amount of Electricity," *Scientific American* [Online]. Oct 2023. Available: https://www.scientificamerican.com/article/the-ai-boom-could-use-a-shocking-amount-of-electricity/.

[16] A. Luccioni, Y. Jernite, E. Strubell, "Power Hungry Processing: Watts Driving the Cost of AI Deployment?" *Arxiv.org* [Online], Nov 2023. Available: https://arxiv.org/abs/2311.16863.

[17] R. Desislavov, F. Martinez-Plumed, J. Hernandez-Orallo, "Trends in AI inference energy consumption: Beyond the performance-vs-parameter laws of deep learning," *Sustainable Comput.: Inform. and Syst.*, vol. 38, pp. 1-17, Apr 2023.

[18] Victor Avelar et al., "The AI Disruption: Challenges and Guidance for Data Center Design," *Schneider Electric* [Online]. Dec 2023. Available: https://download.schneider-electric.com/files?p_Doc_Ref=SPD_WP110_EN.

[19] K. Freund, "Google Cloud Doubles Down on NVIDIA GPUs For Inference," *Forbes* [Online]. May 2019. Available: https://www.forbes.com/sites/moorinsights/2019/05/09/google-cloud-doubles-down-on-nvidia-gpus-for-inference/?sh=39615b346792.

[20] D. Castro, "Rethinking Concerns About AI's Energy Use," *Center for Data Innovation* [Online]. Jan 2024. Available: https://www2.datainnovation.org/2024-ai-energy-use.pdf.

[21] D. Nikelshpur and C. Tappert, "Using Particle Swarm Optimization to Pre-Train Artificial Neural Networks: Selecting Initial Training Weights for Feed-Forward Back-Propagation Neural Networks," in *Student - Faculty Research Day, Pace University*, New York City, NY, Mar 2013, pp. 1-8.

[22] Y. Wang, H. Zhang, and G. Zhang, "cPSO-CNN: An efficient PSO-based algorithm for fine-tuning hyper-parameters of convolutional neural networks," *Swarm and Evol. Comput.*, vol. 49, pp. 114-123, Sep. 2019.

[23] M. Babanezhad et al., "Investigation on performance of particle swarm optimization (PSO) algorithm based fuzzy inference system (PSOFIS) in a combination of CFD modeling for prediction of fluid flow," *Sci. Rep.*, vol. 11, pp. 1-14, Jan 2021.

[24] D. Hernandez and T. Brown, "Measuring the Algorithmic Efficiency of Neural Networks," *Arxiv.org* [Online]. May 2020. Available: https://doi.org/10.48550/arXiv.2005.04305.

[25] J. Hoffman et al., "Training Compute-Optimal Large Language Models," *Arxiv.org* [Online]. Mar 2022. Available: https://arxiv.org/abs/2203.15556.

[26] C. Yao et al., "Evaluating and analyzing the energy efficiency of CNN inference on high-performance GPU," *Concurrency and Comput. Pract. and Experience*, vol 33, Mar 2021.

[27] F. Landola, S. Han, M. Moskewicz, K. Ashraf, W. Dally, and K. Keutzer, "SqueezeNet: AlexNet-level accuracy with 50x fewer parameters and <0.5MB model size," Arxiv.org [Online]. Nov 2016. Available: https://doi.org/10.48550/arXiv.1602.07360.

[28] X. Wang, C. Na, E. Struell, S. Friedler, S. Luccioni, "Energy and Carbon Considerations of Fine-Tuning BERT, in *The 2023 Conf. on*

Empirical Methods in Natural Lang. Process., Singapore, Dec 2023, pp. 9058–9069.

[29] L. Kaack, P. Donti, E. Strubell, G. Kamiya, F. Creutzig, and D. Rolnick, "Aligning artificial intelligence with climate change mitigation," in *Nature Climate Change*, vol. 12, pp. 518–527, Jun 2022.

[30] A. Canziani, A. Paszke, and E. Culurciello, "An analysis of deep neural network models for practical applications," *Arxiv.org* [Online]. Apr 2017. Available: https://doi.org/10.48550/arXiv.1605.07678.

[31] T. Clark, "Why Does AI Consume So Much Energy?" *Forbes* [Online]. Dec 2023. Available: https://www.forbes.com/sites/sap/2023/12/20/why-does-ai-consume-so-much-energy/?sh=5886aa832ede

[32] E. Natarajan, F. Augustin, M. Kaabar, C. Kenneth, and K. Yenoke, "Various defuzzification and ranking techniques for the heptagonal fuzzy number to prioritize the vulnerable countries of stroke disease," *Results in Control and Optim*., vol. 12, , pp. 1-31, Sep 2023.

[33] V. Nayagam and J. Murugan, "Hexagonal fuzzy approximation of fuzzy numbers and its applications in MCDM," *Complex & Intell. Syst.*, vol. 7, pp. 1459-1487, Feb 2021.

[34] L. Velu and B. Ramalingam, "Total Ordering on Generalized 'n' Gonal Linear Fuzzy Numbers," *Int. J. of Comput. Intell. Syst*., vol. 16, pp. 1-19, Feb 2023.

[35] M. Gogoi and R. Chutia, "Fuzzy risk analysis based on a similarity measure of fuzzy numbers and its application in crop selection," in *Eng. Appl. of Artif. Intell*., vol. 107, pp. xxx, Jan 2022.

[36] P. Ejegwa, "Pythagorean fuzzy set and its application in career placements based on academic performance using max-min-max composition," *Complex & Intell. Syst*., vol. 5., pp. 165-175, Feb 2019.

[37] S. Das, B. Roy, M. Kar, S. Kar, and D. Pamucar, "Neutrosophic fuzzy set and its application in decision making," *J. of Ambient Intell. and Humanized Comput*., vol. 11, pp. 5017-5029, Mar. 2020.

[38] E. Ozceylan, B. Ozkan, M. Kabak, and M. Dagdeviren, "A state-of-the-art survey on spherical fuzzy sets," *J. of Intell. & Fuzzy Syst.: Appl. in Eng. and Technol*., vol. 42, pp. 195-212, Dec 2021.

[39] Y. Donyatalab, F. Gundogdu, F. Farid, S. Seyfi-Shishavan, E. Farrokhizadeh, and C. Kahraman, "Novel spherical fuzzy distance and similarity measures and their applications to medical diagnosis," *Expert Syst. with Appl*., vol. 191, pp. 1-15, Apr 2022.

[40] M. Wu, T. Chen, and J. Fan, "Similarity Measures of T-Spherical Fuzzy Sets Based on the Cosine Function and Their Applications in Pattern Recognition," *IEEE Access*, pp. 98181-98192, May 2020.

[41] S. Chan, "The Prospective Threat Vector of a Bounded and Controllable Optimized Computational Approach for Spatio-Temporal Knowledge Graph Completion," in *The Int. Conf. on Big Data Analytics and Practices*, Bangkok, Thailand, 2024, In Press.

[42] B. Cai, Y. Xiang, L. Gao, H. Zhang, Y. Li, and J. Li, "Temporal Knowledge Graph Completion: A Survey," in *Proc. of the Thirty-Second Int. Joint Conf. on Artif. Intell. (IJCAI-23)*, Macao, China, Aug 2023, pp. 6545-6553.

[43] M. Nayyeri, "Dihedron Algebraic Embeddings for Spatio-temporal Knowledge Graph Completion," in *European Semantic Web Conf. (ESWC 2022)*, Hersonissos, Greece, May 2022, pp. 253-269.

[44] H. Ye, N. Zhang, H. Chen, and H. Chen, "Generative Knowledge Graph Construction: A Review," in *Proc. of the 2022 Conf. on Empirical Methods in Natural Lang. Process*., Abu Dhabi, United Arab Emirates, pp. 1-17, Dec 2022.

[45] A. Zeb, S. Saif, J. Chen, A. Haq, Z. Gon, and D. Zhang, "Complex graph convolutional network for link prediction in knowledge graphs," *Expert Syst. with Appl.*, vol 200, Aug 2022.

[46] F. Zhang, Z. Zhang, X. Ao, F. Zhuang, Y. Xu, and Q. He, "Along the Time: Timeline-traced Embedding for Temporal Knowledge Graph Completion," in *Proc. of the 31st ACM Int. Conf. on Inf. & Knowledge Manage. (CIKM'22)*, pp. 2529-2538, Oct 2022.

[47] P. He, G. Zhou, Y. Yao, Z. Wang, and H. Yang, "A type-augmented knowledge graph embedding framework for knowledge graphic completion," *Sci. Rep*., vol. 13, pp. 1-12, Jul 2023.

[48] J. Lu, G. wen, R. Lu, Y. Wwang, and S. Zhang, "Networked Knowledge and Complex Networks: An Engineering View," *IEEE/CAA J. of Automatica Sinica*, vol. 9, pp. 1366-1383, Aug 2022.

[49] J. Grilli, G. Barabás, M. J. Michalska-Smith, and S. Allesina, "Higher- order interactions stabilize dynamics in competitive network models," *Nature*, vol. 548, pp. 210–213, Jul 2017.

[50] S. Terasaki and K. Sato, "Minimal Controllability Problems on Linear Structural Descriptor Systems," *IEEE Trans. on Autom. Control*, vol. 67, pp. 2522-2528, May 2022.

[51] C. Commault and J. Dion, "The single-input Minimal Controllability Problem for structured systems," *Syst. & Control Lett*., vol. 80, pp. 50-55, Jun 2015.

[52] C. Lin, "Structural controllability," *IEEE Trans. Automat. Control*, vol. 19, pp. 201–208, Jun 1974.

[53] S. Alizadeh, M. Posfai, and A. Ghasemi, "Input node placement restricting the longest control chain in controllability of complex networks," *Sci Rep*., vol. 13, pp. 1-14, Mar 2023.

[54] C. Gokler, S. Lloyd, P. Shor, and K. Thompson, "Efficiently Controllable Graphs," *Phys. Rev. Letter*, vol. 118, pp. 1-5, Jun 2017.

[55] G. Lindmark and C. Altafini, "Minimum energy control for complex networks," *Sci. Rep*., vol. 8., pp. 1-14, Feb 2018.

[56] H. Chen and E. Yong, "Optimizing Target Nodes Selection for the Control Energy of Directed Complex Networks." *Sci. Rep*., vol. 10, pp. 1-14, Oct 2020.

[57] L. Gao, G. Zhao, G. Li, L. Deng, and F. Zeng, "Towards the minimum-cost control of target nodes in directed networks with linear dynamics," *J. of the Franklin Inst.*, vol 355, pp. 8141-8157, Nov 2018.

[58] F. Battiston, "The physics of higher-order interactions in complex systems," *Nature Phys*., vol. 17, pp. 1093-1098, Oct 2021.

[59] A. Vazquez, "Complex hypergraphs," *Phys. Rev. E*, vol. 107, pp. 1-8, Feb 2023.

[60] C. Joslyn and K. Nowak, "Ubergraphs: A definition of a recursive graph structure," Arxiv.org [Online], Apr 2017. Available: https://arxiv.org/abs/1704.05547.

[61] H. Sun and G. Bianconi, "Higher-order percolation processes on multiplex hypergraphs," *Phys. Rev. E.,* vol. 104, pp. 1-17, Sep 2021.

[62] F. Battiston et al., "Networks beyond pairwise interactions: Structure and dynamics," *Phys. Rep*., vol. 874, pp. 1-92, Aug 2020.

[63] J. Ding et al., "Artificial Intelligence for Complex Network: Potential, Methodology and Application," *arXiv.org* [online], Feb 2024. Available: https://arxiv.org/abs/2402.16887.

[64] C. Voisin, "Complex Manifolds," in *Hodge Theory and Complex Algebraic Geometry I*, C. Voisin, Cambridge: Cambridge University Press, 2010, pp. 38-62.

[65] "Reading the physics hiding in data," EurekAlert! [Online], Mar 2021. Available: https://www.eurekalert.org/news-releases/690882.

[66] I. Klickstein and F. Sorrentino, "The controllability Gramian of lattice graphs," *Automatica*, vol. 114, Apr 2020.

[67] L. Wang, Y. Chen, W. Wang, and Y. Lai, "Physical Controllability of Complex Networks," *Sci. Rep*., vol. 7, pp. 1-14, Jan 2017.

[68] L. Zhou, C. Wang, and L. Zhou, "Cluster Synchronization on Multiple Sub-networks of Complex Networks with Nonidentical Nodes via Pinning Control," *Nonlinear Dynamics*, vol. 83, pp. 1079–1100, Sep. 2015.

[69] X. Zhang, J. Sun, and G. Yan, "Why temporal networks are more controllable: Link weight variation offers superiority," *Phys. Rev. Res*., vol. 3, pp. 1-5, Aug 2021.

[70] F. Ludice, F. Garofalo, and F. Sorrentino, "Structural Permeability of Complex Networks to Control Signals," *Nature Comm*., vol. 6, pp. 1-6, Sep 2015.

[71] A. Viscardi, "Semi-regular Dubuc-Deslauriers wavelet tight frames," *J. of Comput. and Appl. Math*., vol. 349, pp. 548-562, Mar 2019.

[72] S. Abebe, T. Qin, X. Zhang, and D. Yan, "Wavelet transform-based trend analysis of streamflow and precipitation in Upper Blue Nile River basin," *J. of Hydrology: Regional Stud*., vol. 44, pp.1-18, Dec 2022.

[73] K. Grochenig and A. Rong, "Tight Compactly Supported Wavelet Frames of Arbitrarily High Smoothness," *Proc. of the American Math. Soc*., vol. 126, pp. 1101-1107, Apr 1998.

[74] Y. Zhang, M. Lucas, and F. Battiston, "Higher-order interactions shape collective dynamics differently in hypergraphs and simplicial complexes," *Nature Comm*., vol. 14, pp. 1-8, Mar 2023.

[75] Y. Lee, J. Lee, S. Oh, D. Lee, and B. Kahng, "Homological percolation transitions in growing simplicial complexes," *Chaos*, vol. 31, pp. 1-21, Apr 2021.

[76] J. Jin, Z. Ke, W. Wang, "Phase Transitions for High Dimensional Clustering and Related Problems," *Ann Statist*., vol. 45, pp. 2151-2189, Oct 2017.

[77] M. Halkidi, Y. Batistakis, M. Vazigiannis, "Clustering validity checking methods: part II," *ACM SIGMOD Record*, vol. 31, pp. 19-27, Sep 2002.

[78] A. Shirkhorshidi, S. Aghabozorgi, and T. Wah, "A Comparison Study on Similarity and Dissimilarity Measures in Clustering Continuous Data," *PLOS One*, vol. 10, pp. 1-20, Dec 2015.

[79] I. Sharaf, "A new approach for spherical fuzzy TOPSIS and spherical fuzzy VIKOR applied to the evaluation of hydrogen storage systems," *Fuzzy Syst. and their Math*., vol. 27, pp. 4403-4423, Feb 2023.

[80] J. Ortega, S. Moslem, and D. Esztergar-Kiss, "An Integrated Approach of the AHP and Spherical Fuzzy Sets for Analyzing a Park-and-Ride Facility Location Problem Example by Heterogeneous Experts," *IEEE Access,* vol. 11, pp. 55316-55325, Jun 2023.

[81] M. Akram, A. Al-Kenani, and M. Shabir, "Enhancing ELECTRE I Method with Complex Spherical Fuzzy Information," *Int. J. of Comput. Intell. Syst*., vol. 14, pp. 1-31, Dec 2021.

[82] F. Gundogdu, "A spherical fuzzy extension of MULTIMOORA method," *J. of Intell. & Fuzzy Syst.,* vol. 38, pp. 963-978, Jan 2020.

[83] S. Chan, "AI-Facilitated Ambient Factor-Based Annealment and Resiliency Sufficiency in Austere Locales," in *IEEE Int. Conf. on Smart-Green Technol. in Elec. and Inf. Syst.*," Bali, Indonesia, 2023, pp. 1-6.

[84] M. Hanine et al., "An Intuitionistic Fuzzy Approach for Smart City Development Evaluation for Developing Countries: Moroccan Context," *Math.*, vol. 9, pp. 1-22, Oct 2021.

[85] D. Pamucar, M. Zizovic, and D. Duricic, "Modification of the Critic Method Using Fuzzy Rough Numbers," *Decision Making: Appl. in Manage. and Eng.*, vol. 4, pp. 362-371, Oct 2022.

[86] S. Komsiyah, R. Wongso, and S. Pratiwi, "Applications of the Fuzzy ELECTRE Method for Decision Support Systems of Cement Vendor Selection," *Procedia Comput. Sci*., vol. 157, pp. 379-488, Oct 2019.

[87] J. Laborde, P. Stewart, Z. Chen, Y. Chen, and N. Brownstein, "Sparse clusterability: testing for cluster structure in high dimensions," *BMC Bioinf.*, vol. 24, pp. 1-27, Mar 2023.

[88] V. Vitelli, "A novel framework for joint sparse clustering and alignment of functional data," *Int. Symp. on Nonparametric Statist*., vol. 36, pp. 182-211, May 2023.

[89] S. Deng, "Clustering with Fuzzy C-Means and Common Challenges," *J. Phys.: Conf. Ser*., vol. 1453, pp. 1-6, 2020.

[90] S. Im, M. Qaem, B. Moseley, X. Sun, and R. Zhou, "Fast Noise Removal for k-Means Clustering," in *Proc. of the Twenty Third Int. Conf. on Artif. Intell. and Statist*., vol. 108, Apr. 2020, pp. 456-466.

[91] J. Wang, X. Liu, and H. Shen, "High-dimensional data analysis with subspace comparison using matrix visualization," *Sage J.*, vol. 18, pp. 94-109, Oct 2017.

[92] J. Zhao, J. Tang, T. Fan, C. Li, and L. Xu, "Density peaks clustering based on circular partition and grid similarity," *Concurrency and Comput.: Pract. and Experience*, vol. 32, Nov 2019.

[93] M. Du and F. Wu, "Grid-Based Clustering Using Boundary Detection," *Entropy*, vol. 24, pp. 1-19, Nov 2022.

[94] M. Prasad and T. Srikanth, "A Survey on Clustering Algorithms and their Constraints," *Int. J. of Intelligent Syst. and App. in Eng*., vol. 11, pp. 165-179, May 2023.

[95] S. Buschow, J. Pidstrigach, and P. Friederichs, "Assessment of wavelet-based spatial verification by means of a stochastic precipitation model (wv_verif v0.1.0)," *Geosci. Model Develop*., vol. 12, pp. 3401-3418, Aug. 2019.

[96] M. Du, J. Zhao, J. Sun, and Y. Dong, "M3W: Multistep Three-Way Clustering," *IEEE Trans. on Neural Netw. and Learning Syst*., pp. 1-14, Sep 2022.

[97] P. Wang, "A Three-Way Clustering Method Based on Ensemble Strategy and Three-Way Decision," *Inf.*, vol. 10, pp. 1-13, Feb. 2019.

[98] H. Yu, "A three-way cluster ensemble approach for large-scale data," *Int. J. of Approxi. Reasoning*, vol. 115, pp. 32-49, Dec. 2019.

[99] B. Ali, N. Azam, J. Yao, "A three-way clustering approach using image enhancement operations," *Int. J. of Approx. Reasoning*, vol. 149, pp. 1-38, Oct 2022.

[100] K. Yang, Y. Shi, Z. Yu, Z. Zong, J. Bi, and M. Wang, "Hybrid Clustering Solutions Fusion based on Gated Three-way Decision," *Int. Joint Conf. on Neural Networks*, pp. 1-10, Aug. 2023.

[101] A. Jafari and M. Morradi, "Using Supervised Fuzzy Clustering and CWT for Ventricular Late Potentials (VLP) Detection in High-Resolution ECG Signal," in *11th Mediterranean Conf. on Medical and Biomedical Eng. and Comput*., Ljubljana, Slovenia, Jun. 2007, pp. 99-104.

[102] T. Kumar and C. Swapna, "Segmentation of image with DT-CWT and NLM filtering using fuzzy c-means clustering," in *2014 IEEE Int. Conf. on Comput. Intell. and Comput. Research*, Las Vegas, NV, Dec. 2014, pp. 1-4, doi: 10.1109/ICCIC.2014.7238367.

[103] C. Ko, J. Baek, B. Tavakkol, and Y. Jeong, "Cluster Validity Index for Uncertain Data Based on a Probabilistic Distance Measure in Feature Space," *Sensors*, vol. 23, pp. 1-14, Apr 2023.

[104] Tavakkol, M. Jeong, S. Albin, "Validity indices for clusters of uncertain data objects," *Ann. of Oper. Res*., vol. 303, pp. 321-357, Aug. 2021.

[105] D. Moulavi, P. Jaskowiak, R. Campello, A. Zimek, and J. Sander, "Density-Based Clustering Validation," in *Proc. of the 14th SIAM Int. Conf. on Data Mining*, Philadelphia, PA, Apr. 2014, pp. 839-847.

[106] L. Vendramin, R. Campello, and E. Hruschka, "Relative Clustering Validity Criteria: A Comparative Review," *Statist. Anal. and Data Mining*, vol. 3, pp. 209-235, Jul. 2010.

[107] Y. Liu, "Understanding of Internal Clustering Validation Measures," IEEE Int. Conf. on Data Mining, Sydney, Australia, Dec. 2010, pp. 911-916.

[108] B. Desgraupes, "Clustering Indices," Nov. 2017, [Online]. Available at: https://cran.r-project.org/web/packages/clusterCrit/vignettes/clusterCrit.pdf.

[109] B. Powell, "How I learned to stop worrying and love the curse of dimensionality: an appraisal of cluster validation in high-dimensional spaces," Jan. 2022, [Online]. Available at: https://doi.org/10.48550/arXiv.2201.05214.

[110] M. Du and F. Wu, "Grid-Based Clustering Using Boundary Detection," *Entropy*, vol. 24, pp. 1-19, Nov 2022.

[111] E. Rendon, I. Abundez, A. Arizmendi, and E. Quiroz, "Internal versus External cluster validation indexes," *Int. J. of Comput., Commun., and Control*, vol. 5, pp. 27-34, Jan 2011.

[112] T. Jin, L. Cao, B. Zhang, X. Sun, C. Deng, and R. Ji, "Hypergraph Induced Convolutional Manifold Networks," in *Proc. of the Twenty-Eighth Int. Joint Conf. on Artificial Intell*., Macao, China, Aug. 2019, pp. 2670-2676.

[113] S. Gao, I. Tsang, and L. Chia, "Laplacian Sparse Coding, Hypergraph Laplacian Sparse Coding, and Applications," *IEEE Trans. Pattern Anal. Mach. Intell*., vol. 35, pp. 92–104, Jan 2013.

[114] S. Huang, M. Elhoseiny, A. Elgammal, D. Yang, "Learning Hypergraph-regularized Attribute Predictors," *2015 IEEE Conf. on Comp. Vision and Pattern Recognit. (CVPR)*, Boston, MA, pp. 409-417, Oct 2015.

[115] H. Bauschke and P. Combettes, "Infimal Convolution," *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*, Springer Cham, Apr 2011, pp. 167-180.

[116] C. Aguilar, "Strongly uncontrollable network topologies," *IEEE Trans. on Control of Netw. Syst*., vol. 7, pp. 878-886, Jun 2020.

[117] A. Lambert, D. Bouche, Z. Szabo, and F. D'Alche-Buc, "Functional Output Regression with Infimal Convolution: Exploring the Huber and ε -insensitive Losses," in *Proc. of the 39th Int. Conf. on Mach. Learn. Res*., Baltimore, MD, Jul. 2022, pp. 1-25.

[118] S. Zhang, H. Tong, J. Xu, and R. Maciejewski, "Graph convolutional networks: a comprehensive review," *Comput. Soc. Netw.,* vol. 6, pp. 1-23, Nov 2019.

[119] S. Siami-Namini, N. Tavakoli, and A. Namin, "The Performance of LSTM and BiLSTM in Forecasting Time Series," *2019 IEEE Int. Conf. on Big Data (Big Data)*, Los Angeles, CA, Dec 2019, pp. 3285-3292.

[120] D. Silva and A. Meneses, "Comparing Long Short-Term Memory (LSTM) and bidrectional LSTM deep neural networks for power consumption prediction," *Energy Rep*., vol. 10, pp. 3315-3334, Nov 2023.

[121] X. Hou, R. Ma, L. Yan, and Z. Ma, "T-GAE: A Timespan-aware Graph Attention-based Embedding Model for Temporal Knowledge Graph Completion," *Inf. Sci*., vol. 642, pp. 1-6, Sep 2023.

[122] W. Hamilton, R. Ying, and J. Leskovec, "Inductive representation learning on large graphs," in *The Thirty-first Ann. Conf. on Neural Inf. Process. Syst. (NIPS)*, Long Beach, CA, Dec 2017, pp. 1-11.

[123] T. Jin, L. Cao, B. Zhang, X. Sun, C. Deng, and R. Ji, "Hypergraph Induced Convolutional Manifold Networks," in *Proc. of the Twenty-Eighth Int. Joint Conf. on Artificial Intell*., Macao, China, Aug 2019, pp. 2670-2676.

[124] D. Samuel, Y. Atzmon, G. Chechik, "From generalized zero-shot larning to long-tail with class descriptors," in *2021 IEEE Winter Conf. on Appl. of Comp. Vision (WACV)*, Waikoloa, Hawaii, Jan 2021, pp. 286-295.

[125] M. Buda, A. Maki, and M. Mazurowski, "A systematic study of the class imbalance problem in convolutional neural networks," *Neural Netw*., vol. 106, pp. 249-259, Oct 2018.

[126] J. Sheng et al., "Adaptive Attentional Network for Few-Shot Knowledge Graph Completion," in *Proc. of the 2020 Conf. on Empirical Methods in Natural Lang. Process. (EMNLP)*, Nov 2020, pp. 1681–1691.

[127] L. Lin, J. Liu, F. Guo, C. Tong, L. Zu, and H. Guo, "ERDERP: Entity and Relation Double Embedding on Relation Hyperplanes and Relation Projection Hyperplanes," *Math.*, vol. 10, pp. 1-19, Nov 2022.

[128] P. He, G. Zhou, Y. Yao, Z. Wang, and H. Yang, "A type-augmented knowledge graph embedding framework for knowledge graphic completion," *Sci. Rep*., vol. 13, pp. 1-12, Jul 2023.

[129] Y. Zhang, Q. Yaho, Y.Shao, and L. Chen, "NSCaching: Simple and Efficient Negative Sampling for Knowledge Graph Embedding," in *2019 IEEE 35th Int. Conf. on Data Eng. (ICDE)*, Macau, China, Apr 2019, pp. 614-625.

[130] A. Khuman, Y. Yang, R. John, and S. Liu, "R-fuzzy sets and grey system theory," in *Proc. 2016 IEEE Int. Conf. Syst. Man Cybernetics*, Budapest, Hungary, Oct 2016, pp. 004555–004560.

[131] A. Khuman, "The similarities and divergences between grey and fuzzy theory," *Expert Syst. Appl.*, vol. 186, pp. 1–11, Dec. 2021.

[132] K. Sentz and S. Ferson, "Combination of Evidence in Dempster-Shafer Theory," Sandia Report, Apr. 2002, [Online]. Available at: https://www.stat.berkeley.edu/~aldous/Real_World/dempster_shafer.pdf

[133] B. Izyumov, E. Kalinina, and M. Wagenknecht, "Software tools for regression analysis of fuzzy data," 9th Zittau Fuzzy Colloquium, Zittau, Germany, Sept 2001, pp. 1-10.

[134] R. Kondor, N. Teneva, and P. Mudrakarta, "Parallel MMF: a Multiresolution Approach to Matrix Computation," *Arxiv.org,* Jul. 2015, [Online] Available at: https://doi.org/10.48550/arXiv.1507.04396

[135] T. Hy, R. Kondor, "Multiresolution Matrix Factorization and Wavelet Networks on Graphs," in *Proc. of Topological, Algebraic, and Geometric Learning Workshops*, Baltimore, MD, Jul. 2022, pp. 172-182.

[136] C. Tschida, "Knowledge Graphs and Knowledge Graph Embeddings," *Culminating Projects in Comp. Sci. and Inf. Technol.*, vol. 32, pp. 1-57, May 2020.

[137] E. Medina, M. Petraglia, J. Gomes, and A. Petraglia, "Comparison of CNN and MLP classifiers for Algae Detection in Underwater Pipelines," in *Proc. 7th Int. Conf. Image Process. Theory Tools Appl. (IPTA)*, 2017, pp. 1-6.

[138] M. Moradi A., S. A. Sadrossadat and V. Derhami, "Long short-term memory neural networks for modeling nonlinear electronic components," *IEEE Trans. Compon. Packag. Manuf. Technol.*, vol. 11, no. 5, pp. 840–847, May 2021.

[139] T. Malapane, W. Doorsamy, and B. Paul, "Heart disease prediction approach using machine learning and Multi-Criteria Decision Making methods," *Global Sci. J.*, vol. 10, pp. 1268-1275, Jul 2022.

[140] E. Ackermann and J. Cunningham, "Unsupervised Clusterless Decoding using a Switching Poisson Hidden Markov Model," *BioRxiv*, pp. 1–23, Sep 2019.

[141] B. Khammas, "Ransomware Detection Using Random Forest Technique," *ICT Express*, vol. 6, pp. 325–331, Dec 2020.

[142] A. Roy et al., "Comparative analysis of KNN and SVM in multicriteria inventory classification using TOPSIS," *Int J. Inf. Technol*, Aug 2023.

[143] A. Rayan et al, "Utilizing CNN-LSTM techniques for the enhancement of medical systems," *Alexandria Eng. J.*, vol. 72, pp. 323-338, Apr 2023.

[144] Q. Li et al., "A Hybrid CNN-Based Review Helpfulness Filtering Model for Improving E-Commerce Recommendation Service," *Appl. Sci.,* vol. 11, pp. 1-20, Sep 2021.

[145] H. Madani, N. Ouerdi, A. Boumesaoud, and A. Azizi, "Classification of Ransomware Using Different Types of Neural Networks," *Sci Rep.*, vol.12, pp. 1–11, Mar 2022.

[146] N. Singh, R. Nath, and D. Sing, "Splice-site identification for exon prediction using bidirectional LSTM-RNN Approach," *Biochemistry and Biosphysics Reports*, vol. 30, pp. 1-7, Jul 2022.

[147] H. Salman, G. Yang, H. Zhang, C. Hsieh, and P. Zhang, "A convex relaxation barrier to tight robustness verification of neural networks," *Adv. in Neural Inf. Process. Syst.*, vol. 30, Dec 2019.

[148] J. Gomes, et al. "Surveying Emerging Network Approaches for Military Command and Control Systems," ACM Computing Surveys, vol. 56, pp. 1-38, Jan 2024.

[149] S. Chan, "Inference-Optimized Metaheuristic Approach for a Prospective AI Training/Inference Inversion Paradigm in Optimized Energy-Aware Computing," 2024 IEEE World AI IoT Congress (AIIoT), Seattle, WA, USA, 2024, pp. 370-379.

[150] L. Elmhadhbi, M. Karray, B. ArchimAlde, J. Otte, and B. Smith, "A semantics-based common operational command system for multiagency disaster response," *IEEE Trans. on Eng. Manage.*, pp. 1-15, Aug 2020.

[151] W. Aqqad and X. Zhang, "Modeling command and control systems in wildfire management: Characterization of and design for resiliency," in *2021 IEEE Int. Symp. on Technol. For Homeland Security (HST),* Nov. 2021, pp. 1-5.